

Jacques Ganoulis

Risk Analysis of Water Pollution

Second, Revised and Expanded Edition



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The Author

Prof. Dr. Jacques Ganoulis
UNESCO Chair on Water
Aristotle University of Thessaloniki
Department of Civil Engineering
Division of Hydraulics and
Environmental Engineering
54124 Thessaloniki
Greece

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Preface to the Second Edition

This second edition of the book brings in new concepts, and approaches to environmental risk analysis with emphasis on water pollution, which have been developed during the last 15 years. The book deals with the quantitative analysis of environmental issues related to the water quality of natural hydrosystems like rivers, lakes, groundwaters and coastal waters. More specifically, issues concerning risk and the reliability of water quality are analysed, mainly from an engineering point of view, and a methodology is developed to evaluate environmental impacts on rivers, groundwater and coastal areas from wastewater disposal and alternative water resources management plans.

According to the new paradigm of water pollution, water quality is closely connected to aquatic ecological and biological characteristics. This is reflected in the new European Union Water Framework Directive (EU WFD 2000/60), where the ecological health of aquatic ecosystems is also an important indicator of 'good water status'. In a living environment where there are many risks and where unexpected events may occur, an attempt to apply a rigorous analysis to uncertain and complex environmental issues may appear ambitious. For example what would the effect on algal blooms and eutrophication in a coastal area be, if the pollutant loads from a river doubled? Even in an abiotic environment, issues related to the quality of water resources are complex, unstable and difficult to understand. Even more complicated is the quantitative prediction of coastal water quality from possible climate change (e.g. doubling of the atmospheric CO_2).

It is usually impossible to accurately describe water pollution problems, because available data is incomplete. Mathematical modelling faces difficulties, because of the different types of processes involved, such as hydrodynamic, physico-chemical and biological interactions. Furthermore, the multitude of parameters necessary to describe physico-chemical processes, their physical meaning and their variability in space and time, raises many challenging and intriguing questions.

The study of changes in water quality and the environmental impact of projects related to water resources require adequate methodological tools. Risk and reliability analysis provides a general framework to identify uncertainties and quantify risks. As will be detailed in this book, two main methodologies have so far been developed to analyse natural risks: (a) the stochastic approach and (b) the fuzzy set theory. Stochastic variables and probability concepts are based on frequencies and require

large amounts of data. Questions of independence between random variables and validation of stochastic relationships, such as the well-known statistical regression, are usually difficult to resolve. Fuzzy set theory and fuzzy calculus may be used as a background to what should be called 'imprecision theory'. In cases of lack of information or very little data, this book demonstrates how fuzzy numbers and variables may be used for modelling risks. The use of fuzzy regression is a good alternative when statistical regression fails.

Analysis of uncertainties and quantification of risks is not sufficient to formulate and realise environmental projects aiming to improve water quality. It is also important to consider incremental variations in the benefits and costs as functions of risk. This is the risk management issue which will be discussed in relation to the consequences of risk and the decision-making process.

This book started out as lecture notes for a graduate course on risk and reliability in water resources, held at the Division of Hydraulics and Environmental Engineering, Department of Civil Engineering, Aristotle University of Thessaloniki (AUTH). Some of the examples, case studies and research related to this topic go back two decades and are related to the author's PhD thesis, in which probabilistic modelling was applied to evaluate the risk of the intrusion of a non-wetting fluid into a porous medium.

This book is not exhaustive, nor does it cover all types of water pollution problems. For example questions of water pollution in lakes and reservoirs have been omitted, although such problems are similar to those in semi-enclosed coastal bays and lagoons.

Parts of the first edition of this book were written while the author was on sabbatical leave at the University of Melbourne, Australia, Department of Civil and Agricultural Engineering (1991) and at the "Laboratoire Énergétique des Phénomènes de Transfert" – ENSAM, Bordeaux, France (1992) and accordingly special thanks are given to Professor T. MacMahon, Melbourne, Australia and to Professor M. Combarous, Bordeaux, France for their support.

The concept of the application of fuzzy set theory to water resources problems emerged from informal discussions with Professor Lucien Duckstein, University of Arizona and Professor Istvan Bogardi, University of Nebraska, USA. I am very grateful for all the information they have provided me with on this matter. I would also like to thank Dr Hans-Joachim Kraus, VCH Publishing Division III, for giving me the opportunity to publish the first edition of this book, and Dr Frank Weinreich, manager of VCH's Water and Environmental books programme for the opportunity to publish this second edition. Thanks also to Lesley Belfit, Project Editor at VCH, for her help with the design of the book's cover.

Parts of the first edition of this book were typed by Ms Efi Meimaroglou, Department of Civil Engineering, AUTH, to whom I am very grateful. For the present second edition, my appreciation goes to Petros Anagnostopoulos at the Department of Civil Engineering, AUTH and especially to Katie Quartano at the UNESCO Chair, AUTH for their constructive remarks and technical assistance while reviewing and proof-reading the manuscript.

Thessaloniki, Greece
March 2009

Jacques Ganoulis

Preface to the First Edition

This book deals with a quantitative analysis of environmental issues related to the water quality of natural hydrosystems. More specifically the questions of risk and reliability in water quality are analysed, from the engineering point of view and a methodology is developed to evaluate environmental impacts on rivers, groundwater and coastal areas from wastewater disposal.

In a biological environment with many risks and unexpected events, an attempt to apply a rigorous analysis to uncertain and complex environmental issues may appear ambitious, if not utopic. In fact, environmental problems related to water resources are complicated, unstable and difficult to understand. What would be, for example, the effect on algal blooms and eutrophication in a coastal area if the pollutant loads from a river doubled? Even more complicated is the quantitative prediction of coastal water quality from a possible climate change (e.g. doubling of the atmospheric CO_2).

Accurate description of water pollution problems is, most of the time, impossible, because available data is incomplete. The different type of the processes involved, such as hydrodynamic, physico-chemical and biological interactions, raise difficulties for mathematical modelling. Furthermore, the multitude of parameters, which are necessary to describe ecosystem's kinetics, their physical meaning and variability in space and time raise a multitude of challenging and intriguing questions.

The study of changes in water quality and the environmental impact of projects related to water resources, requires adequate tools. Engineering risk and reliability analysis provides a general framework to identify uncertainties and quantify risks. As it is shown in this book two main methodologies have been developed so far to analyse natural risks: (a) the stochastic approach and (b) the fuzzy set theory. Stochastic variables and probability concepts are based on frequencies and require large amounts of data. Questions of independence between random variables and validation of stochastic relations, such as the well known statistical regression, are most of the time difficult to resolve. Fuzzy set theory and fuzzy calculus may be used as a background of what we should call "imprecision theory". In this book it is demonstrated how, in case of lack of information or very little data, fuzzy numbers and variables may be used for modelling risks. The use of fuzzy regression is a very good alternative, when statistical regression fails.

Analysis of uncertainties and quantification of risks is not sufficient to formulate and realize environmental projects aiming to improve water quality. It is also

important to consider incremental variations of benefits and costs as functions of risk. This is the risk management issue, which is discussed in the book in relation to the consequences of risk and the decision-making process.

The writing of this book started as lecture notes for a graduate course on risk and reliability in water resources, in the Department of Hydraulics and Environmental Engineering, School of Civil Engineering, Aristotle University of Thessaloniki (AUT). Some of the examples, case studies and research related to this topic go back two decades: in relation to the author's PhD Thesis, probabilistic modelling was applied to evaluate the risk of intrusion of a non-wetting fluid into a porous medium.

In this book, no attempt has been made to be exhaustive and cover all types of water pollution problems. For example, questions of water pollution in lakes and reservoirs have been left out, although a similarity exists between such problems and semi-enclosed coastal bays and lagoons.

Parts of the book were written while on sabbatical leave at the University of Melbourne, Australia, Department of Civil and Agricultural Engineering (1991) and "Laboratoire Énergétique des Phénomènes de Transfert" – ENSAM, Bordeaux, France (1992).

Special thanks go to Prof. T. MacMahon, Melbourne, Australia and to Prof. M. Combarrous, Bordeaux, France for their help while I was in these Departments.

The application of fuzzy set theory on water resources problems has emerged, as a concept, from friendly discussions with Prof. Lucien Duckstein, University of Arizona and Prof. Istvan Bogardi, University of Nebraska, USA. I am really thankful for all the information they have provided for me on this matter in the form of papers and lecture notes.

I would also like to thank Dr. Hans-Joachim Kraus, VCH Publishing Division III, for the opportunity he gave me to publish this book.

Parts of the book have been typed by Ms Efi Meimaroglou, Department of Civil Engineering, AUT, to whom I am very grateful. Last but not least, my appreciation goes to Anastassia Papalopoulou, Petros Anagnostopoulos, Stephen Richardson and especially to Stelios Rafailidis at the Department of Civil Engineering, AUT for their constructive remarks and technical assistance while reviewing and proof-reading the manuscript.

Thessaloniki, Greece
May 1994

Jacques Ganoulis

1

Water Resources: Quantity and Quality

Water pollution, together with loss of biodiversity, climate change, energy and socio-economic issues, is one of the main threats and challenges humanity faces today. Human activities and human-related substances and wastes introduced into rivers, lakes, groundwater aquifers and the oceans modify the environmental water quality and make huge quantities of water unsuitable for various uses. This is the case not only for human-related uses such as drinking, bathing, agricultural irrigation and industrial production but also for terrestrial and aquatic ecosystems for which clean, fresh water is a prerequisite for life.

Water pollution is a serious problem for human health and the environment. The extent of the problem has been confirmed by many reports from UN organisations and related statistics. For example the Global Environment Outlook report (2000) produced by the United Nations' Environment Programme (UNEP) included the following statistics:

- Already one person in five has no access to safe drinking water.
- Polluted water affects the health of 1.2 billion people every year, and contributes to the death of 15 million children less than 5 years of age every year.
- Three million people die every year from diarrhoeal diseases (such as cholera and dysentery) caused by contaminated water.
- Vector-borne diseases, such as malaria, kill another 1.5–2.7 million people per year, with inadequate water management a key cause of such diseases.

Water pollution contributes to the so-called global 'water crisis', because it reduces the available amount of freshwater resources for both people and ecosystems. Freshwater scarcity is already a reality in many parts of the world, not only in developing countries like India, China and many African countries, but also in countries and regions traditionally considered as water rich, such as the USA and Europe. The United Nations (UN) predicts that two-thirds of the world's population will live in water-scarce regions by 2025. The increase in water demand, together with the increase in population in many parts of the world, but mainly the over use of water in areas like agriculture, together with water pollution and climate change are the main driving forces behind this phenomenon.

The quality of water resources and aquatic ecosystem preservation are very much related to the design and operation of hydraulic engineering structures, such as dams, reservoirs and river levees. Until now the design of these structures has paid far greater attention to cost, benefit and safety than to issues of environmental impact. Technical projects such as wastewater treatment plants, management of waste disposal and remediation of contaminated sites, which aim to treat wastewaters and therefore improve water quality, also produce various environmental hazards and risks.

To face real situations of water resources pollution, the efficient application of an environmental impact assessment, including data acquisition, risk analysis and examination of institutional aspects of water resources management, is of crucial importance. In this book the term 'water resources' covers fresh surface water and groundwater, as well as coastal water resources.

Many new techniques for risk assessment and management have been developed recently both in the USA and Europe (Duckstein and Plate, 1987; Ganoulis, 1991c; Haimes *et al.*, 1992; Morel and Linkov, 2006; Hlavinek *et al.*, 2008). These techniques aim to quantify the risks arising from the various uses of water, for example urban water supply, irrigation and industrial processes. However, few of these developments have filtered into academic curricula, and even fewer into engineering practice. The main objective of this book is to present, in a unified framework, methods and techniques of risk and reliability analysis for evaluating the impact on environmental water quality from different water uses, wastewater disposal and water resources management planning.

Risk and reliability analysis has also been used in fields other than engineering, for example in social, economic and health sciences. Risks have been analysed within these disciplines in relation to public policy, administration, financing or public health. Public risk perception, social behaviour and attitudes under risk, risk costs and exposure assessment are some of the major topics of study.

In this book environmental risk and reliability analysis is discussed, as applicable specifically to water pollution in the natural environment. Risk and reliability analysis may also provide a general methodology for the assessment of the safety of water-related engineering projects. In water pollution problems, risk is related to various uncertainties in the fate of pollutants. Thus, risk and reliability assessment of water pollution is a useful tool to quantify these uncertainties and evaluate their effect on water resources. In this respect, the important technical aspects are the management of hydrosystems (rivers, lakes, aquifers and coastal areas) taking into account water quality and environmental impacts, the design of environmental amenities, the management of waste disposal, the optimum operation of wastewater treatment plants and the remediation of contaminated sites.

Important features covered in this book are:

- Uncertainty Analysis of Water Quantity and Quality.
- Stochastic Simulation of Hydrosystems: model selection, water quantity and quality assessment and changes in water quality due to possible climate change in coastal waters, risk of groundwater and river pollution.
- Application of Fuzzy Set Theory in Engineering Risk Analysis.

- Decision Theory under Uncertainty: risk management, risk–cost trade-offs.
- Case Studies.

Environmental water pollution could lead to public health hazards (risk to human health), deterioration of water quality and damage to ecosystems (environmental risk) or may cause economic consequences (economic risk). In this sense, environmental risk and reliability analysis is an interdisciplinary field, involving engineers, chemists, biologists, toxicologists, economists and social scientists. Although there is a strong interaction between these disciplines and for specific applications only team work is appropriate, this book focuses mainly on the technical and engineering aspects of environmental risk.

In this introductory chapter the role of engineering risk and reliability analysis in water pollution problems is further clarified. After stressing the importance of both natural water resources and water quality, environmental risk assessment and management are explained and the organisation of material presented in the following chapters is summarised.

1.1

Water Pollution and Risk Analysis

Risk and reliability have different meanings and are variously applied in different disciplines such as engineering, statistics, economics, medicine and social sciences. The situation is sometimes confusing because terminologies and notions are transferred from one discipline to another without modification or adjustment. This confusion is further amplified as scientists themselves can have different perceptions of risks and use different tools to analyse them.

Risk has different definitions in engineering, economic, social and health sciences. *Risk analysis* is mainly based on the quantification of various uncertainties which may occur in the evolution of different processes. The use of modelling techniques to quantify such uncertainties is an essential part of risk analysis. Furthermore, because preventive and remedial actions should be based on predictions of how processes might develop under uncertainty in the future, probabilistic approaches are more appropriate for this purpose than deterministic methods. Probabilities, and more recently the fuzzy set theory, are suitable tools for quantifying uncertainties which may induce a risk of failure.

Water quantity and quality problems are very much inter-related and should be studied within an integrated framework. Furthermore, water quality is related to the integrity of ecosystems and these should be analysed together. This unified approach has been adopted in this book. After reviewing the importance of water resources and the need for good water quality for sustainable economic development, the management of water resources is analysed. The latter is based on both the design and decision making processes, in which various uncertainties may exist. The concept of quantification of these uncertainties and how one may proceed from the assessment to the management of risks are presented in the following pages and discussed in detail in Chapters 2 and 3.

1.1.1

A Systemic View of Water Resources

The total volume of water on Earth is estimated at 1360 million cubic kilometers or $1338 \times 10^6 \text{ km}^3$ (Gleick, 1996 and USGS). This number was derived from a long-term assessment of the average amount of water stored in the hydrosphere, that is, that part of the Earth covered by water and ice, the *atmosphere* and the *biosphere* (all living organisms on Earth). About 70% of the Earth's surface is covered by oceans. The salt water in the seas and oceans represents 97% of the total water on Earth, the remaining 3% being fresh water.

Freshwater is distributed in different components (glaciers, rivers, lakes, groundwater, atmosphere and biosphere) as shown in Table 1.1. From this table it can be seen that the greatest part (68.7%) of total freshwater is trapped in polar glaciers and ice sheets, and is therefore not directly accessible for use. Only 0.3% of the freshwater on Earth is surface water, in the form of lakes (87%) and rivers (2%).

Table 1.1 Distribution of freshwater on Earth.

Source of freshwater (estimate)	Percentage of the total freshwater
Glaciers and permanent snow cover	68.7%
Groundwater	30.1%
Freshwater lakes	0.26%
Rivers	0.006%
Atmosphere	0.004%
Biosphere	0.003%

Water exists in three states: liquid, solid (ice and snow) and gas (water vapour). Due to the energy supplied by the sun, water is permanently being transformed from one state to another, and is in constant motion between oceans, land, atmosphere and biosphere. As shown in Figure 1.1, water in motion constitutes *the hydrologic cycle* through the following hydrological processes, which take place in a permanent manner (UNESCO glossary):

- *Evaporation*: emission of water vapour by a free surface at a temperature below boiling point.
- *Transpiration*: transfer of water vapour from vegetation to the atmosphere.
- *Interception*: process by which precipitation is caught and held by vegetation (canopy and litter structures) and which may then be lost by evaporation without reaching the ground.
- *Condensation*: the change in water phase from a vapour state into a liquid state.
- *Precipitation*: liquid or solid products of the condensation of water vapour falling from clouds or deposited from the air onto the ground. For example rain, sleet, snow, hail.
- *Runoff*: that part of precipitation that appears in surface streams.
- *Infiltration*: flow of water through the soil surface into a porous medium.
- *Groundwater flow*: movement of water in an aquifer.

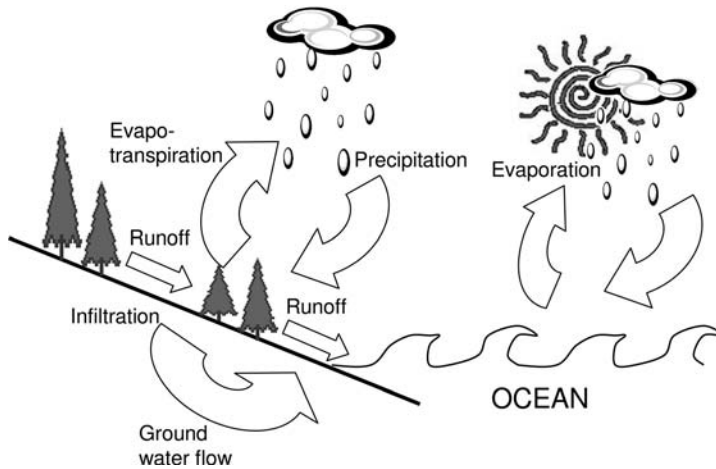


Figure 1.1 The hydrological cycle.

For water resources management in a given hydrological area or at the catchment scale it is necessary to quantify the available water resources for a given time scale. The *water balance* or the *water budget* of a region is the quantification of the individual components of the water cycle during a certain time interval.

What is important for the development of water resources is not the amount of precipitation in an area but rather the so-called *efficient precipitation*. This is the amount of runoff water remaining when *evapotranspiration* is subtracted from the total precipitation. This amount represents the potential water resource and includes the overland flow and water infiltrating the soil. For the EU the mean annual volume of precipitation water is estimated at $1375 \text{ km}^3/\text{year}$ (97 cm/year) and the efficient precipitation at $678 \text{ km}^3/\text{year}$ (48 cm/year) (Bodelle and Margat, 1980).

1.1.1.1 Examples of Application

Annual Water Budget of Romania (Table 1.2).

Table 1.2 Annual water budget of Romania (National Institute of Meteorology and Hydrology, Regional Office, Timisoara).

Precipitation	850 mm/year
Runoff	300 mm/year
Evaporation	550 mm/year

Annual Water Budget of Bulgaria (Table 1.3).

Table 1.3 Annual water budget of Bulgaria (Geography of Bulgaria, monograph, Bulgarian Academy of Sciences, 1989).

Precipitation	690 mm/year
Runoff	176 mm/year
Evaporation	514 mm/year

In today's complex economy water resources play a key role. In addition to the fact that fresh water is essential to all kinds of life, it is also used in agriculture and industrial processes. Fresh water is used in settlements to meet domestic demands (Figure 1.2) and also in municipal waste water systems, industrial wastewater treatment plants in agriculture, and for the dissolution and removal of dirt and waste.

A sufficient supply of fresh water has become a necessary condition to ensure economic growth and development. Since it takes 1000 tons of water to produce 1 ton of grain, importing grain is the most efficient way to import water. Countries are, in effect, using grain or other agricultural products to balance their water resources budget.

As demand for water for different uses increases and pollution deteriorates water quality, economic development is put under stress and conflicts result between different 'direct' and 'indirect' users (Figure 1.2). The problem is further exacerbated in regions where long-term droughts have decreased the available amount of water, while the needs for water have increased. At the same time, preservation of good water quality in rivers, lakes, aquifers and coastal waters is necessary to protect public health and ecosystems.

The importance of water resources and problems of water quantity and quality may be better perceived by analysing the economic importance of water and the new

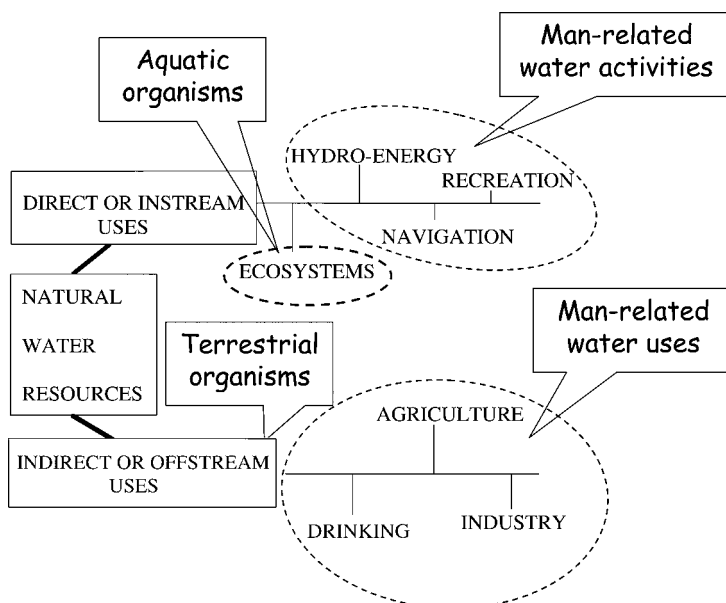


Figure 1.2 Direct and indirect uses of water resources by man and ecosystems.

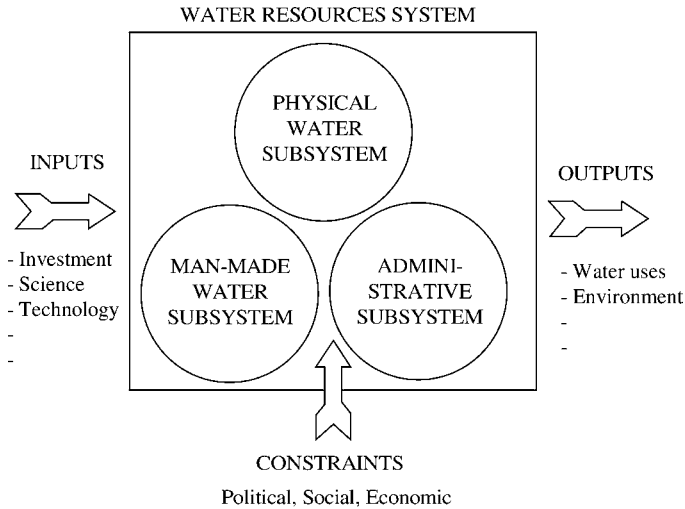


Figure 1.3 Description of a water resources system.

opportunities in the *water market*. In the EU it has been estimated (Williams and Musco, 1992) that the costs of running municipal water supply and wastewater systems alone are 14 billion Euros per year. For the implementation of the municipal wastewater and drinking water directives in the EU, including its new members, several hundred billion Euros will be needed in the near future. To face the problems of future water demand and to combat growing pollution it is expected that the already huge market for water will be expanded further with new technologies, new investments and new management methods.

When considering management issues of water-related problems it becomes apparent that besides scientific and technical components there are also social, economic and institutional components involved (Figure 1.3).

If water resources are defined as a system (Figure 1.3), apart from the natural water subsystem, man-made water subsystems (channels, distribution systems, artificial lakes, etc.), as well as the administrative system, should also be included. These three subsystems are interconnected and are subject to various social, political and economic constraints (Figure 1.3). Inputs to the system are data, investment, science and technology and outputs are water uses, environmental protection, new technologies, and so on.

1.1.2

The New Paradigm of Water Quality

In water resources management water quality plays an increasingly important role, just as important as that of water quantity. In fact, as pollution of surface, coastal and groundwater increases, it has become essential to adopt an integrated approach encompassing both water quantity and quality (Figure 1.4).

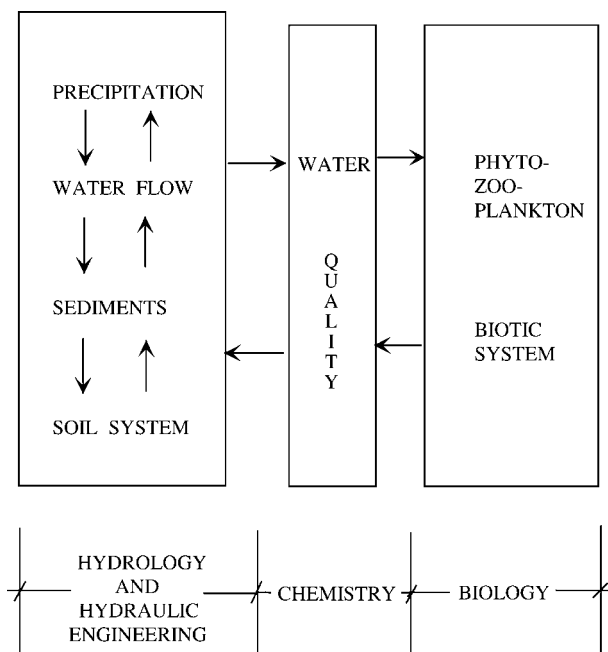


Figure 1.4 Elements of abiotic and biotic water systems.

Furthermore, according to the new paradigm of water quality, the ecological status of a water body should also be taken into consideration. This means that a good status of water biology and healthy aquatic organisms are necessary for obtaining a good status of water quality, and vice versa optimum physico-chemical conditions of water resources are necessary to sustain healthy ecosystems.

This integrated definition of the 'good status of water' was adopted in the new EU-Water Framework Directive 2000/60, which means that the environmental protection of water resources requires joint investigations of both abiotic and biotic elements.

For example in coastal regions, the most serious environmental problems in order of priority are:

- (1) Decrease in water transparency as a result of high concentrations of organic elements, suspended matter and nutrients.
- (2) Oxygen depletion, due to excessive demand for oxygen from organic matter, nitrogen and phosphorus. As oxygen is an essential requirement for both predatory and non-predatory organisms, a low oxygen concentration may comprise the existence of marine life.
- (3) Bacteriological contamination, which poses a threat not only to water but also to shellfish and oysters. This represents a major danger to public health.
- (4) Loss of habitat and invasion of tropical species. In the Mediterranean, the appearance of new species of algae is attributed to excessive pollution.

- (5) Eutrophication phenomena due to the increase of nutrients, such as nitrogen and phosphorus.

The social causes of these problems are mainly due to the increase in coastal populations and also intensive agricultural, industrial and harbour activities in coastal areas. Preserving water quality in this integrated manner safeguards human well-being and health and at the same time maintains diversity in the biota. In coastal area, fishing and other aquacultures are traditional and very important economic activities, employing and feeding large populations, especially on islands. Tourism forms an important part of the economy in many European countries and is directly related to the quality of marine resources. The importance of these aspects is discussed below.

1.1.2.1 Human Well-being and Health

Although water quality has a direct impact on the actual health of urban populations, there are also extremely important indirect impacts through the food chain. Catches of fish and oyster farming in polluted coastal areas may introduce bacterial or toxic metal contamination into the human food chain, causing epidemiological occurrences. Even in cases where contamination remains tolerable, the presence of pollutants may cause abnormal growth of certain algae in the water body, causing oxygen depletion (eutrophication). Fish feeding on these algae may suffer adverse changes in flavour or odour, and become unsuitable for human consumption. In addition, decaying algae produce H_2S and other odorous substances which may affect the well-being of the population living along the water body. The important interplay between water quality and human settlements on the coast is exemplified by the total absence of permanent habitation around the Dead Sea. The quality of water there is so poor that not only does it not attract people, but it actually turns them away.

1.1.2.2 Ecological Impacts and Biodiversity

A rich variety of organisms inhabit the world's fresh, coastal and oceanic waters. Generally, these may be divided into producers (e.g. phytoplanktonic diatoms, flagellates, etc.) and consumers of organic matter (e.g. zooplankton, nekton, benthos, etc.). In addition, there are also different types of bacteria, in concentrations ranging from one per litre to more than 10^8 per millilitre. Generally, bacteria do not contribute significantly to nutrient recycling in the water column but mainly in the sediments (Odum, 1971).

Areas containing water play an important role in trapping solar energy and in the transformation of biological matter. Species diversity in the water column is directly related to water quality. Studies by Copeland and Bechtel (1971) have shown a paucity of biodiversity in areas close to effluent outfalls, with the effect diminishing with distance. Also, water toxicity was found to be inversely related to species diversity in the water body.

Copeland (1966) has reported that in polluted waters the levels of various industrial wastes found in fish increase, even when the effluent has not yet reached toxic levels. This is because the reduction in the concentration of dissolved oxygen, caused by the discharge of biological matter around the outfall, forces fish to pump more water through their gills and thus absorb greater quantities of pollutants. This may then have a knock-on effect on the rest of the biota through the food chain.

1.1.2.3 Fishing and Oyster Farming

Water quality is very important for fishing and the aqua culture industry, especially shellfish farming. It is well known that organisms living in water accumulate pollutants from the surrounding water in their flesh and pass them into the food chain. This is particularly so for mussels, oysters and other stationary marine animals growing in polluted waters. For this reason, for some time now legislation has stipulated the allowable quality of water for oyster farming.

1.1.2.4 Tourism

Regions having a pleasant climate and a rich cultural heritage usually attract tourists. The Mediterranean countries, for example enjoy substantial tourist influxes. It is estimated that as much as one-third of the world tourist traffic concentrates there (Golfi *et al.*, 1993). The coastal strip has become a major attraction for tourist recreation, in the form of bathing, sport fishing and water sports. As a result, tourism has become a major contributor to the local economy.

The tourist economy in these areas, however, is jeopardised by inadequate water infrastructure, such as municipal water supplies and efficient wastewater treatment facilities. This frequently results in deterioration of the quality of coastal water, which was one of the primary factors attracting tourists in the first place. An example of the problems which may result from unsatisfactory water resources management was the damage to the tourist industry on the North Adriatic coast in the late 1980s, due to the occurrence of severe seasonal algal blooms, caused by abnormally high eutrophication and warm ambient temperatures.

If it were not for the substantial amounts of man-made pollution discharged into water bodies in modern times, nature itself would be able to provide a continuous recycling of biological matter in natural waters.

Groundwater contamination is the most critical among the various types of pollution that can occur in the water cycle, because of the long time scales involved and the irreversible character of the damage caused. Due to the very slow movement of groundwater, pollutants can reside for a very long time in the aquifer, and even if the pollutant sources are no longer active the groundwater can remain polluted for centuries. At the same time, because of the complex interaction between pollutants, soil and groundwater, the remediation of contaminated subsurface is a very delicate operation. Usually it is necessary to totally remove and clean the contaminated soil or for biological techniques to be applied over a long period of time.

For surface water resources, in addition to the inherent biological loading from natural recycling of carbonaceous matter, further inputs from the land may arise in the form of

- large amounts of sediments, resulting from increased soil erosion due to the substantial deforestation in historical times, especially in Mediterranean countries;
- inorganic and organic pollutants, mainly nitric or phosphoric fertilisers, pesticides or herbicides used in farming. These result in a substantial contribution and are estimated to account for most of the overall water pollution (USEPA, 1984; ASIWPCA, 1985);
- organic, microbial or toxic man-made pollutants such as heavy metals or greases discharged from sewers.

Of these loads, heavy metals and toxic constituents tend to be chemically inactive and are removed mainly by mechanical or physical processes (e.g. sedimentation, adsorption onto solid particles or surfaces immersed in the water, etc.), whereas organic and other inorganic substances decay via numerous and very complicated chemical and biological processes.

All pollution loads, whether natural or man-made, are subject to the influence of water circulation currents. This results in advection and turbulent dispersion in the water body, following the laws of conservation of mass for each constituent substance in the system.

Advection occurs by turbulent mass transport within the water, while additional diffusion and turbulent dispersion of pollutants takes place. In addition, the pollutants are subject to different types of decay, such as

- chemical, as a result of the oxidising effect of oxygen dissolved in the water, and by mutual neutralisation between acidic and alkaline pollutants;
- biological, arising from metabolism by microbes, phyto- or zooplankton.

Overall, all these processes are extremely complicated and with the exception of water advection and circulation, not understood in any great detail. Therefore, much of the following discussion is based predominantly on empirical findings from experiments.

According to Rafailidis *et al.* (1994) of particular interest to engineers in the field of surface water resources are the concentrations

- The *Carbonaceous Biochemical Oxygen Demand* (CBOD). This is an indicator of the overall 'loading' of the aquatic system due to the oxidation requirements of organic pollutants. It also includes the respiration demand of marine microbes which metabolise organic and fix inorganic matter (e.g. nitrates, inorganic phosphorus, etc.).
- The *Dissolved Oxygen* content (DO). This parameter is more critical because it shows whether there is sufficient oxygen in the water for marine life to survive. The actual DO content reflects the equilibrium between re-aeration at the surface added to photosynthetic oxygen generated by chlorophyll in the water body, minus the biological and any chemical oxygen demand. Generally, most marine fauna will swim away from waters in which DO has fallen to less than about 5 mg/l. Nevertheless, some types of worms have been found to survive in virtually anoxic sediments in river deltas or heavily polluted areas around effluent outfalls.

- The *concentration of nutrients* (ammonia, nitrates, phosphates, inorganic nitrogen or phosphorus) is linked directly to non-point source runoff from agricultural watersheds as a consequence of soil fertilisation, insecticide or pesticide spraying, and so on. Nutrients are metabolised by marine microorganisms and the inorganic elements are fixed to more complex compounds. Algae play a very important role in these processes, enhancing water denitrification (release of N_2 to the atmosphere) or nitrification (capture of N_2 from air).
- *Ammoniac* compounds are antagonistic to nitrates, as both compete for algal uptake. On the other hand, the simultaneous presence of phosphorus enhances algal growth, leading to eutrophication, that is, abnormal growth of algae and marine flora. This is particularly troublesome in enclosed waters (e.g. lakes and lagoons) but also occurs in coastal areas suffering from large pollution inflows and suppressed natural circulation and flushing.
- The *coliform bacteria concentration*. Although these microorganisms are not pathogenic and exist naturally in human intestines, their presence indicates pollution due to urban sewage effluents. However, doubts have been voiced about the suitability of this parameter as an indicator of *pathogenic* potential in coastal waters (Sobsey and Olson, 1983). This is because pathogenic viruses have lower decay rates than coliforms, and can also cause infection at smaller doses. Furthermore, whereas coliforms are of human origin, some opportunistic pathogens (e.g. *Pseudomonas Aeruginosa*, *Legionella Pneumophila*) also often originate from non-fecal sources and can grow naturally in various waters (Bowie *et al.*, 1985). Upon discharge into the water body environmental conditions such as temperature and sunlight determine the eventual fate of coliform bacteria through a multitude of processes (e.g. photo-oxidation, sedimentation, pH, predators, algae, bacteriophages, etc.).
- Apart from the above pollutants, sediments in the water column may also cause environmental problems as they bury benthic flora, or choke the gills of marine invertebrates. In fact, coastal areas at the deltas of large rivers typically suffer from anoxic conditions (Nelsen, 1994) due to oxygen demand from the large sediment and that required for the transport of biological matter. This is in addition to the polluting effects of any other organic or inorganic nutrients carried by the river.

In summary, CBOD in surface waters indicates the overall organic pollution of the water, DO shows whether marine life may be sustained there, and nutrient concentration gives the potential for eutrophication. Coliform counts indicate the danger of disease for humans using the water for bathing or recreation.

1.1.2.5 Algal and Chlorophyllic Photosynthesis

Phytoplankton exists in many different forms (e.g. diatoms, green algae, blue-green algae, dinoflagellates, etc.) and form an important part of the water ecosystem determining eventual water quality. Algae are primarily responsible for the uptake of nutrients, which are then recycled through algal respiration and decay.

Photosynthesis by algae in the euphotic zone produces oxygen; this is reversed at night due to respiration.

On the other hand, algae which settle in deeper, oligophotic waters contribute to oxygen depletion there. Algae either take up dissolved CO₂ or produce CO₂ as a by-product of respiration, thus changing water pH and the subsequent chemical interactions in the water. Their presence increases water turbidity, reducing the euphotic depth in the column. On the other hand, phytoplankton constitutes the foundation of the food chain of higher species, virtually supporting the marine animal biota.

Because of the variety of different algae, it is customary to consider algal concentrations in terms of chlorophyll-a concentrations. Algal growth is a function of temperature, solar insolation and nutrients (phosphorus, nitrogen, carbon and silicon for diatoms). Other essential nutrients such as iron, manganese, sulphur, zinc, copper, cobalt, molybdenum and vitamin B₁₂ may also be important, especially in oligotrophic waters.

1.1.2.6 Zooplankton Growth

Zooplankton are part of the same biomass pool as phytoplankton. Zooplankton dynamics depend on growth, reproduction, respiration, excretion and non-predatory mortality. In contrast to phytoplankton, zooplanktonic organisms are mobile, so settling does not occur. Furthermore, zooplankton migrate vertically following the diurnal cycle, adding another complication to the analysis.

Growth, consumption, respiration and non-predatory mortality are direct functions of temperature. Zooplanktonic animals are typically filter feeders. Therefore, zooplankton growth may be simply considered as being proportional to the available food concentration. Predation is also related to the rates of consumption by higher predators.

1.1.2.7 Bacteria

Coliform growth and decay depend mainly upon environmental conditions through a variety of mechanisms. These include physical (e.g. photo-oxidation, coagulation, sedimentation), physico-chemical (e.g. pH, osmotic effects) and biochemical-biological (e.g. nutrient levels, predators, algae, bacteriophages) factors.

The interplay between these factors is poorly understood, especially quantitatively. Because of this limitation, coliform growth and decay have traditionally been assessed on the basis of the first-order approach of the T-90 measured values, that is, the exposure required to ensure 90% mortality. Care must be taken, however, because bacterial decay in the dark is only approximately half of that at midday (Bowie *et al.*, 1985). Apart from this sensitivity to light, measurements have also detected a sensitivity to water salinity. As in all other physico-bio-chemical processes, temperature also plays an important role (Ganoulis, 1992).

1.1.3

Integrated Water Resources Management

Water Resources Management is traditionally defined as a process of effectively allocating an appropriate amount of water to a given sector, such as urban water

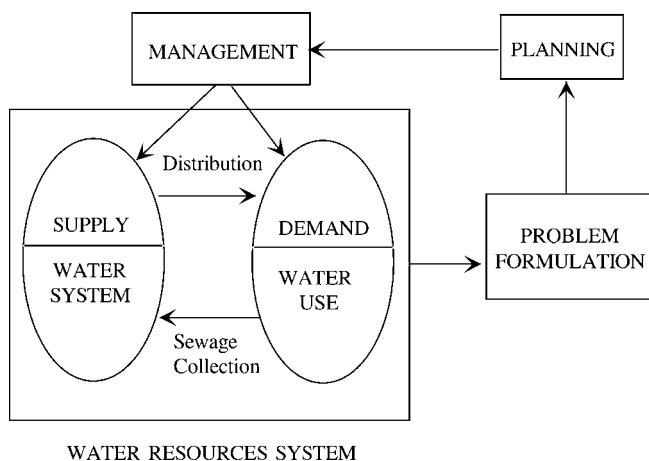


Figure 1.5 Urban water resources management.

supply, agriculture or industry. Adequate decisions should be made and measures taken in order to satisfy the demand for water both in terms of quantity and quality.

In urban water supply for example, decisions on structural and non-structural measures should be taken in order to ensure good drinking water for all citizens. As shown in Figure 1.5, satisfying the demand whilst collecting municipal wastewater and providing adequate treatment for environmental protection, constitute parts of the urban water management problem. Management of urban water resources involves addressing not only technical issues but also many social factors, institutions and administrative procedures.

The main objective of water resources management is to satisfy the demand, given the possibilities and limitations of the water supply. The balance between supply and demand should take into consideration both water quantity and quality aspects as well as the protection of the environment.

As seen in Figure 1.5 water resources management involves problem formulation, planning, implementation of appropriate measures, regulation of both water demand and supply and finally decision making. The various steps involved in this process are described schematically in Figure 1.6. Implementation of the design and decision stages involves the following processes:

- Step 1: Identify the problem. Analyse important factors and variables.
- Step 2: Determine the objectives in terms of the above variables.
- Step 3: Develop a mathematical model correlating input–output variables.
- Step 4: Identify alternative options.
- Step 5: Select the optimum solution.
- Step 6: Employ sensitivity analysis, that is, examine the influence on the results of any change in the value of parameters and the assumptions made.

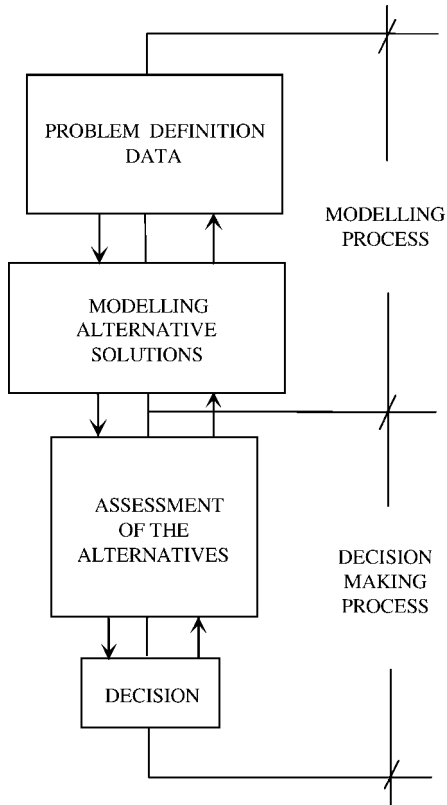


Figure 1.6 Modelling and decision making processes in water resources management.

At the river catchment scale, different uses of water, such as for drinking, agricultural irrigation, hydropower production and industry, often lead to conflicting situations. For example industries producing large amounts of untreated wastewater may pollute groundwater in the surrounding aquifer, which in turn affects the quality of water pumped for drinking purposes. The increase in water pollution from industrial activities may also affect the quality of river water used for irrigation. When groundwater is over-pumped from a series of wells, the groundwater table is lowered and could affect agricultural production, as less water will be available for crop roots. Lowering the water table in a coastal zone may also increase seawater intrusion and soil salinisation, leading to a negative impact on agriculture and ecosystems.

Obviously, when actions are taken for different water uses, as can be seen in the examples above, there is a need to coordinate the various aspects of the related activities, such as between different:

- *sectors of water uses* (water supply, agriculture, industry, energy, recreation, etc.);
- *types of natural resources* (land, water and others);
- *types of water resources* (surface water, groundwater);

- *locations in space* (local, regional, national, international);
- *variations in time* (daily, monthly, seasonal, yearly, climate change);
- *impacts* (environmental, economic, social, etc.);
- *scientific and professional disciplines* (engineering, law, economy, ecology, etc.);
- *water-related institutions* (government, private, international, NGOs, etc.);
- *decision-makers, water professionals, scientists and stakeholders.*

As shown in Figure 1.7, Integrated Water Resources Management (IWRM) can be achieved by coordinating different topics, areas, disciplines and institutions, which can be categorised as being either natural (type of resources, space and time scales) or man-related (sectors, scientific disciplines, impacts, institutions, participants). There is no general rule about the optimum degree of integration and how to achieve it. Concerning the spatial scale and taking into account the hydrological cycle and the water budget, the area of the river basin is the most relevant water management unit. The effect of possible climate change should also be taken into account, although major uncertainties still persist for quantifying such effects.

The need for coordination in Water Resources Management was first recognised in the 1970s and the actual term 'Integrated Water Resources Management' (IWRM) was first coined in 1977 at the UN Conference in Mar del Plata. The term is very broad, and is therefore subject to different definitions.

In the Background Paper No. 4 produced by its Technical Committee (TEC), The Global Water Partnership (GWP) – an NGO based in Stockholm – defines IWRM as 'a process which promotes the coordinated development and management of water, land and related resources to maximise the resultant economic and social welfare in an equitable manner without compromising the sustainability of vital ecosystems'. (GWP, 2000). The 'Tool Box' being developed by GWP promotes IWRM and makes recommendations on how it can be achieved (GWP, 2002, 2004).

The World Water Council (WWC) stated that IWRM is a 'Philosophy that holds that water must be viewed from a holistic perspective, both in its natural state and in balancing competing demands on it – agricultural, industrial, domestic and environmental. Management of water resources and services needs to reflect the interaction between these different demands, and so must be coordinated within and across sectors. If the many cross-cutting requirements are met, and if there can be horizontal and vertical integration within the management framework for water resources and services, a more equitable, efficient and sustainable regime will emerge' (Global Water Partnership, Framework for Action 1999).

At the World Summit in Rio (1992), a special reference was made to IWRM. In the action programme known as Agenda 21 adopted at the Conference, in Chapter 18, Paragraph 18.6 it is stated that "... the holistic management of freshwater as a finite and vulnerable resource, and the integration of sectoral water plans and programmes within the framework of national economic and social policy, are of paramount importance for action in the 1990s and beyond". The fragmentation of responsibilities for water resources development among sectoral agencies is proving, however, to be an even greater impediment to promoting integrated water management than had been anticipated. Effective implementation and coordination mechanisms are required.

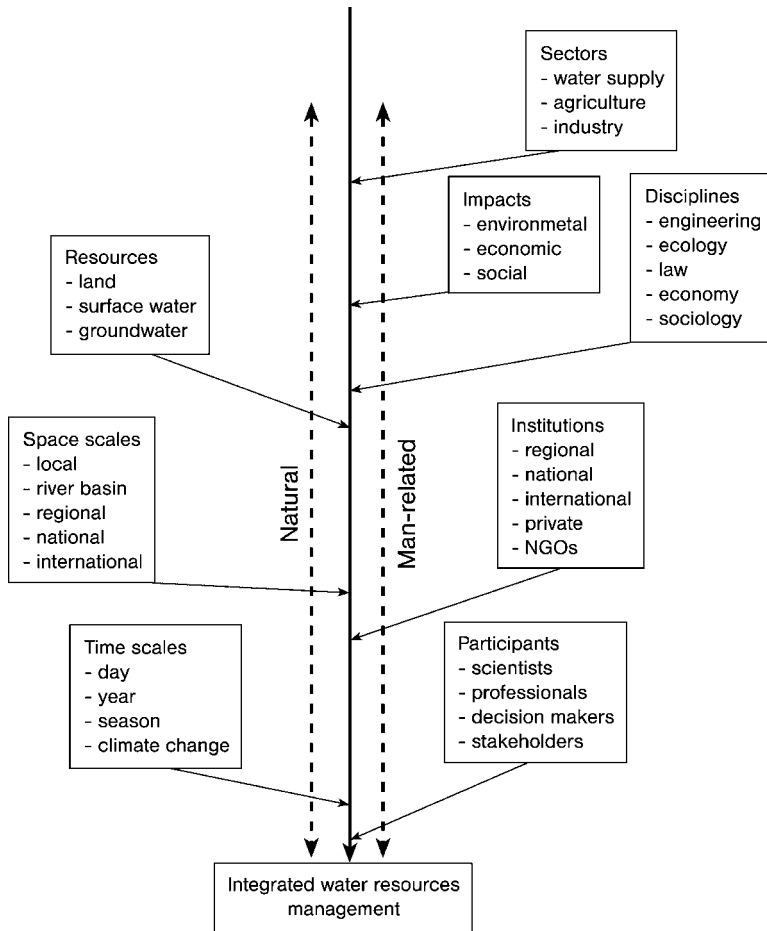


Figure 1.7 Natural and man-related issues of IWRM.

Water resources development and protection, and more particularly IWRM, is one of the main elements for achieving 'sustainable development'. According to the Brundtland Commission (1987) sustainable development should satisfy current needs without compromising the requirements of future generations. For water resources, sustainable management may be defined as using water for various needs without compromising its hydrologic, qualitative and ecological integrity. Sustainability can be achieved by resolving environmental, economic and social issues related to water management.

Sustainability may be viewed as an ultimate goal, but it is one which is very difficult to achieve. It is therefore important to define quantitative sustainability indices in order to measure and record the progress achieved or the degradation observed in different domains. A risk assessment approach using four risk indices (technical or engineering, economic, environmental and social) is proposed in order to monitor

quantitatively the degree to which IWRM achieves sustainable water resources management and sustainable development (Ganoulis, 2001).

1.2

Water Pollution in Transboundary Regions

Two types of border dividing the territory of different states within a river basin are shown in Figure 1.8:

- (1) Borders cross the river at a point and divide the river catchment in two areas, the upstream and the downstream. In this case, there is no joint sharing of one river section by the two states. This is the case of the border between Hungary and Serbia at points crossed by the Danube and Tisza Rivers or the border between Greece and the Former Yugoslav Republic of Macedonia (FYROM) crossed by the Vardar/Axios) River near the city of Gevgelia (Figure 1.8a).
- (2) Rivers serve as borders between states, as in the case of the lower course of the Danube River, which serves as the border between Bulgaria and Romania (Figure 1.8b); and borders that follow and also cross international rivers.

How the interstate borders follow and/or cross international rivers, and how they divide rivers and river basins, will determine what type of water resources problems exist or will likely arise and need bilateral or multilateral interstate solutions. For transboundary waters, a large number of international agreements for solving various types of interstate water resources problems are available. The most important international treaty is the United Nations Economic Commission for Europe (UNECE) Convention.

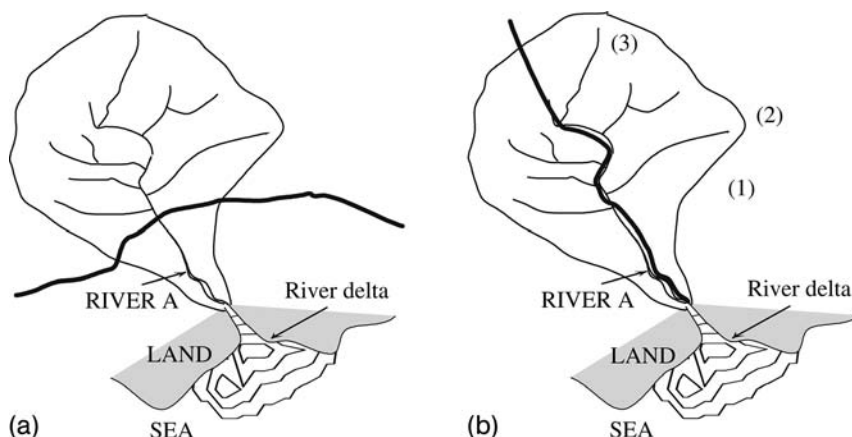


Figure 1.8 Schematic representation of two types of interstate borders: (a) crossing and (b) following a river.

1.2.1

The UNECE Convention (Helsinki, 1992)

Legal name: Convention on the protection and use of transboundary watercourses and international lakes.

The Convention obliges Parties to prevent, control and reduce water pollution from point and non-point sources. It is intended to strengthen national measures for the protection and ecologically sound management of transboundary surface waters and groundwaters. Multilateral cooperation for the protection of natural resources to prevent, control and reduce transboundary impact of surface or groundwaters which mark, cross or are located on boundaries between two or more States.

Transboundary impact means any significant adverse effect on the environment resulting from a change in the conditions of transboundary waters caused by a human activity, the physical origin of which is situated wholly or in part within an area under the jurisdiction of another Party. The Convention also includes provisions for monitoring, research and development, consultations, warning and alarm systems, mutual assistance, institutional arrangements, and the exchange and protection of information, as well as public access to information. In taking protective measures the Parties are advised to be guided by the following principles:

- (a) *The precautionary principle*, by virtue of which action to avoid the potential transboundary impact of the release of hazardous substances shall not be postponed on the grounds that scientific research has not fully proved a causal link between those substances on the one hand, and the potential transboundary impact on the other.
- (b) *The polluter-pays principle*, by virtue of which costs of pollution prevention, control and reduction measures shall be borne by the polluter.
- (c) *Sustainability*: Water resources shall be managed so that the needs of the present generation are met without compromising the ability of future generations to meet their own needs.

The Convention requires that the limits of discharges should be based on *best available technologies* for hazardous substances. Municipal wastewater needs to be biologically treated and best available technologies should be used to reduce nutrient discharges. Appropriate measures and *best environmental practices* must be used for the reduction of nutrients and hazardous substances from non-point sources.

1.3

The EU Water Framework Directive

EU environmental regulation aims at coordinating different measures taken at Community level to tackle particular environmental problems in order to meet

established objectives. Key examples of such regulation are the Urban Waste Water Treatment Directive, the Nitrates Directive and the Integrated Pollution Prevention and Control Directive.

In 2000, the EU issued the Water Framework Directive (WFD) in order to ensure an analysis of the state of water bodies and 'a review of the impact of human activity on the status of surface waters and on groundwater'. The analysis and review are to be conducted so as to determine how far each body of water is from the objectives (Directive 2000/60/EC).

The overall objective of the WFD is a 'good status' for all waters to be achieved by December 2015. For surface waters, 'good status' is determined by a 'good ecological' and a 'good chemical status'. This is determined by hydro-morphological (e.g. the condition of habitats), physico-chemical and biological monitoring and analysis. The WFD aims to establish a framework for the protection of inland surface waters, transitional waters, coastal waters and groundwater which:

- Prevents further deterioration and protects and enhances the status of aquatic ecosystems.
- Promotes sustainable water use based on the long-term protection of available water resources.
- Aims to enhance protection and improvement of the aquatic environment.
- Ensures the progressive reduction of pollution of groundwater and prevents its further pollution.
- Contributes to mitigating the effects of floods and droughts.

Key elements of the WFD include:

- Technical considerations: monitoring, river basin planning, and management.
- Institutional: adopt the river basin as a single system for water management.
- Environmental: water quality and ecosystems.
- Water economics.
- Public participation.

The WFD requires that River Basin Management Plans (RBMPs) are produced for each River Basin District (RBD) by 2009. These will be strategic management documents, developed via the river basin planning process, which will integrate the management of the water and land environment. Preparation will involve a process of analysis, monitoring, objective setting and consideration of the measures to maintain or improve water status.

Under the WFD, environmental monitoring programmes are required and specific objectives for water quality are set up. The WFD operates using a cyclical management process. This process begins by identifying water bodies in each RBD and describing their natural characteristics. The second stage is to assess the pressures and impacts on the water environment. This assessment identifies those water bodies that are unlikely to achieve the environmental objectives set out in the Directive by 2015.

The Directive calls for the application of economic principles (e.g. the recovery of the costs of water services and the polluter pays principle), approaches and tools (e.g. cost effectiveness analysis), and for the consideration of economic instruments (e.g. water pricing) for achieving its environmental objective in the most effective manner.

The WFD recognises the value and importance of involving all those with an interest in the water and land environment in how the WFD is put into practice. In certain areas (e.g. the development of RBMPs), stakeholder involvement is an inherent part of the Directive.

1.4 Uncertainties in Water Resources Management

Although rather the exception, there are situations in water resources engineering that may be considered as deterministic. As the uncertainties are low, a deterministic approach relating input and output suffices in these cases. Take, for example, the water supply from a reservoir operated with a gate. As shown in Figure 1.9, there is a deterministic relationship between the flow rate and the water height in the reservoir. In such cases there is no reason to use risk and reliability techniques, because the situation is predictable.

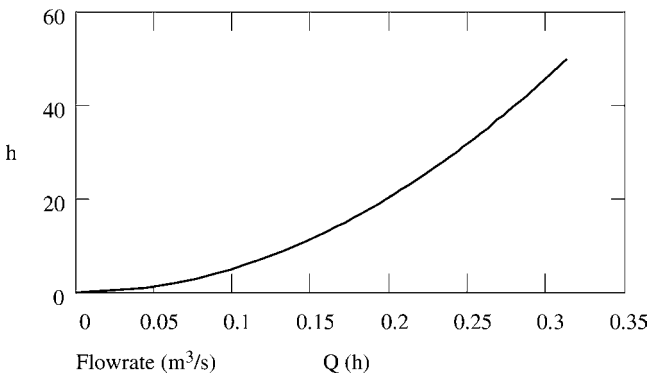


Figure 1.9 Water height expressed as a deterministic function of flow rate.

When the reservoir is filled by an inflow which varies with time, uncertainties in the variation of water height in the reservoir is no longer deterministic, as shown in Figure 1.10.

When uncertainties are important and influence the output of the water system, it becomes more appropriate to use risk analysis. Otherwise, traditional engineering modelling and simulation should be applied. Water resources engineering risk and reliability can be classified into three main categories:

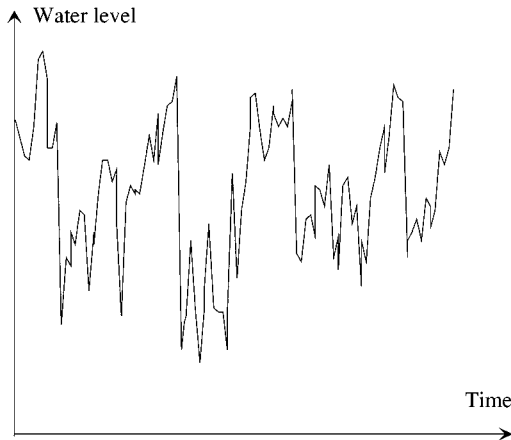


Figure 1.10 Stochastic variation of the water level in a reservoir.

- (1) structural reliability (dams, flood levees and other hydraulic structures),
- (2) water supply reliability (problems of water quantity),
- (3) water pollution risk (problems of water quality).

In all three areas, uncertainties are mainly due to the spatial and temporal variability associated with hydrological variables. In addition to these uncertainties, which arise from the definition of the physical problem, other types of uncertainties are added, such as those related to the use of methods and tools to describe and model the physical problem (i.e. sampling techniques, data acquisition, data analysis and mathematical modelling). This book deals with the third group of problems, the risk to environmental water quality.

Four different types of uncertainties may be distinguished:

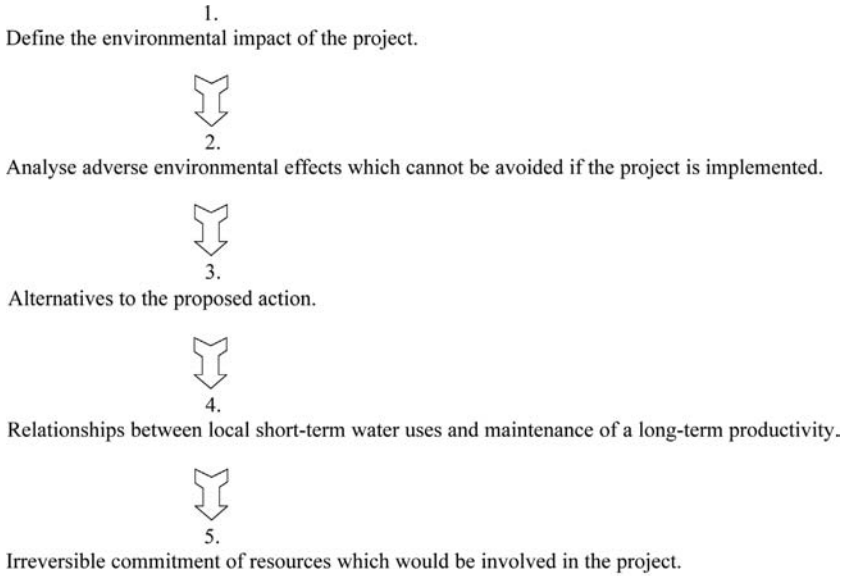
- (1) Hydrologic uncertainty
This refers to the various hydrological events such as precipitation, river flow, coastal currents, water quality, and so on.
- (2) Hydraulic uncertainty
These are uncertainties related to hydraulic design and hydraulic engineering structures.
- (3) Economic uncertainty
This refers to all fluctuations in prices, costs and investments that may affect the design and optimisation processes.
- (4) Structural uncertainty
This means all deviations due to material tolerances and other possible technical causes of structural failure.

Methods and tools able to quantify such uncertainties should be incorporated into the design and decision processes.

1.5

Environmental Risk Assessment and Management

According to the European Commission (EC) directive 85/337 relative to environmental impact studies, the design of water resources management projects should proceed in five steps:



As shown in Figure 1.11, application of environmental risk analysis consists of two main parts

- (1) the assessment of risk, and
- (2) risk management.

The assessment of risk is mainly based on modelling of the physical system, including forecasting its evolution under risk. Although the main objective of risk

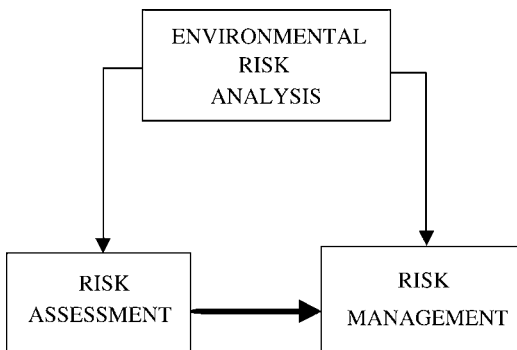


Figure 1.11 Risk assessment and risk management as elements of environmental risk analysis.

analysis is the management of the system, it is not possible to do this if risk has not been quantified first.

The risk assessment phase involves the following steps:

- Step 1: Risk or hazard identification
- Step 2: Assessment of loads and resistances
- Step 3: Uncertainty analysis
- Step 4: Risk quantification

When it is possible to assess risk under a given set of assumptions, the process of risk management may begin.

The various steps needed for risk management are:

- Step 1: Identification of alternatives and associated risks
- Step 2: Assessment of costs involved in various risk levels
- Step 3: Technical feasibility of alternative solutions
- Step 4: Selection of acceptable options according to the public perception of risk, government policy and social factors
- Step 5: Implementation of the optimal choice

Because of the human and social questions involved, risk management is the most important part of the whole process and is also the most difficult to develop. From an engineering point of view, theories and algorithms of optimisation under uncertainty, multi-criterion optimisation and decision making under risk are all applicable.

Apart from the stochastic approach, the fuzzy set theory may be applied. Although the use of fuzzy sets in sequential decision-making was formulated more than two decades ago by Bellman and Zadeh (1970), no realistic application has yet been put forward for hydrology and water resources engineering. The reason for this may be that a solid background in both fuzzy set theory and water resources engineering is required. However the calculations are much simpler than in classical dynamic programming, as applied for example, in reservoir operation or groundwater management. It is quite natural to describe extreme events (especially droughts) as fuzzy or hybrid numbers.

Here we may distinguish between

- (a) design or planning problems, and
- (b) operational problems.

In *planning* problems, a set of discrete alternatives (also called options, actions, schemes, decisions, etc.) is defined and rated with a set of *performance indices* or *figures of merit* which are usually deterministic, stochastic or fuzzy. These usually also include non-numerical criteria. The recommended multi-objective procedure (Teclé *et al.*, 1988) consists of

- (1) defining the type of objectives-specifications-criteria-preferences (weights, scales, etc.);
- (2) defining the alternatives, which should be distinct, not just marginally different;
- (3) using the input uncertainty characterisation/quantification to rate each alternative in terms of the selected criteria, including risk-related figures of merit. Using value functions to quantify the non-numerical criteria;

- (4) selecting at least one multi-criterion decision-making technique to rank the alternatives;
- (5) evaluating the ranking, for example by comparing the results of application of different techniques (Duckstein *et al.*, 1991), and performing a sensitivity analysis.

The trade-off between criteria or indicators may be carried out in a hierarchical manner: at first, ecological indicators may be traded off to yield a composite index of risk, and then the economic indicators may be followed to yield a composite cost index. At the second stage the two composite indices are then traded off. At each level, fuzzy numbers may be combined or compared, using for example, a measure of the distance between them.

In *operational* problems, it is customary to include only numerical criteria and use a sequential decision making scheme such as dynamic programming (Parent *et al.*, 1991).

Taking into account the above considerations, the presentation of material in this book is arranged along two main paths. The first comprises the main elements of risk analysis, which rank from the simpler to the more advanced such as

- (1) identification of hazards,
- (2) risk quantification, and risk management.

The second path follows the traditional engineering approach, that is

- (1) analysis of inputs,
- (2) modelling, and evaluation of outputs.

This book then combines the two paths. The material illustrates recent concepts and techniques of risk and reliability analysis in water resources engineering planning and management with emphasis on water quality problems.

1.6

Aim and Organisation of the Book

Risk and reliability is still a relatively new subject as applied to water resources and environmental engineering. As a contribution to understanding and using this powerful tool, the present volume aims to serve as a textbook. This is the main reason for having included numerical examples of applications, questions and problems on the various topics covered by the book and also characteristic case studies, illustrating the use of risk assessment techniques in environmental water quality. The general framework of engineering risk analysis is presented from the traditional engineering (conceptual modelling) rather than from the systems engineering point of view. Methods and tools from stochastic modelling and fuzzy set theory are applied to environmental problems related to water pollution.

In May 1985, a NATO Advanced Study Institute (ASI) was held in Tucson, Arizona, USA for the purpose of classifying various concepts of engineering reliability and risk

in water resources. In the edited volume of the meeting (Duckstein and Plate, 1987) a general systems engineering framework is provided for the calculation of engineering risk and reliability in water resources. Reliability investigations are presented in two groups: reliability in hydraulic structures and reliability in water supply systems. In the same volume, the last chapter is devoted to decision making under uncertainty and under multiple objectives.

Six years later (May 1991), another NATO ASI was held in Porto Carras, Greece. This ASI may be considered as a continuation and extension of the initial endeavour. The main purpose was to provide a unified approach to risk and reliability in both water quantity and water quality problems, reflecting at the same time concepts and techniques that have emerged recently. The book published after the meeting (Ganoulis, 1991c), illustrates the steps to be followed in a systematic framework for the analysis and management of risks in water resources engineering problems. Methods and tools for risk quantification and management, recent developments, new techniques and case studies are presented for risk-based engineering design in water resources and water pollution problems. Whilst less importance is placed on structural reliability and standard techniques for reservoir management under 'non-crisis' conditions, more attention is placed on the methodologies for the quantification and management of risks related to a broad spectrum of problems. Such methodologies range from the hydrologic estimation of exceeding probability (Bobée, Ashkar and Perreault, 1991) and the stochastic estimation of pollution risk in rivers (Plate, 1991), coastal waters (Ganoulis, 1991d) and groundwater (Bagtzoglou, Thompson and Doudherty, 1991) to new techniques, such as the 'envelope' approach for dynamic risk analysis (Haines *et al.*, 1991) and the fuzzy set approach (Duckstein and Bogardi, 1991). These techniques appear to be applicable to both scientific and decision making aspects of water resources and environmental engineering.

Although many theoretical developments have occurred in recent years (Morel and Linkov, 2006; Hlavinec *et al.*, 2008), progress made both in the understanding and application of Risk and Reliability analysis in Water Resources and Environmental Engineering remains slow. The main reasons for this seem to be the large amount of data required and the lack of engineers trained to deal with phenomena of a stochastic nature, including optimum cost/benefit decisions under uncertainty. To the author's knowledge, no other textbook is actually available in the current literature that presents the various aspects of risk and reliability in environmental impact analysis and water quality problems in a unified and comprehensive framework.

The purpose of this book is to present in a unified manner, methods and techniques to evaluate the impacts and risks on environmental water quality from alternative water management plans and wastewater or pollutant disposal into the aquatic environment. The book covers uncertainty analysis of water quantity and quality data, stochastic simulation in hydraulic/water resources/environmental engineering, decision theory under uncertainty and case studies. Methods for risk analysis of extremes in hydrology and risk assessment of groundwater, river and coastal pollution are also presented. In this second edition, questions and numerical exercises are added at the end of each chapter and information to help answer these questions and resolve the numerical applications are given in Appendix C.

This book may be of interest to engineers (civil, chemical and environmental), hydrologists, chemists, biologists, graduate students, researchers and professionals working on the issues of environmental water quality.

The assessment of risk in water resources problems should be based on the proper identification of the particular situation. This means that the most significant loads, parameters and boundary conditions of the problem should be identified, together with uncertainties which may give rise to a risk of environmental threat. This process is that of *risk identification* and it is analysed in Chapter 2. Two main methodologies have been developed so far for uncertainty analysis, namely the probabilistic approach and the fuzzy set theory. Basic concepts and the main rules for stochastic and fuzzy calculus are presented in this chapter, together with illustrative examples mainly taken from water quality and water pollution applications.

Risk assessment may be accomplished by the *quantification of risk*. This is very important for engineering applications and forms the background for risk management. Methods and techniques to quantify risks, not only in water resources but in a broader area of engineering, are presented in Chapter 3. In this chapter 'loads' and 'resistances' are described either as stochastic or as fuzzy variables. With the exception of some simple cases, where direct calculation of risk is possible, the environmental system is usually modeled by means of either the stochastic or the fuzzy set approach. The general frameworks for stochastic and fuzzy modelling are described in Chapter 3 and methodologies, such as the Monte Carlo simulation, are also illustrated for risk quantification.

Chapter 4 deals more specifically with the *risk assessment* of water pollution. The assessment of pollution risks in coastal, river and aquifer systems is analysed by appropriate mathematical modelling, describing transport, dispersion and physico-chemical reactions of the pollutants. To quantify uncertainties due to different variabilities such as advection, dispersion and initial conditions the random walk simulation is used.

Chapter 5 deals with *risk management*. Here the risks have been identified and, as far as possible, quantified. Various criteria are defined to characterise risk, including performance indices related to the effects of uncertainty. Some of these criteria may be probabilistic or fuzzy. In any case, risk management provides the means with which to investigate the mitigation of the consequences of risks. For this purpose, trade-offs may be made at increasingly high levels between the various risk indicators. For example at one level an environmental risk index may be traded off against a technical risk index, and at a higher level an overall risk index may be traded off against an overall economic risk index.

A very important demonstration of how risk analysis can be useful when facing new and challenging problems, such as the implications in engineering works from possible coastal pollution and eutrophication due to climate change, is given in Chapter 6, with the case of the Gulf of Thermaikos (Macedonia, Greece). Some other characteristic risk-related *case studies* are presented in this chapter, namely the coastal pollution in the Gulf of Thermaikos and its interactions with the optimum design of the sewage treatment plant of the city, the

risk assessment of pollution with nitrates from the river Axios (Macedonia, Greece) and the groundwater salinisation of the Campaspe aquifer (Victoria, Australia).

1.7

Questions and Problems – Chapter 1

Water Pollution and Risk Analysis

A Systemic View of Water Resources

- (a) What percentage of the Earth's surface is covered by oceans?
 - (b) What percentage of the total water on Earth is fresh water?
 - (c) What percentage of the total freshwater on Earth is surface water?
 - (d) What percentage of the total freshwater on Earth is groundwater?
 - (e) Explain why water is the most valuable resource on Earth.
 - (f) What is the main characteristic of the hydrological cycle?
 - (g) Define efficient precipitation.
 - (h) Is the water balance equation generally valid and what are its limitations?
- (1) In a catchment area of 0.7 Gm^2 , the mean annual rainfall is 670 mm and the correspondent evapotranspiration 520 mm. (a) Assuming negligible storage, determine the total runoff (surface and underground) in mm and km^3 ; (b) determine the catchment's annual water budget in km^3 ; (c) calculate the evapotranspiration rate as a percentage of the rainfall. What can you conclude if you compare this to the global average percentage evapotranspiration rate?
 - (2) The surface area of a reservoir is $0.9 \times 10^6 \text{ m}^2$. The average inflow is $0.15 \text{ m}^3/\text{s}$ and the mean annual evaporation rate is 1500 mm/year. Calculate: (a) the daily evaporation rate in mm and m^3 ; (b) the change in storage per year in mm and m^3/year . Does the storage capacity increase or decrease? (c) The time needed to raise the water level by 1 m.

The New Paradigm of Water Quality

- (a) How are water quantity and water quality interrelated?
- (b) What is the meaning of the new water quality paradigm?
- (c) How is biological assessment a useful tool for assessing the status of water quality?

Integrated Water Resources Management (IWRM)

- (a) What is the best space scale at which IWRM should take place?
- (b) What are the main reasons for adopting the IWRM process?
- (c) Give examples of the benefits of using IWRM.
- (d) Do you consider water to be a human right or a human need?
- (e) In your opinion should water be considered as a commodity or a social requirement?

Water Pollution in Transboundary Regions

- (a) How can 'fair' water allocation be implemented in a transboundary river basin?
- (b) Equity and efficiency are notions closely related to water allocation. What are the parameters that should be taken into account in order to achieve equity and efficiency?
- (c) Should higher value uses of water take priority over lower value uses?

The EU Water Framework Directive

- (a) What is the definition of 'good status of water' in the EU WFD?
- (b) How can 'good water status' be achieved?
- (c) Enumerate the key elements of the EU-WFD.

Uncertainties in Water Resources Management (WRM)

- (a) What are the causes of uncertainties in WRM?
- (b) What is the relationship between uncertainties and risk?
- (c) Describe at least four different types of uncertainties in WRM.

Environmental Risk Assessment (ERA) and Environmental Risk Management (ERM)

- (a) What is the difference between ERA and ERM?
- (b) Describe at least four steps necessary to achieve ERA.
- (c) Describe at least five steps necessary to achieve ERM.

2

Risk Identification

The first step in engineering risk and reliability analysis is to identify different situations in which uncertainties can generate risk of failure. Risk identification is essential when formulating problems involving uncertainties. Uncertainties are inherent to natural processes (aleatory uncertainties or randomness) and are also associated with data, physical variables, parameters and boundary conditions (epistemic uncertainties or imprecision).

Take as an example the case of coastal eutrophication. This is related to the abnormal growth of algae leading to different adverse environmental consequences, such as (i) reduction of water transparency in the euphotic zone near the free surface of the sea, (ii) decrease of solar penetration and subsequent changes in chlorophyll concentration below the euphotic depth, (iii) uptake of nutrients and oxygen production in the euphotic zone, and (iv) settling of algae in deeper waters and oxygen reduction in the oligotrophic, deeper water layers. First, different kinds of uncertainties should be investigated in relation to various relevant processes such as coastal circulation, turbulent dispersion and complex interactions with nutrients and other physical and chemical elements. The complex phytoplankton kinetics are also affected by temperature fluctuations on diurnal, seasonal or long-term scales. Depending on the specific algal type (i.e. diatoms, dinofragellates, green or blue algae), the phytoplankton growth rate generally increases with temperature. Incidences of high temperature in spring or summer, together with moderate winds and weak water circulation may produce eutrophication and algal blooms and normally lead to oxygen depletion. This could affect the biological and chemical equilibrium in coastal waters and induce detrimental risks to marine organisms and fishes consuming oxygen.

Not only is it important to identify different situations involving risks, but also the uncertainties generating these risks need to be analysed, so that the risks involved may be quantified. Only with this information may adverse consequences be properly managed.

This chapter starts by defining risk. Although a very broad description of the notion of risk can generally be agreed upon, the particular definition of risk depends on the socio-economic context, the particular scientific discipline and also on the methodology used for analysing uncertainties. From a mathematical point of view, two basic methodologies are used:

- (1) the probabilistic approach, in which risk is defined as a function of probabilities, both of failure and consequences, and
- (2) the fuzzy set theory, in which fuzzy numbers are introduced to define risk.

Basic theoretical notions of probability and fundamentals of fuzzy set theory are briefly reviewed in Appendices A and B. The main theoretical features are presented and several numerical applications are discussed in order to better illustrate the theory.

After classifying various types of risks, uncertainties related to environmental water quality and related risks are also described in this chapter. Critical pollutant concentrations, leading to risks of water pollution are introduced by means of environmental quality standards that are presented and discussed at the end of the chapter.

2.1

Definition of Risk

The Society of Risk Analysis (SRA) is a multidisciplinary association of scientists based in the US (www.sra.org) with regional chapters in several parts of the US and the rest of the world (e.g. SRA-Europe, www.sraeurope.org). One of the SRA's first actions was to establish a special committee with the task of defining the word 'risk'. After 4 years of deliberation, the committee concluded that it was better not to come out with just one definition of risk. It recommended that every author use his own definition, provided that a clear explanation of its meaning was given (Kaplan, 1997). According to the Merriam-Webster's Online English Dictionary, risk is defined as *the possibility of loss or injury*.

Risk has different connotations and interpretations depending on the socio-economic context and the historical developments of specific scientific disciplines. Different societies have developed their own perceptions, beliefs and modalities to interact with uncertainties, to manage unforeseen incidents and to deal with potential losses.

It is interesting to follow the historical development of the etymology of the word 'risk' (Cline, 2004). In ancient Greece it was commonly believed that everyone's destiny was pre-ordered by fate, however both humans and gods could influence the future by taking initiatives involving dangers and chances. In Homer's rhapsody of Odyssey 'Sirens, Scylla and Charybdee' the verb '*peirao*' was used to describe how Odysseus tried to save himself when heavy seas conjured up by the god Zeus destroyed his ship. The word '*peirao*' meaning in Greek 'to attempt, to try to do' was translated in Latin by the word '*periculum*' meaning 'a passage through, experiment, proof, danger, which comes from an attempt'. *Periculum* evolved into the Latin word '*resicum*' in the fourteenth century (Andrews, 1879) again meaning danger, venture and then into the Italian word '*riscare*', which means *to endanger*. At the same time this verb took on the double meaning; taking a risk means both the possibility of loss and also the possibility of benefit or opportunity, hence 'nothing ventured, nothing gained'. In 1611 the French word '*risque*' was defined as 'danger, chance,

adventure'. In 1654 the French mathematician Blaise Pascal introduced the background of what is known as probability theory (Appendix A) and completely changed the way we deal with uncertainty.

Engineering risk (Duckstein and Plate, 1987), environmental, ecological or health risks may have different verbal or mathematical expressions. A distinction should be made between hazard, risk, reliability and vulnerability.

- *Hazard*: means the potential source of harm. The term evolved from the Arabic 'al zahr', meaning the dice. The negative connotation and the associated notion of danger come in Western Europe, from games first learned during the Crusades in the Middle East, where the use of loaded dice usually resulted in losses.
- *Risk*: the possibility of adverse effects like loss and injury caused by exposure to a hazard.
- *Reliability*: the probability of the system being able to perform under a given hazard.
- *Vulnerability*: denotes the susceptibility of the system to cope under a given hazard.

The vulnerability of a system may be estimated by measuring the *possible degree of damage* to it or the *severity of the consequences* if a given incident occurs, for example a natural hazard (flood, earthquake, tsunami, etc.).

If we consider the first definition of risk as being *the possibility of losses* (injuries, deaths, economic losses, environmental damages), we may see that two elements are essential for describing risk: (i) the severity of the hazard and (ii) the susceptibility of the system to sustain the hazard. This is why risk may be defined by the following expression (Living with Risk, 2004):

$$\text{RISK} = \text{HAZARD} \times \text{VULNERABILITY}$$

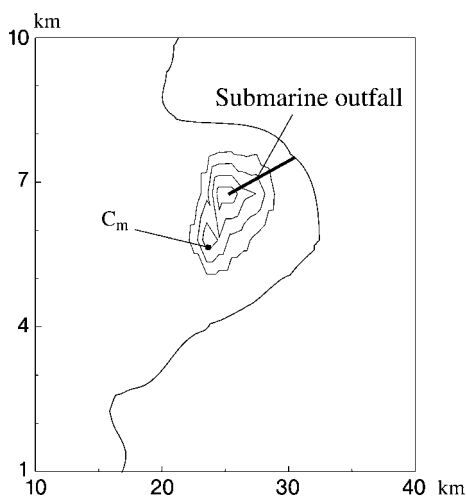
Table 2.1 summarises some disciplinary definitions and mathematical estimations of risks.

- In economic sciences risk is related to possible economic losses. The expected economic loss or the variance of loss may be used to estimate the economic risk.
- In social sciences the possible loss of revenues and the expected number of jobs lost are referred to.
- In public health we talk about the number of people infected per thousand or million of population.
- In environmental sciences we may distinguish between a-biotic and biotic or ecological environment, as shown in Table 2.1.
- In engineering sciences the probability of an incident and the related consequences are taken into account.

The risk of water pollution may be described by means of a characteristic variable expressing the state of water quality. According to the new paradigm of 'good status of water', which was adopted in the EU WFD 2000/60, the quality of a water body, such as a river, lake, coastal area or a groundwater aquifer, is described not only in terms of the concentration of specific physico-chemical substances but also by biological indices indicating the status of the aquatic ecosystems. For example, the

Table 2.1 Risk definition and risk estimation in different scientific disciplines.

Discipline	Risk definition: possible loss of ...	Risk estimation
Economy	Money, capital, investment	Expected loss: $E[x < 0]$ or deviation from target t : $t - E[x < t]$
Social sciences	Revenues, jobs, social cohesion	Expected loss of revenues, possible number of jobs lost
Public health	Lives, human health	Number of deaths or casualties per million of the population
Ecology	Species	Biodiversity index, Eco-integrity characteristics
Environmental (a-biotic)	Air, water or soil quality characteristics	Expected deviation from defined quality standards, possible consequences
Engineering	Technical security, reliability	Probability of incident, possible consequences

**Figure 2.1** Two-dimensional field of pollution concentration at the sea surface from a submarine outfall.

maximum pollutant concentration of heavy metals or coliform bacteria at a specific site may be selected as a characteristic variable. This is a function of various conditions, such as wastewater loadings, flow characteristics, kinetic transformations and ecosystem distribution controlling the water quality.

Consider the case of a submarine outfall discharging wastewater to a coastal area (Fischer *et al.*, 1979; Ganoulis, 1991c). As shown in Figure 2.1, at the sea surface

a two-dimensional field of pollutant concentration may be measured, corresponding to the chemical composition of the pollutants. The latter could be nitrates, phosphates, phytoplankton or coliform bacteria, found in variable concentration in municipal wastewaters with or without preliminary treatment. The maximum concentration of a specific pollutant C_m may be chosen to indicate some of the adverse effects produced in the water environment.

Depending on the specific use of seawater, for example bathing, oyster farming, fishing and so on, environmental quality standards specify the *maximum allowed concentration* in seawater C_0 . This concentration should be considered as the *capacity* of the marine ecosystem to sustain pollution.

A critical condition producing adverse effects and risk for seawater pollution occurs when the maximum pollutant concentration C_m exceeds the receiving capacity of the system, that is when $C_m \geq C_0$. Otherwise the system is safe, as far as the pollution is concerned. So we have

$$\text{FAILURE or POLLUTION : } C_m \geq C_0$$

$$\text{SAFETY or RELIABILITY : } C_m < C_0$$

As explained in Ganoulis (1991b) we should define as *load* ℓ a variable reflecting the behaviour of the system under certain external conditions of stress or loading. There is a characteristic variable describing the capacity of the system to overcome an external load. We should call this system variable *resistance* r . There should be a *failure* or an *incident* when the load exceeds this resistance, that is

$$\text{FAILURE or INCIDENT : } \ell \geq r \quad (2.1)$$

$$\text{SAFETY or RELIABILITY : } \ell < r \quad (2.2)$$

Loads and resistances are terms used in structural engineering. In the field of water resources engineering and environmental water quality these two variables have a more general meaning, as is explained in Table 2.2. Failure or safety of the system may also be considered in relation to the consequences of failure, such as loss of lives or economic damage.

To illustrate the concepts of loads, resistances, failures, incidents and consequences of failure, a hydraulic structure is considered as an example. Let us examine the safety of an earth dam, shown in Figure 2.2. The question is how to determine safely the height of the dam, when the level of water in the reservoir fluctuates according to the local hydrologic conditions. In this particular example, the load ℓ is the height of water in the reservoir h and the resistance r of the system is the height of the dam H . There should be an incident or a failure when, because of a flood, the value of h becomes greater or equal to H , that is

$$\text{FAILURE CONDITION : } h \geq H$$

on the contrary, the design will be successful if we have

$$\text{SAFETY CONDITION : } h < H$$

Table 2.2 Examples of ‘loads’ and ‘resistances’ in water resources engineering.

Physical system	Scientific discipline	‘Load’	‘Resistance’	Type of failure
Hydraulic structure (gate, dam, flood levee, ...)	Civil and hydraulic engineering	<ul style="list-style-type: none"> • Force, stress • Wind load • Flood rate 	<ul style="list-style-type: none"> • Resisting stress • Dam height 	<ul style="list-style-type: none"> • Structural failure
Water system (lake, aquifer, river, coastal area, ...)	Water resources and environmental engineering	<ul style="list-style-type: none"> • Water demand • Pollutant load • Pollutant concentration 	<ul style="list-style-type: none"> • Water supply • Reservoir capacity • Receiving capacity 	<ul style="list-style-type: none"> • Water shortfall • Water pollution
Water system	Hydrology	<ul style="list-style-type: none"> • Flow rate • Flood • Rainfall 	‘T-years’ <ul style="list-style-type: none"> • Flow rate • Flood • Rainfall 	<ul style="list-style-type: none"> • Hydrologic exceedance • Floods
Ecosystem	Biological sciences	<ul style="list-style-type: none"> • Exposure 	<ul style="list-style-type: none"> • Ecosystem capacity 	<ul style="list-style-type: none"> • Ecosystem damage
Human organism	Health sciences	<ul style="list-style-type: none"> • Exposure 	<ul style="list-style-type: none"> • Human capacity 	<ul style="list-style-type: none"> • Health damage

From the previous example we can see that the terms ‘load’ and ‘resistance’ may take on different meanings, depending on the specific problem being faced. They can mean pollutant concentrations, water heights, capacity to receive pollution or the height of a dam. Table 2.2 summarises different definitions of ‘loads’ and ‘resistances’ according to the physical system they are applied to, in the context of Water Resources Engineering.

What is important, as for example for the earth dam shown in Figure 2.2, is to take into account the consequences of an incident or failure. As shown in Figure 2.2, in case of failure, a flood wave will be generated and propagated downstream (Ganoulis, 1987). The loss of property and perhaps human lives caused by the flood are important parameters, which need to be taken into consideration during the design of the dam.

The consequences of failure together with the perception of risks by the public are considered in the management of risks. This is the step following the identification and quantification of risks and will be developed in Chapter 5.

In a typical problem of failure under conditions of uncertainty, we usually face three main questions which are addressed in three successive steps:

STEP 1: WHEN SHOULD THE SYSTEM FAIL?

STEP 2: HOW OFTEN IS FAILURE EXPECTED?

STEP 3: WHAT ARE THE LIKELY CONSEQUENCES?

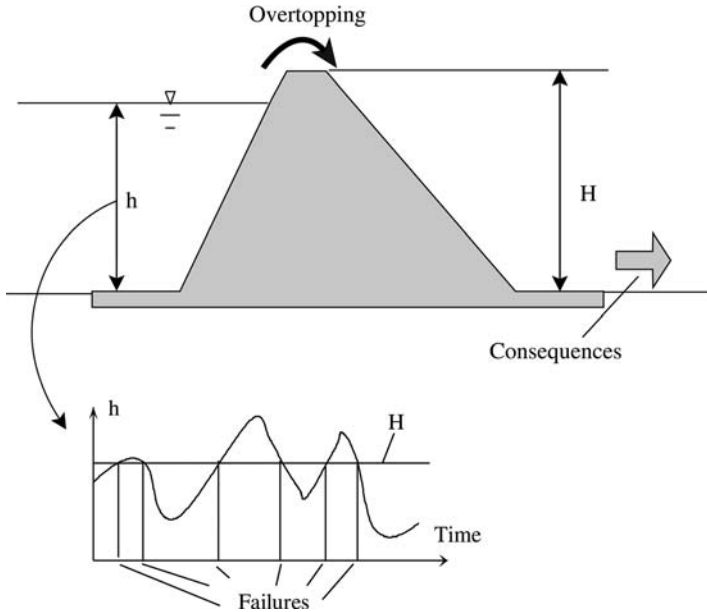


Figure 2.2 Risk of failure of an earth dam under hydrologic uncertainty conditions.

The two first steps are part of the uncertainty analysis of the system. The answer to question 1 is given by the formulation of the critical condition in Equation 2.1, that is, when the load ℓ exceeds the resistance r of the system.

To provide an adequate answer to question 2 it is necessary to consider the variables of the problem, such as the load ℓ and resistance r , as non-deterministic. When time is also taken into consideration, we are then referring to *unsteady* or *time-dependent* risk and reliability analysis. When loads and resistances are considered constant at time t , we are then referring to *static* risk and reliability analysis. In a probabilistic framework, ℓ and r are taken as *random* or *stochastic* variables (see Appendix A). In probabilistic terms, the chance of having a failure or the probability of failure is generally considered as a first definition of risk. For example, in the case of the dam in Figure 2.2, we will have

$$\text{RISK} = \text{probability of failure} = P(h \geq H)$$

$$\text{RELIABILITY} = \text{probability of success} = P(h < H)$$

More generally, risk is a function of the probability of failure and the consequences. Risk is usually taken as the ‘mean’ or ‘expected value’ of consequences or damages expressed by the product of probability and its consequences, that is

$$\text{RISK} = (\text{Probability of event } i) \times (\text{Consequences of event } i)$$

The risk associated with a number of events is

$$\text{RISK} = \sum_{\text{all } i \text{ events}} (\text{Probability of event } i) \times (\text{Consequences of event } i)$$

When the consequences are damages denoted by D , the risk may be generally expressed as

$$\text{RISK} = (\text{Probability}) \times (\text{Damage}) = P \times D$$

If fuzzy logic is used (see Appendix B), ℓ and r are considered as fuzzy numbers, then risk and reliability are defined by means of appropriate fuzzy measures, which will be introduced later on.

2.2

Typology of Risks and the Precautionary Principle

Hazards and risks, depending on their origin, may be classified in two main categories

- *Natural risks* are those occurring in the biosphere and caused by natural events inducing dangers (natural hazards). Natural risks may have one of the following origin
 - Hydro-meteorological (Atmospheric, Hydrological or Oceanographical)
 - Geological (Endogenous or Tectonic and Exogenous)
 - Biological
- *Anthropogenic or man-made risks* are related to human activities like industrial accidents, infrastructure failure, economic or social disruption and other man-related hazards.

Table 2.3 summarises the typology of risks and provides some characteristic examples (adapted from Living with Risk, 2004).

2.2.1

Unacceptable versus Acceptable Risks

Due to the severity of the damage caused, most of the risks indicated in Table 2.3 are *unacceptable or catastrophic*. In this case, if the hazards are natural we are talking about *natural disasters*. *Acceptable risks* may be taken when the consequences are limited, which means affordable by the human community. This is the case for the design of storm sewers in inhabited areas, which are calculated for evacuating floods of return periods of 2 or 5 years. This means that the risk of flooding once every 2 or 5 years is acceptable, because the consequences, maybe only a few millimeters of water in the streets, are acceptable. Otherwise the cost of choosing sewers larger in diameter would increase the cost tremendously, while the reduction of risk produces a negligible benefit. In the case of acceptable risks, the main question is who decides which risks may be considered as being affordable, and how is that decision reached.

Table 2.3 Types and examples of risks.

	Origin of risk	Examples
Natural	Hydro-meteorological	<ul style="list-style-type: none"> • Floods, debris and mudflows • Tropical cyclones, storm surges, wind, rain and lightning • Drought, desertification, forest fires, temperature extremes, sand or dust storms • Permafrost, snow avalanches
	Geological	<ul style="list-style-type: none"> • Earthquakes, tsunamis • Volcanic eruptions • Landslides, submarine slides
	Biological	<ul style="list-style-type: none"> • Epidemic diseases • Plant contagion • Animal infection
Man-made	Technological	<ul style="list-style-type: none"> • Industrial explosion • Infrastructure failure • Dam failure
	Socio-economic	<ul style="list-style-type: none"> • Bankruptcy • Stock market disruption • Social unrest
	Terrorist attacks	<ul style="list-style-type: none"> • Bomb explosion • Kidnapping

2.2.2

Controllable versus Uncontrollable Risks

Depending on the intensity of the hazard and the severity of the consequences, risks may be classified as controllable or not. Structural measures like dams and flood levees are usually built in order to reduce and control the risk of flooding. However not all risks are controllable (e.g. strong tsunamis) and most of the time the question is how much will barriers and other structures to control risks cost.

2.2.3

Gradual versus Sudden Risks

Depending on the time-frame, risks may be classified as gradual, sudden or abrupt. For example it may take several days for a river to flood its banks, but an earthquake takes only a few seconds.

If the risk is defined by the *product of probability P and the consequences or damages D* , then in the two-dimensional representation shown in Figure 2.3, curves of equal risk are of hyperbolic form $P \times D = \text{Constant}$.

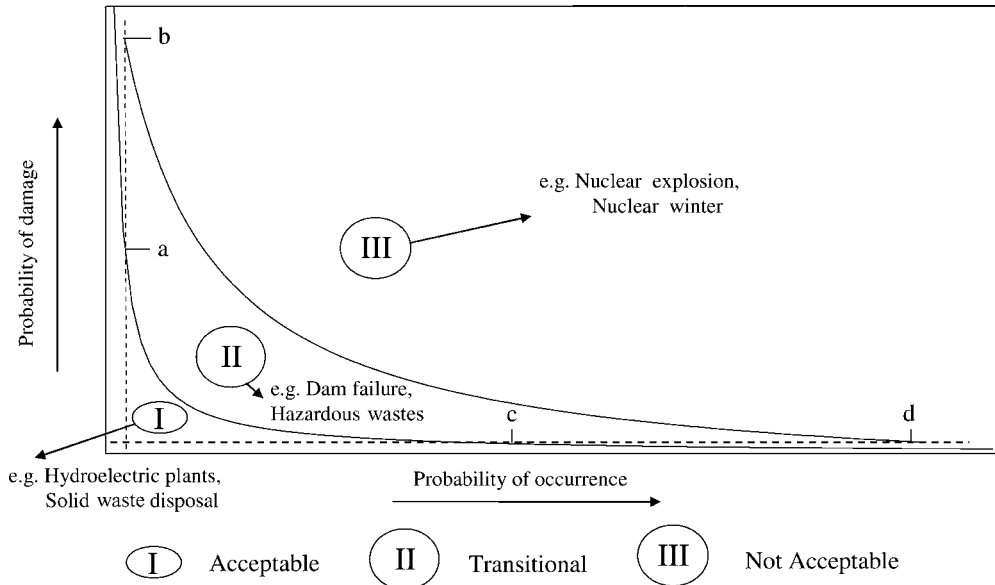


Figure 2.3 Acceptable, transitional and unacceptable risks.

The product form of risk, $P \times D$ is not valid for very small or very high values of P and D . As shown in Figure 2.3 domains of acceptable, transitional and unacceptable risks may be defined.

2.2.4

The Precautionary Principle

The main question for all kind of risks is about the policy that should be developed in order to avoid or reduce the risks. Besides Risk Analysis (RA), which is the scientific discipline for assessing and managing risks, it is useful to know about the so-called 'precautionary principle' and under which circumstances this principle should be used.

Reference is made to the Rio declaration on Environment and Development (Rio Declaration, 1992) where it is stated that:

Where there are threats of serious or irreversible environmental damage, lack of full scientific certainty shall not be used as a reason for postponing cost effective measures to prevent environmental degradation.

There is no universally accepted definition of the precautionary principle. The above statement from the Rio declaration on environmental risk may be extended to any risk, such an economic, social or ecological risks. Considering the risk as the product of (probability) \times (the consequences), scientific knowledge about these two components of risk is synonymous with the degree of certainty

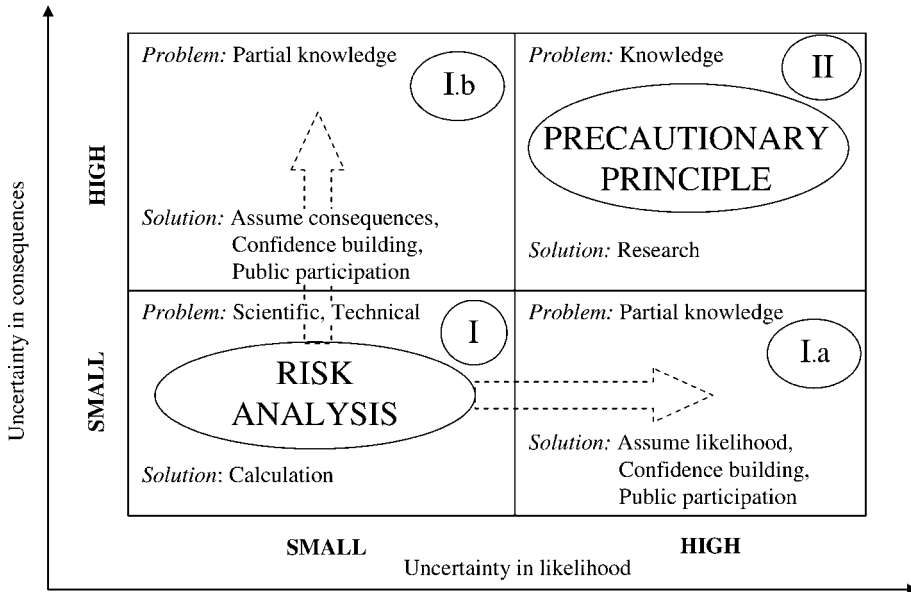


Figure 2.4 Domains of application of risk analysis and the precautionary principle.

in defining them. As our ignorance or the uncertainty in probability and the consequences increase, as shown in Figure 2.4 we move from the scientific domain of Risk Analysis to the domain of application of the precautionary principle (Figure 2.4).

In terms of policy to be applied in order to mitigate different risks, the following logical diagram shown in Figure 2.5 may be used in practice. We can distinguish the cases where risk analysis may be used as an analytical tool from those where our limited knowledge on risks makes use of the precautionary principle for deciding on alternative preparedness measures.

When risk analysis is applied, risk management which follows the assessment of risks becomes the tool for adopting the best prevention measures. Risk management is analysed in Chapter 5.

A residual or unexpected risk, which has a very small but never zero probability of being realised is always possible. In this case restoration and rehabilitation plans should be developed in advance and effectively applied in case of unexpected natural or technological disasters (e.g. the 1986 Chernobyl nuclear explosion in Europe or the August 2005 hurricane Katrina and the resulting catastrophic floods in New Orleans, USA).

2.3

Uncertainties in Water Pollution Problems

In the disposal of effluents and pollutant substances in the water environment, the risk of water contamination is subject to several types of uncertainty. These are caused

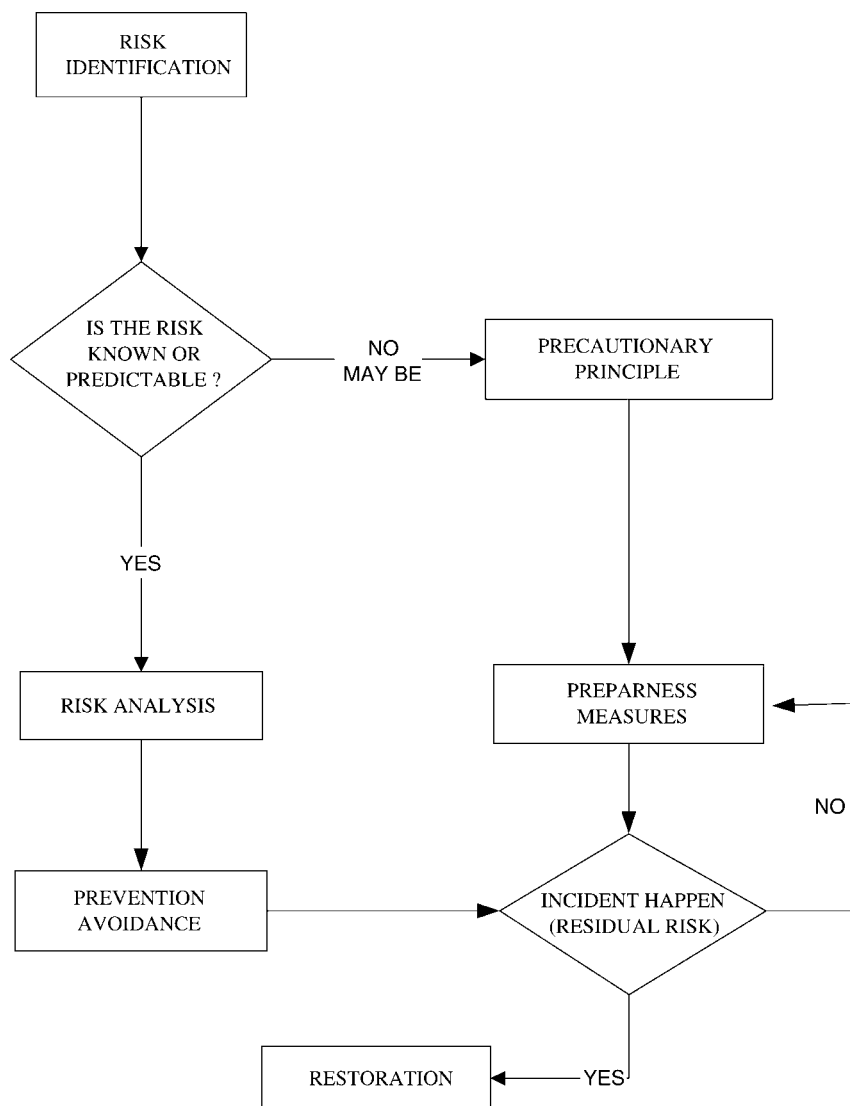


Figure 2.5 Flow diagram for risk policy.

by the high variability in space and time of the hydrodynamic, chemical and biological processes involved.

Actually, uncertainties are due to the lack of knowledge about the structure of various physical and biochemical processes and also to the limited amount of data available (Bogardi and Duckstein, 1978; Duckstein and Plate, 1987; Plate, 1991; Ganoulis, 1991c). Several authors have analysed different types of uncertainties

and made various distinctions, such as between objective and subjective, basic and secondary, natural and technological uncertainties.

Distinction should be made between

- (1) aleatory or natural uncertainties or randomness and
- (2) epistemic or man-induced or technological uncertainties.

2.3.1

Aleatory Uncertainties or Randomness

It is postulated that natural uncertainties are inherent to the specific process and they cannot be reduced by use of an improved method or more sophisticated models. Uncertainties due to natural randomness or aleatory uncertainties may be taken into account by using the stochastic or fuzzy methodologies, which are able to quantify uncertainties.

2.3.2

Epistemic or Man-induced Uncertainties

There are various types of man-induced uncertainties: (a) data uncertainties, due to sampling methods (statistical characteristics), measurement errors and methods of analysing the data, (b) modelling uncertainties, due to the inadequacy of the mathematical models in use and to errors in parameter estimation, and (c) operational uncertainties, which are generally related to the construction, maintenance and operation of engineering works. Contrary to natural randomness, man-induced uncertainties may be reduced by accumulating more information or by improving the mathematical model. As we will see later in this chapter, in a Bayesian framework, prior information may be increased into posterior information by use of additional information or data. Alternatively, when data are scarce, the fuzzy set theory may be used to handle and quantify imprecision.

The fate of pollutants in a water-receiving body, such as a river, is influenced by the combination of three mechanisms: (a) advection by currents, (b) turbulent diffusion and (c) chemical, biological or other interactions. As a result, data relating to physical and chemical parameters show high variability in time, as seen in Figure 2.6 for typical time series of water temperature and nitrate concentration.

This is the case not only for water quality parameters but also for hydrodynamic quantities, such as the flow rate of rivers. A typical time series of flow rate measured in a river is shown in Figure 2.7. This figure, as well as the previous one, is based on collected data. They both include natural randomness and imprecision due to data collection, sampling and subsequent laboratory analyses.

An incident or failure is produced if the concentration C of a characteristic substance exceeds the limits given by water quality standards. For example, as shown in Figure 2.8, dissolved oxygen (DO) may fall below 5 mg/l, which is generally accepted as the minimum concentration of DO necessary for the

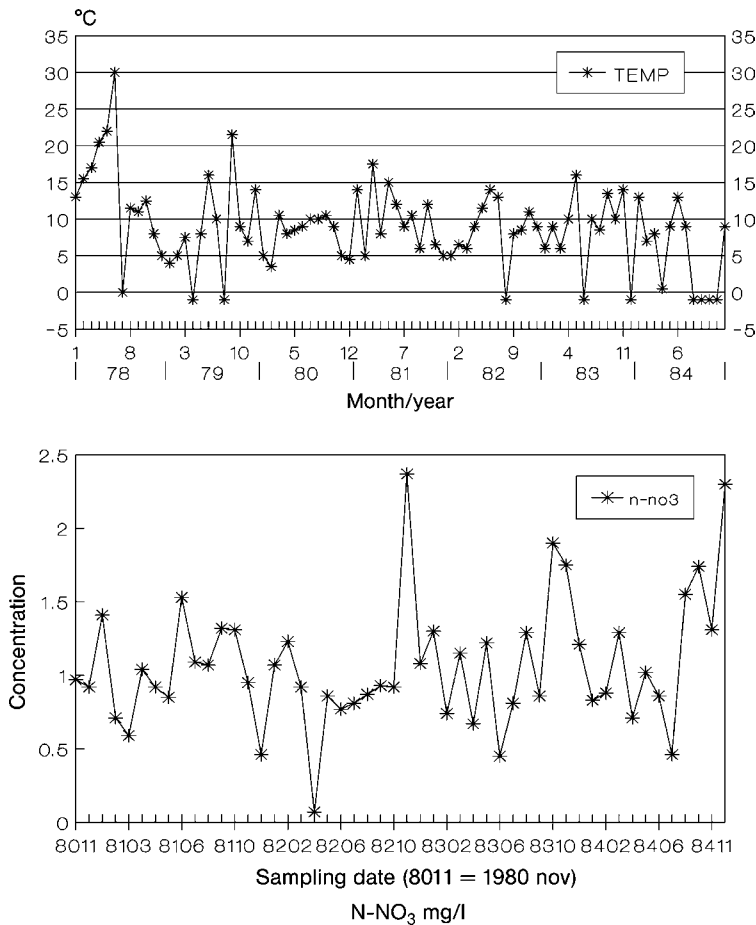


Figure 2.6 Typical time series of water temperature TEMP (°C) and concentration of nitrate-nitrogen N-NO₃ (mg/l) measured in a river.

survival of different marine organisms. Also, values of concentration of total organic carbon (TOC) shown in Figure 2.8 may exceed the maximum allowed concentration.

Risk analysis of water pollution may proceed in the following steps:

- (1) Different types of uncertainties are identified and different scenarios are set, depending on the combination of various kinds of uncertainties (risk identification).
- (2) Conditions involving incidents or failures are identified, that is C smaller or larger than C_0 .
- (3) Risk is quantified under different scenarios, and risk is compared to water quality standards and the reliability of the system is evaluated.

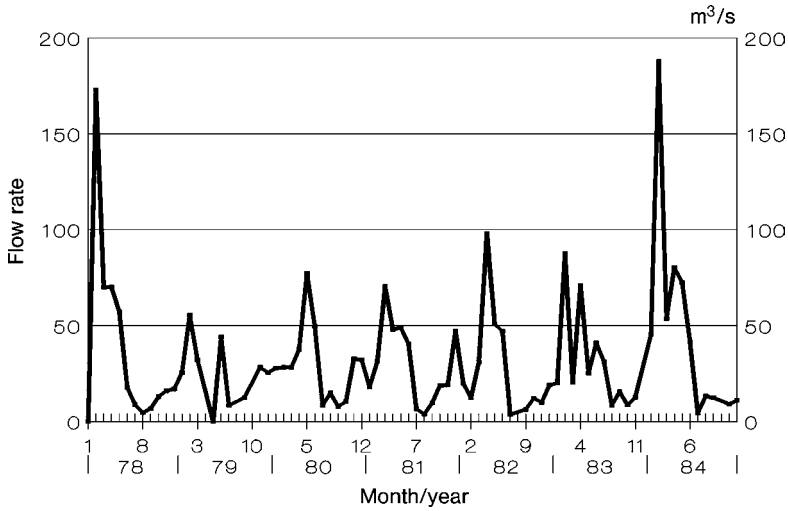


Figure 2.7 Typical time series of flow rate (m^3/s) measured in a river.

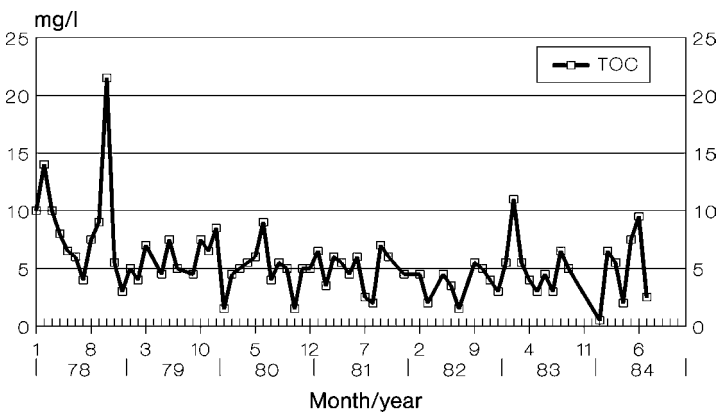
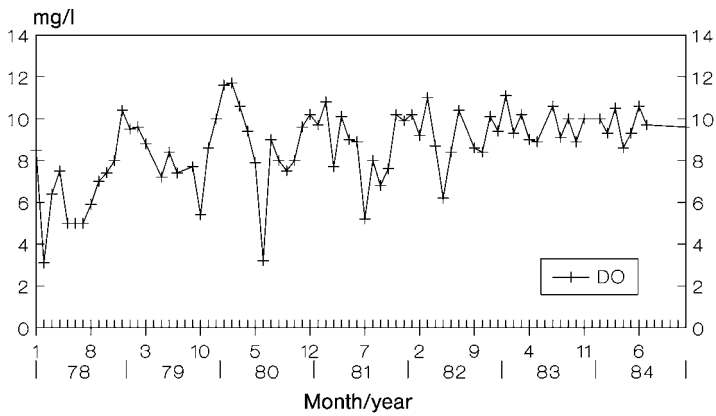


Figure 2.8 Typical time series of dissolved oxygen (DO) and total organic carbon (TOC) (in mg/l), measured in a river.

Some water quality standards, especially within the EC, are given at the end of this chapter. Before that, the two principal methodologies for quantifying risks, that is, the probabilistic approach and the fuzzy set theory are briefly reviewed.

2.4

Water Quality Specifications

Risk in water quality problems is defined in terms of maximum allowed concentration of a given pollutant, as stated by water quality standards. In order to understand the reasons which led to the development of specifications concerning environmental water quality, it is helpful to summarise the elements of the problem.

Efforts to preserve a satisfactory level of quality in fresh and coastal waters need to take into consideration issues concerning public health as well as any ecological risk. The problem may be formulated in terms of the 'maximum receiving capacity' of the water body, in terms of a specific pollutant (e.g. urban sewage), and the aim will therefore be to design a sanitary engineering system configured so that this maximum capacity is not exceeded.

Risks from fresh and marine water pollution also affect human beings, and are incurred primarily from drinking, bathing and consumption of produce from the sea (especially if consumed raw e.g. some shellfish). Therefore, regulations are generally formulated as series of standards concerning 'drinking', 'bathing' and 'shellfish farming', expressed in values of maximum allowable levels of pollutants.

2.4.1

Water Quality Standards

The distinction between environmental and man-related risks from different water uses leads to standards, expressed as the maximum receiving capacity of the water for specific categories of pollutants most frequently discharged into the water environment.

Water quality standards for the protection of freshwater and marine environments have been developed by several countries. Quality criteria for freshwater, bathing water and, to a lesser extent, shellfish-growing water have been issued by these countries, the European Economic Community and also international organisations. Similar criteria are now under development for the protection of waters for other reasons such as ecology, aesthetics or wildlife.

It is evident that schemes for wastewater disposal into the water environment should be designed primarily to protect the beneficial uses in the area affected by the discharge. Therefore, water quality criteria derived from these uses are the principal parameters in the design of sanitary engineering structures (e.g. a wastewater treatment plant or a submarine outfall).

To be useful in the design and computations of such structures, water quality criteria need to fulfil the following basic characteristics:

- (a) The criteria need to be expressed in terms of parameters and values which may be incorporated directly into the design.
- (b) Criteria and parameters should be relevant to the beneficial uses of the water body. They need to be associated with sanitary and ecological consequences, either through a direct cause–effect relationship or through a clearly-stated statistical relationship.
- (c) Criteria should be attainable by normal technical procedures and should take into account the natural baseline concentrations in regional waters.
- (d) Although only average values are mainly used for the design, in order to take into account the natural variability and changes in environmental parameters, water quality criteria should be defined in a statistical form.

To preserve *fisheries* in freshwaters such as rivers and lakes, recommended water quality criteria are given in Table 2.4, according to the EC directives (16/6/75).

In 1985, Mediterranean states adopted interim criteria for bathing waters, based mainly on faecal coliforms, though faecal streptococci constitute an important additional parameter.

Recommended quality criteria for *bathing* water, which can be used as parameters for the design of engineering structures in coastal regions, are listed in Table 2.5.

In the case of *shellfish farming*, criteria and standards in current use are based on bacterial concentrations in the shellfish themselves, as opposed to the actual waters (Table 2.6).

Because of the concentration factor and variations in uptake, no definite correlation has been established so far between concentrations in the actual shellfish and the surrounding water. A recommendation made by WHO and UNEP in 1986 proposed a maximum concentration of 10 faecal coliforms per 100 ml in at least 80% of the samples, and a maximum concentration of 100 faecal coliforms per 100 ml in 100% of the samples. The quality criteria adopted on a joint

Table 2.4 Recommended freshwater water quality criteria for fisheries protection (EC Directive 16/6/75).

Quality criteria	Unit	Desirable limit	Higher allowable limit
Dissolved oxygen	ppm	50% ≥ 8 100% ≥ 5	50% ≥ 7 100% ≥ 4
Suspended solids	ppm	25	
BOD5	ppm	6	
Total ammonia (NH ₄ ⁺)	ppm	0.2 (0.16) ^{a)}	1 (0.78) ^{a)}
Nitrites (NO ₂ ⁻)	ppm	0.03	
Total phosphorus (P)	ppm	3	
Dissolved copper (Cu)	ppm	0.04	

^{a)}Nitrogen ammonia N/NH₄⁺.

Table 2.5 Recommended bathing water quality criteria for design purposes (UNEP/WHO, 1985).

Parameter	Unit	Percentiles		Remarks
		50%	90%	
A. Bacteriological				
1. Faecal coliforms	n/100 ml	100	1000	Bathing
2. Faecal streptococci	n/100 ml	100	1000	Bathing
B. Physical				
3. Colour	mg Pt-Col/l	10	30	a)
4. Suspended solids	mg/l	1.3NV	1.5NV	b)
C. Chemical				
5. Dissolved oxygen	mg/l	6	5	Surface
6. Nitrogen ammonia	mg N/l	0.05	0.12	
7. Dissolved orthophosphate	mg P/l	0.02	0.05	

^{a)} To be observed at the plume surfacing point.^{b)} NV = Normal value in the area before the discharge.

basis by Mediterranean states in 1987 imposed a maximum concentration of 300 faecal coliforms per 100 ml of shellfish (flesh + intervalvular fluid) in at least 75% of the samples.

For the calculation and control of the impact from wastewater disposal, faecal coliforms and faecal streptococci can be considered as non-conservative pollutants subject to exponential bacterial decay. Dissolved oxygen should be evaluated taking into account the oxygen consumption due to bacterial degradation of organic matter. Finally nitrogen ammonia and dissolved orthophosphate should be considered as conservative pollutants, while colour, suspended solids and pH criteria are applied in the upper parts of the rising plume. All these criteria are presented in Tables 2.4 to 2.6, mainly as technical recommendations.

Water quality criteria can also be used as a tool for monitoring domestic wastewater discharges, coastal and freshwater areas and also for the control and evaluation of the efficiency of sanitary engineering works. They are included here only for reference

Table 2.6 Water quality criteria for oyster farming waters (EC Directive 30/10/79).

Quality criteria	Unit	Desirable limit	Higher allowable limit
Dissolved oxygen	% of saturation	≥80%	70–110%
Faecal coliforms (waters)	n/100 ml	<70/100 ml	
(in the flesh)	n/100 ml	≤300/100 ml	

Table 2.7 Normal effluent standards for coastal disposal (UNEP, 1994).

Contaminants	Units	Values for the limits					
		Open areas (%)			Enclosed areas (%)		
		50	90	Maximum	50	90	Maximum
1. Greases and oil	mg/l	25	40	75	5	10	30
2. Settling solids	mg/l	1	1.5	3	2	3	6
3. Turbidity	FTU	75	100	250	50	75	150
4. pH		–	–	6–9	–	–	6–9
5. BOD ₅	mg/l	300	400	600	100	150	250
6. Organic nitrogen ^{a)}	mgN/l	–	–	–	5	7	20
7. Oxidised nitrogen ^{a)}	mgN/l	–	–	–	3	6	12
8. Total phosphorus ^{a)}	mgP/l	–	–	–	1	1.5	3
9. Colour	^{b)}	–	–	1:40	–	–	1:20

^{a)} These limits will be observed in areas where eutrophication is possible.

^{b)} Should not be detected over 10 cm with the indicated dilution more than 10% of the reference value.

and should not replace any existing national standards and regulations when designing sanitary engineering structures.

2.4.2

Effluent Standards

The discharge of raw or pre-treated wastewaters should be restricted to domestic effluents which do not contain high loads of persistent, bio-accumulative or toxic substances. Industrial discharges should always be subjected to treatment before discharge into the water environment.

In order to remain below the receiving capacity of environmental waters, it will normally be sufficient that the conditions mentioned earlier are maintained when considering all discharges into the affected area.

As a further guarantee that the discharge does not exceed the receiving capacity of the marine environment, some basic effluent standards can be applied to medium and large outfalls of cities of more than 50 000 inhabitants. A set of these standards is proposed in Table 2.7. These effluent standards are expressed in a statistical form to allow them to be monitored by the corresponding water authority.

2.5

Probabilistic Risk and Reliability

By considering the system variables as random, uncertainties can be quantified on a probabilistic framework. Loads ℓ and resistances r previously defined are taken as random variables L and R , having the following probability distribution and

probability density distribution functions (see Appendix A)

$$F_L(\ell), f_L(\ell) : \text{load}$$

$$F_R(r), f_R(r) : \text{resistance}$$

There are different definitions of risk in a probabilistic framework. The simpler one is the probability that load exceeds resistance. This is the probability of failure p_F of one component in a steady state system. Risk is given by the following relationship:

$$p_F = P(R \leq L) \quad (2.3)$$

If the empirical definition of risk is considered, then noting

N_F : number of times the system fails

N_S : number of times the system succeeds

$N = N_F + N_S$: total number

p_F is the asymptotic limit, theoretically for $N \rightarrow \infty$, of the ratio

$$p_F = \lim_{N \rightarrow \infty} \left(\frac{N_F}{N_F + N_S} \right) = \lim_{N \rightarrow \infty} \left(\frac{N_F}{N} \right) \quad (2.4)$$

Example 2.1

Every year 200 people die accidentally in the US by electrocution. Assuming that the population of the US is 200 million people, the risk of death by accidental electrocution according to Equation 2.4 is

$$\text{annual risk of accidental electrocution} = \frac{200}{200 \times 10^6} = 10^{-6}$$

Taking 70 years as the average life-time of an individual, we have

$$\text{risk of electrocution during life-time} = 70 \times 10^{-6}$$

So, the *mortality risk*, that is, the probability of somebody dying by electrocution is, following the result obtained above, equal to 70 per million of the population.

The quantity p_F is obtained in terms of the joint probability density function $f_{LR}(\ell, r)$ of the random variables R and L (see Appendix A). As shown in Figure 2.9, the risk p_F may be estimated by integrating the function $f_{LR}(\ell, r)$ above the bisecting line $L = R$.

By performing the integration in the above domain, the following equation is obtained

$$p_F = P(L \geq R) = \int_0^\infty \left(\int_0^\ell f_{LR}(\ell, r) dr \right) d\ell \quad (2.5)$$

This is a general expression for quantifying risk in a probabilistic framework. However, Equation 2.5 seems to be rather difficult to use, because most of the time the joint density probability function $f_{LR}(\ell, r)$ is unknown. Simplifications include the assumption of

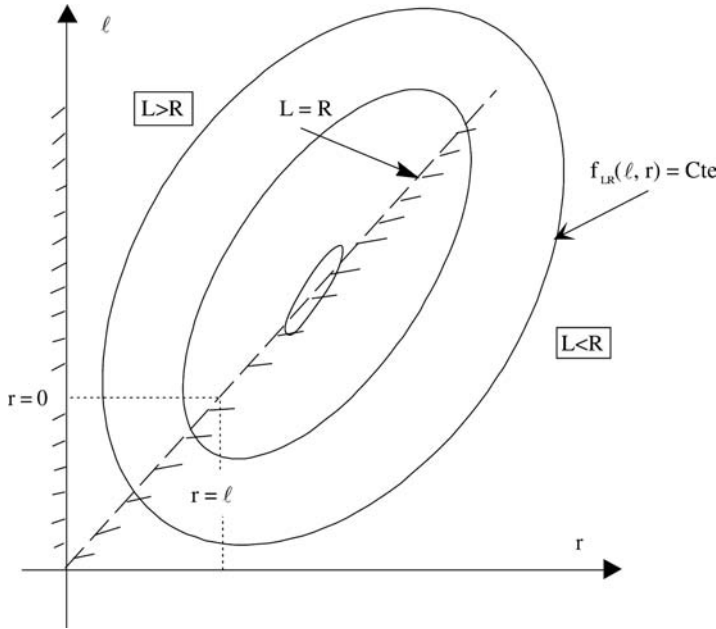


Figure 2.9 Definition of probabilistic risk.

independence between load and resistance, or the case when one of the two variables is deterministic.

2.6

Fuzzy Risk and Reliability

If an event or realisation of a hazard is described by means of fuzzy logic, then the reliability of that event may be calculated as a fuzzy number (see Appendix B). Consider now that the system has a resistance \tilde{R} and a load \tilde{L} , both represented by fuzzy numbers. A *reliability measure* or a *safety margin* of the system may be defined by the difference between load and resistance (Shrestha *et al.*, 1990). This is also a fuzzy number given by

$$\tilde{M} = \tilde{R} - \tilde{L} \quad (2.6)$$

Taking the h -level interval of \tilde{R} and \tilde{L} (see Appendix B) as

$$R(h) = [R_1(h), R_2(h)]$$

$$L(h) = [L_1(h), L_2(h)]$$

then, for every $h \in [0, 1]$, the *safety margin* $M(h)$ is obtained by subtracting $L(h)$ from $R(h)$ that is

$$M(h) = R(h) - L(h), \forall h[0, 1]$$

Two limiting cases may be distinguished, as shown in Figure 2.10:

(a) Safety $M(h) > 0$ and (b) Failure $M(h) \leq 0, \forall h[0, 1]$

A *fuzzy measure of risk*, or *fuzzy risk index* R_i may defined as the relative area of the fuzzy safety margin, where *values of M are negative*. Mathematically this index may be calculated as follows

$$R_i = \frac{\int_{m \leq 0} \mu_{\tilde{M}}(m) dm}{\int_m \mu_{\tilde{M}}(m) dm} \quad (2.7)$$

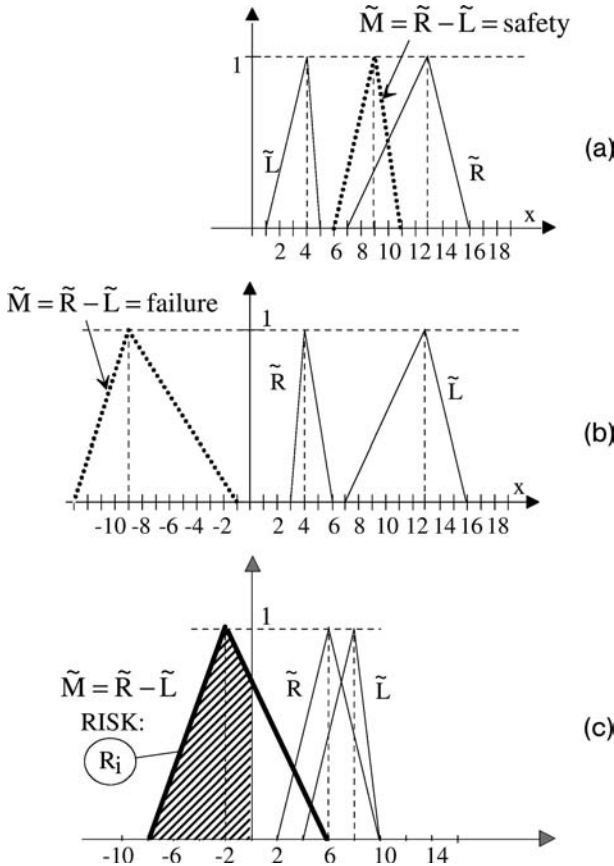


Figure 2.10 Absolute safety (a), absolute failure (b) and fuzzy risk index (c).

The fuzzy measure of reliability, or *fuzzy reliability index* R_e is the complement of R_i , i.e.

$$R_e = 1 - R_i = \frac{\int_{m>0} \mu_{\tilde{M}}(m) dm}{\int_m \mu_{\tilde{M}}(m) dm}$$

2.7

Questions and Problems – Chapter 2

Definitions of Risk

- What is the difference between a 'hazard' and a 'risk'?
- Using the terms 'hazard', 'vulnerability', 'probability of failure' and 'consequences of failure' formulate two different and general definitions of risk.
- How can you define the 'water pollution risk'?

Typology of Risks and the Precautionary Principle

- What are the two main categories of risks?
- How can we distinguish between 'acceptable' and 'unacceptable' risks?
- Explain why the risk definition as the product of (Probability) \times (Damage) is not valid for very high values of (Probability) and (Damage).

Uncertainties in Water Pollution Problems

- What is the difference between 'aleatory' and 'epistemic' uncertainties?
- How can we reduce the two types of uncertainty cited above?
- Can you evaluate the risk of water pollution given a time series of a pollutant concentration?

Water Quality Specifications

- Why are water quality standards defined statistically?
- Why do wastewater effluent standards differ from the water quality standards of the receiving water body?
- How can we define the 'receiving capacity' of a water body?

Probabilistic Risk and Reliability

- In the past 200 years four Presidents of the USA have been assassinated. What is the annual per capita risk per 100 000 of population of being assassinated whilst President of the USA? Compare this risk to the annual accidental mortality per capita risk of an airline pilot, which is estimated as 10 per 100 000 inhabitants or 10^{-4} .
- Formulate the general expression of risk:
 - when load and resistance are two independent variables and
 - when the resistance is constant.
- Research indicates that for human beings the permissible daily dose of nitrate concentration in drinking water is 5 mg/kg body weight/day. If a 5 kg

baby absorbs 0.40 l of water on a daily basis, check the drinking quality of water containing a nitrate concentration of 70 ppm.

Fuzzy Risk and Reliability

- (a) Fuzzy risk may be considered as a generalisation of interval-based uncertainty. Suppose that resistance and load are given by the following two intervals:

$$\bar{R} = [2, 6] \quad \text{and} \quad \bar{L} = [-3, 4]$$

Calculate an interval-based index of risk and of reliability.

- (b) Generalise the case (a) above assuming that \tilde{R} and \tilde{L} are two fuzzy triangular numbers $\tilde{R} = (2, 4, 6)$, $\tilde{L} = (-3, 3, 4)$, i.e. with 0-confidence level intervals the same as in case (a) above and the most confident values 4 and 3 respectively.

3

Risk Quantification

Risk quantification is one step further on from the formulation of the problem and the analysis of different uncertainties that may cause risk of failure (risk identification). Quantification of risk is very important in engineering because simulation, prediction and engineering design are based on quantitative rather than qualitative concepts.

Because, by definition, risk analysis is related to uncertainties, quantification of risks should be based on methodologies that take uncertainties into account. Two main theoretical approaches are available for doing this (i) the probabilistic approach and (ii) the fuzzy set theory. This chapter will describe how these two methodologies may be utilised for risk quantification. Static reliability analysis is considered first, when loads and resistances are assumed to be constant at a given time. Time-dependent reliability and risk quantification in systems with several components are reviewed at the end of this chapter.

Although the stochastic approach is relatively well established, fitting probability laws and analysing dependencies between random variables need large quantities of data which are not always available. If load and resistance are assumed to be independent, direct integration may be applied to quantify risk and reliability. Available data may be used to determine extreme values and the risk of exceedance, such as the hydrologic risk. Another possibility for quantifying risks is the formulation of stochastic differential equations. Monte Carlo simulation is a powerful technique for numerical representation of the system and subsequent risk quantification.

When very small samples of data are available or when variables are not easy to measure, fuzzy sets and fuzzy calculus may be utilised for analysing imprecision. This approach is relatively new in engineering, but it has proven to be very useful so far in informatics and control theory. In this chapter, by means of the basic concepts of fuzzy set theory, measures for fuzzy risk and reliability, direct evaluation, fuzzy simulation and fuzzy regression are shown to be useful tools for risk quantification.

3.1

Stochastic Approach

3.1.1

Direct Evaluation

Loads and resistances are considered here to be positive scalars, applied on a single component of a system and being time-independent. The latter assumption is necessary to enable direct integration for risk quantification. The above approach is generally known as static reliability analysis. Unsteady reliability of systems having several components is considered later in this chapter. The results to be presented below may also be generalised to situations where loads and resistances are vectors with several components.

Let us consider the *load* or *exposure* ℓ as a random variable L . In this case uncertainties associated with the estimation of L are quantified by means of probabilistic methods. Alternatively, fuzzy set theory may be used to compute uncertainties in the load or exposure by considering ℓ as a fuzzy number \tilde{L} . The *resistance* or *capacity* r is expressed in the same units as exposure. Because in many cases it is also uncertain, probabilistic methods or fuzzy set theory may be used to describe resistance either as a random variable R or a fuzzy number \tilde{R} .

Suppose now that both ℓ and r are positive random variables and that probabilistic methods are utilised to quantify risk. Recalling Equation 2.5, in the general case the risk p_F may be computed as follows:

$$p_F = P(L \geq R) = \int_0^\infty \left\{ \int_0^\ell f_{LR}(\ell, r) dr \right\} d\ell \quad (3.1)$$

Introducing the conditional probability $f_{LR}(\ell/r) = f_{LR}(\ell, r)/f_R(r)$ into Equation 3.1 we obtain

$$p_F = \int_0^\infty f_{LR}(\ell/r) \left\{ \int_0^\ell f_R(r) dr \right\} d\ell \quad (3.2)$$

Let L and R be independent random variables, that is $f_{LR}(\ell/r) = f_L(\ell)$. In this case, as shown in Figure 3.1, the integral takes non-zero values in the overlap between the two curves $f_L(\ell)$ and $f_R(r)$, and Equation 3.2 yields

$$p_F = \int_0^\infty f_L(\ell) \left\{ \int_0^\ell f_R(r) dr \right\} d\ell \quad (3.3)$$

Introducing the distribution function $F_R(r)$ into Equation 3.3, where $F_R(\ell) = \int_0^\ell f_R(r) dr$, the following equation is obtained

$$p_F = P(L \geq R) = \int_0^\infty F_R(\ell) f_L(\ell) d\ell \quad (3.4)$$

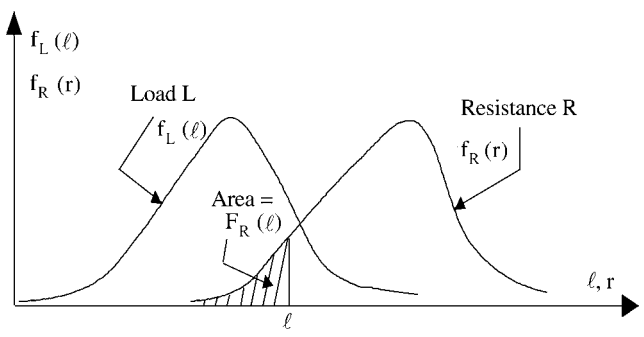


Figure 3.1 Direct quantification of risk by use of probability distribution functions.

The successive steps for quantifying risk by means of Equation 3.4 are shown in Figure 3.2.

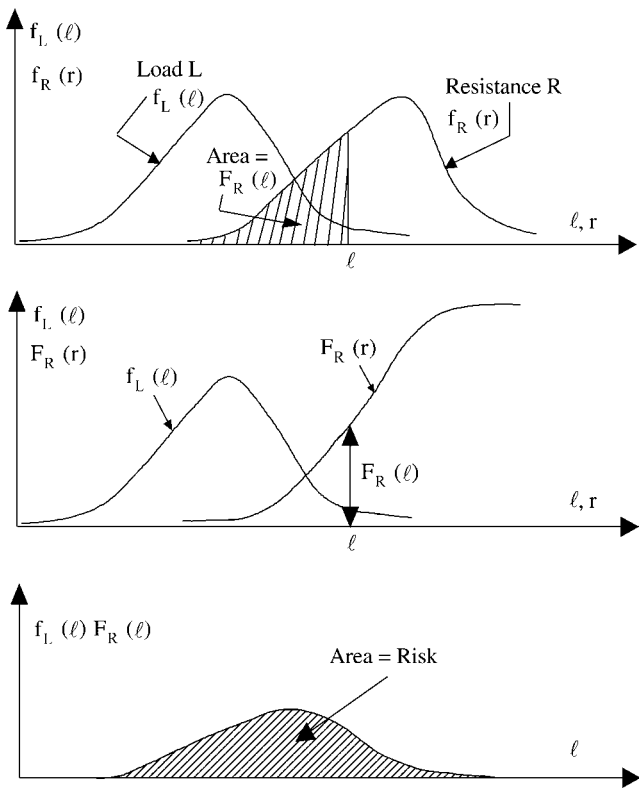


Figure 3.2 Steps for direct quantification of risk.

Example 3.1

Assume that the load and resistance are exponentially distributed, that is

$$f_R(r) = \lambda_R e^{-\lambda_R r}, \quad r > 0 \quad (3.5)$$

$$f_L(\ell) = \lambda_L e^{-\lambda_L \ell}, \quad \ell > 0 \quad (3.6)$$

By introducing expressions 3.5 and 3.6 into the Equation 3.4, the risk is calculated as

$$\begin{aligned} p_F = P(L \geq R) &= - \int_0^\infty \left\{ \int_0^\ell \lambda_L e^{-\lambda_L \ell} d\ell \right\} \lambda_R e^{-\lambda_R r} dr \\ &= \int_0^\infty \{1 - e^{-\lambda_L \ell}\} \lambda_R e^{-\lambda_R r} dr \\ &= \frac{\lambda_R}{\lambda_L + \lambda_R} = \frac{1}{\frac{\lambda_L}{\lambda_R} + 1} = \frac{1}{1 + \frac{E(R)}{E(L)}} = \frac{1}{1 + \frac{R_0}{L_0}} \end{aligned} \quad (3.7)$$

where $E(R) = R_0 = 1/\lambda_R$ and $E(L) = L_0 = 1/\lambda_L$ are the mean values of R and L . The result (Equation 3.7), shown graphically in Figure 3.3, indicates that for exponential distributions of L and R , a unique relation exists between the risk of failure and the ratio R_0/L_0 . The latter is called *central safety factor*.

In general, the risk is not only a function of this ratio but also depends on the values of the variances σ_L^2 and σ_R^2 .

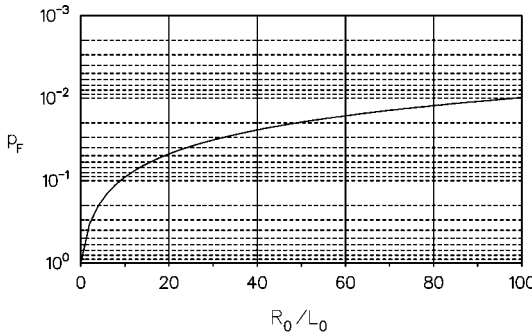


Figure 3.3 Relationship between risk and $E(R)/E(L)$ for exponential distribution.

Risk may be considered as a *performance index* of the system; this idea will be further developed in Chapter 5, where more details about the management of risk are given. Other *performance variables* and performance indices of the system are the *margin of safety* and the *safety factor*.

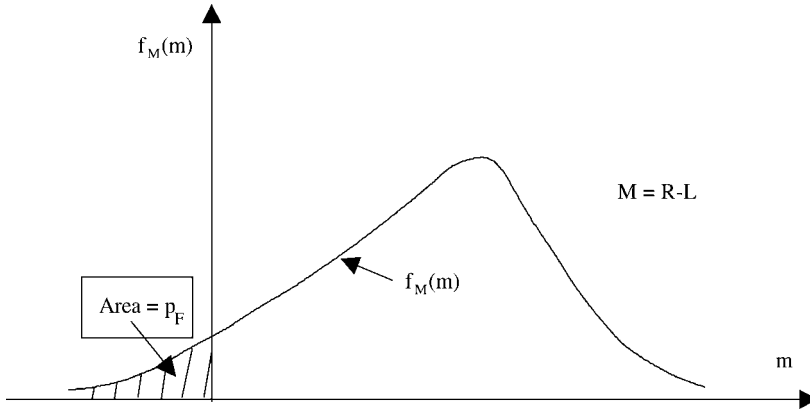


Figure 3.4 Probability density distribution of safety margin $M = R - L$.

3.1.1.1 Margin of Safety

This is a performance variable defined as follows

$$M = R - L \quad (3.8)$$

Because L and R are random variables, M is also a random variable with a probability density distribution function $f_M(m)$. The condition for an incident or failure to happen is written as follows

$$M \leq 0 \quad \text{or} \quad R - L \leq 0 \quad (3.9)$$

and the risk of failure is expressed by the following equation

$$p_F = \int_{-\infty}^0 f_M(m) dm = F_M(0) \quad (3.10)$$

As shown in Figure 3.4, p_F represents the area below the curve $f_M(m)$, for $m \leq 0$.

Example 3.2

Find the risk and the reliability in terms of the safety margin M , if we suppose that the load L and resistance R are normally distributed, that is

$$L = N(L_0, \sigma_L^2) \quad \text{and} \quad R = N(R_0, \sigma_R^2) \quad (3.11)$$

If L and R are independent, then the safety margin $M = R - L$ is also a normal random variable, with mean value $M_0 = R_0 - L_0$ and variance $\sigma_M^2 = \sigma_L^2 + \sigma_R^2$.

The reduced variable $M' = (M - M_0)/\sigma_M$ also has a normal probability density with zero mean and unit variance, that is $M' = N(0, 1)$. The risk may be calculated as

$$p_F = P(M \leq 0) = P(R - L \leq 0) \quad (3.12)$$

Subtracting $M_0 = R_0 - L_0$ from both sides of the inequality in Equation 3.12 and dividing both sides by σ_M we obtain

$$(3.13)$$

function, defined as

$$\Phi(x) = (1/\sqrt{2\pi}) \int_{-\infty}^x \exp(-x^2/2) dx$$

Numerical values of Φ are given in statistical handbooks and reported in Table 3.1.

Table 3.1 The standard normal distribution function $\Phi(x) = (1/\sqrt{2\pi}) \int_{-\infty}^x \exp(-x^2/2)dx$.

[illegible]

From Equation 3.13 we have

$$p_F = \Phi\left(-\frac{M_0}{\sigma_M}\right) = 1 - \Phi\left(\frac{M_0}{\sigma_M}\right) = 1 - \Phi\left(\frac{R_0 - L_0}{\sqrt{\sigma_L^2 + \sigma_R^2}}\right) \quad (3.14)$$

To illustrate the meaning of these results, a numerical application is given for the case of river pollution. According to the environmental quality standards, which have been recommended by the environmental protection agency, there is pollution in the river when the concentration of pollutant $C(t)$ exceeds a critical concentration C_0 . The permissible risk of exceeding the standards, that is the risk which does not affect ecosystems, is set at 10%. This means that we should have

$$p_F = P(C(t) \geq C_0) \leq 0.10$$

If we multiply both terms in the above inequality by the river flow rate $Q(t)$ we obtain

$$p_F = P(M_L(t) \geq M_R(t)) \leq 0.10$$

where

$M_L(t) = C(t) Q(t)$ is the pollutant load (mass per time)

$M_R(t) = C_0 Q(t)$ is the allowed pollutant mass rate or the resistant load (mass per time)

Suppose now that both loads $M_L(t)$ and $M_R(t)$ are normally distributed, e.g. $M_R = N(45, 7.5^2)$ and $M_L = N(35, 5^2)$. Application of Equation 3.14 and use of values in Table 3.1 gives

$$p_F = 1 - \Phi\left(\frac{R_0 - L_0}{\sqrt{\sigma_L^2 + \sigma_R^2}}\right) = 1 - \Phi\left(\frac{45 - 35}{\sqrt{7.5^2 + 5^2}}\right) = 1 - \Phi(.864) = 0.136 > 10\%$$

We conclude that there is risk of pollution.

The reliability $R_e = 1 - p_F$ of the system is

$$R_e = \Phi\left(\frac{R_0 - L_0}{\sqrt{\sigma_L^2 + \sigma_R^2}}\right) \quad (3.15)$$

It can be seen that both risk and reliability are functions of the coefficient

$$\beta = \frac{R_0 - L_0}{\sqrt{\sigma_L^2 + \sigma_R^2}} \quad (3.16)$$

known as the *reliability index*.

Using Table 3.1, which gives values of the function Φ , and Equations 3.14 and 3.16, the risk may be expressed as a function of the reliability index. The numerical values obtained are summarised in the Table 3.2.

The reliability index β is a characteristic performance index of the system. From Equation 3.14 it can be concluded that risk is a function of both the relative

Table 3.2 Risk p_F as a function of the reliability index β .

β	0	1.28	2.33	3.10	3.72	4.25	4.75
p_F	0.5	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}

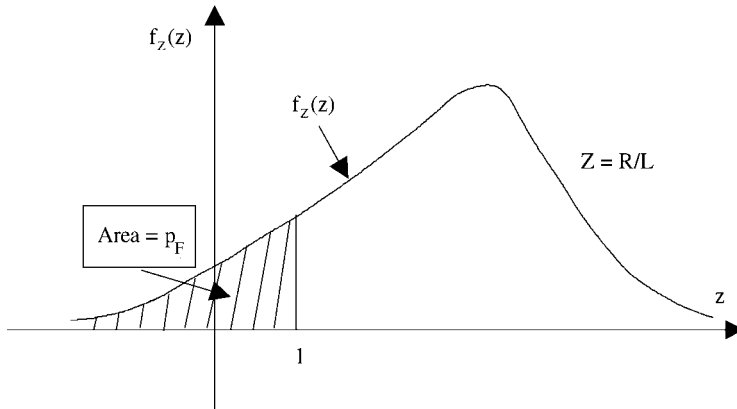
position of the $F_R(r)$ and $F_L(\ell)$ as measured by the mean safety margin $M_0 = R_0 - L_0$ and the degree of dispersion measured by $\sigma_M = \sqrt{\sigma_L^2 + \sigma_R^2}$.

3.1.1.2 The Safety Factor

Another performance index, very well known in engineering, is the safety factor $Z = R/L$. When L and R are random variables, Z is also a random variable with a probability density distribution function $f_Z(z)$.

The failure condition is $Z \leq 1$ and the risk is specified by $P(Z \leq 1)$. As shown in Figure 3.5, the risk is represented by the area under $f_Z(z)$ for $0 < Z \leq 1$. This area may be computed by means of the following integral

$$p_F = \int_0^1 f_Z(z) dz = F_Z(1.0) \quad (3.17)$$

**Figure 3.5** Probability density distribution of the safety factor $Z = R/L$.

Example 3.3

Find the risk and reliability, when L and R are independent, log-normal random variables or variates. R and L are log-normal variates when their logarithmic transforms

$$X = \ln R \quad \text{and} \quad Y = \ln L$$

follow normal distributions with parameters $\mu_{\ln R}$, $\sigma_{\ln R}^2$ and $\mu_{\ln L}$, $\sigma_{\ln L}^2$.

By use of the transformation given in Appendix A (Equation A.29), the probability density distributions of the log-normal variates L and R may be obtained as

$$f_L(\ell) = \frac{1}{\ell(\sqrt{2\pi})\sigma_{\ln L}} \exp \left[0.5 \left(\frac{\ln \ell - \mu_{\ln L}}{\sigma_{\ln L}} \right)^2 \right]$$

and

$$f_R(r) = \frac{1}{r(\sqrt{2\pi})\sigma_{\ln R}} \exp \left[0.5 \left(\frac{\ln r - \mu_{\ln R}}{\sigma_{\ln R}} \right)^2 \right]$$

$$p_F = P \left(Z = \frac{R}{L} \leq 1 \right) = P \left(\ln \left\{ \frac{R}{L} \right\} \leq 0 \right) \quad (3.18)$$

the random variable $\ln Z = \ln L - \ln R$ is also a normal variate with parameters

$$\mu_{\ln Z} = \mu_{\ln R} - \mu_{\ln L} \quad (3.19)$$

and

$$\sigma_{\ln Z}^2 = \sigma_{\ln R}^2 + \sigma_{\ln L}^2 \quad (3.20)$$

The random variable $Z' = \frac{\ln Z - \mu_{\ln Z}}{\sigma_{\ln Z}}$ is a normal variate with zero mean and unit variance. By means of Equation 3.18 we obtain

$$p_F = P \left(Z' < -\frac{\mu_{\ln Z}}{\sigma_{\ln Z}} \right) = \Phi \left(-\frac{\mu_{\ln Z}}{\sigma_{\ln Z}} \right) = 1 - \Phi \left(\frac{\mu_{\ln Z}}{\sigma_{\ln Z}} \right)$$

Taking into account the relationships shown in Equations 3.19 and 3.20 the following result is obtained

$$p_F = 1 - \Phi \left(\frac{\mu_{\ln Z}}{\sigma_{\ln Z}} \right) = 1 - \Phi \left(\frac{\mu_{\ln R} - \mu_{\ln L}}{\sqrt{\sigma_{\ln R}^2 + \sigma_{\ln L}^2}} \right) \quad (3.21)$$

As a numerical example, find the risk if R and L are log-normal variates with the same parameters as in Example 3.2, that is $\mu_R = 45$, $\sigma_R = 7.5$ and $\mu_L = 35$, $\sigma_L = 5.0$.

Between mean values and variances of the variates $\ln R$ and $\ln L$ and R and L the following relationships are valid (Ang and Tang, 1975)

$$\mu_{\ln R} = \frac{1}{2} \ln \left[\frac{\mu_R^2}{1 + \left(\frac{\sigma_R}{\mu_R} \right)^2} \right] \quad \sigma_{\ln R}^2 = \ln \left[\left(\frac{\sigma_R}{\mu_R} \right)^2 + 1 \right]$$

and

$$\mu_{\ln L} = \frac{1}{2} \ln \left[\frac{\mu_L^2}{1 + \left(\frac{\sigma_L}{\mu_L} \right)^2} \right] \quad \sigma_{\ln L}^2 = \ln \left[\left(\frac{\sigma_L}{\mu_L} \right)^2 + 1 \right]$$

For $\mu_R = 45$, $\sigma_R = 7.5$ and $\mu_L = 35$, $\sigma_L = 5.0$.

We find that

$$\mu_{\ln R} = 3.793, \quad \sigma_{\ln R}^2 = 0.027 \quad \text{and} \quad \mu_{\ln L} = 3.545, \quad \sigma_{\ln L}^2 = 0.02$$

By means of Equation 3.21 it follows that

$$p_F = 1 - \Phi\left(\frac{3.793 - 3.545}{\sqrt{0.027 + 0.02}}\right) = 1 - \Phi(1.138) = 1 - 0.871 = 0.122$$

This result should be compared with Equation 3.14 expressing the risk for normal variates in terms of the reliability index β .

3.1.2

Second-Moment Formulation

In most cases the probability distribution functions of load and resistance are not known and it is very difficult to obtain complete information about them without substantial further effort. Usually the available data are scarce and only estimations can be made about the first and second moments of the probability distributions.

Let us assume that, although the probability distribution laws are unknown, good estimates are available from data on mean values $E(\ell)$, $E(r)$ and variances $\sigma_L^2 = \text{Var}(\ell)$, $\sigma_R^2 = \text{Var}(r)$ of load L and resistance R . Consider the reduced variables

$$L' = \frac{L - E(L)}{\sigma_L}, \quad R' = \frac{R - E(R)}{\sigma_R} \quad (3.22)$$

The failure condition of the system may be expressed in terms of a performance index. Take, for example, the safety margin M . The critical condition is

$$M = R - L = 0 \quad (3.23)$$

Introducing the reduced variables (Equation 3.22) into the critical condition (Equation 3.23) we obtain the limit-state equation

$$\sigma_R R' - \sigma_L L' + E(R) - E(L) = 0 \quad (3.24)$$

As shown in Figure 3.6 the geometrical representation of Equation 3.24 in the space of reduced variates L' and R' is a straight line. This line divides the plane

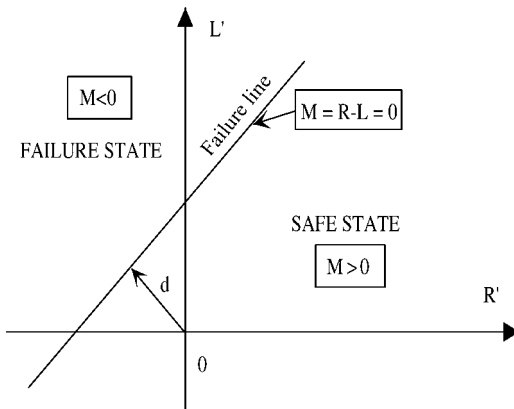


Figure 3.6 Reliability condition obtained by use of second moment formulation.

in two parts (i) the upper part, where $M < 0$ represents the failure state of the system and (ii) the rest of the plan indicates safe conditions ($M > 0$). All points on the straight line correspond to values of R and L for which the failure or critical condition is valid (failure line).

The distance d between the origin and the failure line is a measure of the reliability of the system. By means of analytical geometry we obtain

$$d = \frac{E(R) - E(L)}{\sqrt{\sigma_R^2 + \sigma_L^2}} \quad (3.25)$$

This equation should be compared with the expression shown in Equation 3.16 for the reliability index β . If L and R are normal variates, the distance d is equal to β . In this case the reliability R_e and the risk p_F may be computed in terms of the distance d as follows

$$R_e = \Phi(d) \quad \text{and} \quad p_F = 1 - \Phi(d) \quad (3.26)$$

where Φ is the cumulative normal function.

The second moment method has been generalised when R and L are functions of other variables X_i of the system. Take for example the case of a dam breaking due to malfunctioning of the spillway or to an accidental breach (Ganoulis, 1987).

The load is the inflow rate Q_L , which is a function of the peak flow upstream Q_p . The resistance is the flow capacity of the spillway Q_c , given as a function of the width b and the elevation z of the crest and also the water level h , by the following expression

$$Q_c = C\sqrt{2gb}(h-z)^{3/2} \text{ with } C = \text{constant.}$$

The *performance function* or *state function* in this case is

$$g(\mathbf{X}) = Q_L - Q_c = f(Q_p) - C\sqrt{2gb}(h-z)^{3/2}$$

where \mathbf{X} is the vector of the variables X_i , which, for this example, are the variables Q_p and h .

In general, the state function may be written as

$$g(\mathbf{X}) = g(X_1, X_2, \dots, X_n)$$

and the limit-state equation of the system is expressed by $g(\mathbf{X}) = 0$. The *safe state* is characterised by the condition $g(\mathbf{X}) > 0$ and the *failure state* by $g(\mathbf{X}) < 0$.

If $f_X(\mathbf{X})$ is the joint probability density distribution function, the reliability R_e and risk p_F may be computed by

$$R_e = \int_{g(\mathbf{X}) > 0} f_X(\mathbf{X}) d\mathbf{x} \quad \text{and} \quad p_F = \int_{g(\mathbf{X}) < 0} f_X(\mathbf{X}) d\mathbf{x}$$

More details about the general case of performance function with correlated and non-correlated system variables X_i can be found in the book by Ang and Tang (1984).

3.1.3

Frequency Analysis of Data

Statistical analysis of data, with the aim to quantify extreme values of physical variables and associated risks, has been traditionally applied in hydrology. The main objective in this case is the definition of the *hydrologic risk* or the evaluation of the probability of *hydrologic exceedance*. Water engineering structures are designed to operate for a certain period of time. During their life-time, they should be reliable, that is fulfil their purpose and withstand the applied loads. For example, a municipal water supply system is designed to satisfy the demand for water supply over 35 years of operation. This should be the case irrespective of uncertainties related to the various operating conditions of the system, such as an increase in population or availability of water supply. A flood levee is provided to resist the largest flood over a 50-year life-time. In the same way, a submarine outfall should safeguard water quality specifications under risk conditions during its life-time.

To evaluate hydrologic risks, or more generally risks caused by natural phenomena, it is important to have large time series information about extreme values of loads and resistances. For example, time series of maximum annual flow rate in a river, maximum annual precipitation of a given duration and maximum daily pollutant concentration in a river are characteristic extreme values of hydrologic and water quality parameters.

Let us consider, as shown in Figure 3.7, a time series of daily observations of flow rates or pollutant concentrations in a river over several years. The *maximum observed annual values* (X_1, X_2, \dots, X_N) over N years may be considered to be independent random variables and to have the same probability distribution function $F_X(x)$.

If x_0 is a characteristic value, then the *annual hydrologic risk* or *annual risk of exceedance* is the probability that the maximum annual value X exceeds x_0 , that is (Figure 3.8)

$$P(X \geq x_0) = p_F \quad (3.27)$$

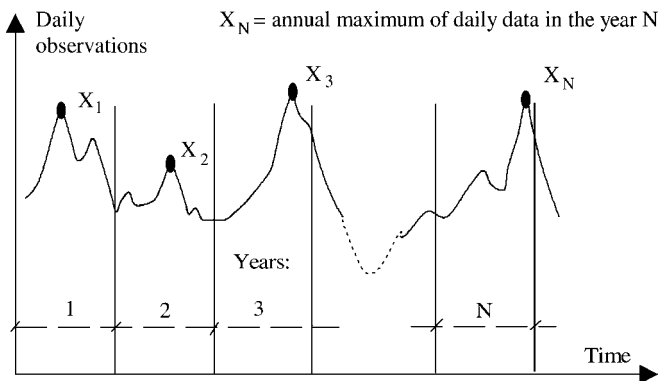


Figure 3.7 Time series of annual maximum values X_i of daily observations.

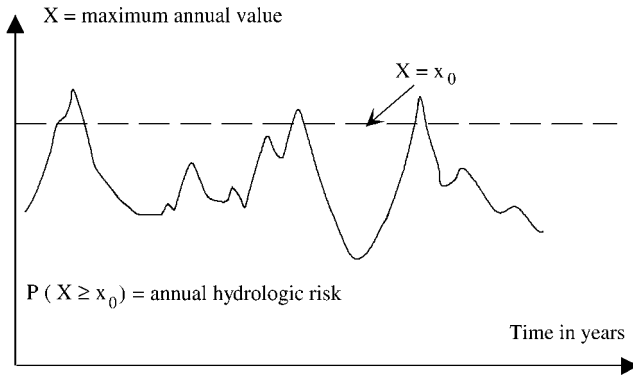


Figure 3.8 Annual hydrologic risk of exceedance.

The condition shown by Equation 3.27 describes the risk of having a *hydrologic failure* or *incident* and it is easily computed in terms of the probability distribution of X . However, most of the time this probability distribution is not known and in any case would be difficult to estimate.

Another method of evaluating hydrologic risk has been developed, which does not make reference to any specific probability distribution of hydrologic loads X . This method introduces the concept of the *return period*.

Let T , a characteristic time period known as the *interval of occurrence* or *return period*, be defined as the number of years until the considered load X equals or exceeds *on average* a specified value x_T only once, x_T is called the T -year event.

For example a 50-year flood is by definition a flood which may occur *on average* only once in 50 years. This does not necessarily imply that the above flood will occur only after 50 years: it may occur next year or several times in the next 50 years or not at all for 100 years. Of course, the probability of exceedance of x_T depends on the considered interval of time, but the fundamental result is that the annual risk of exceedance is equal to $1/T$.

$$P(X \geq x_T) = p_F = \frac{1}{T} \quad (3.28)$$

For example the probability that the 50-year flood may be exceeded in a given year is $1/50$.

The above result expressed by Equation 3.28 is the consequence of two main assumptions (Ang and Tang, 1975):

- (a) occurrences of random variables X are independent,
- (b) the hydrologic events are time invariant.

Using the above two assumptions, it follows that the occurrence of the events is a Bernoulli sequence (Ang and Tang, 1975).

To prove the result (Equation 3.28), the time V , in years, between two consecutive exceedances should be considered. V is a random variable whose mean value is the return period T .

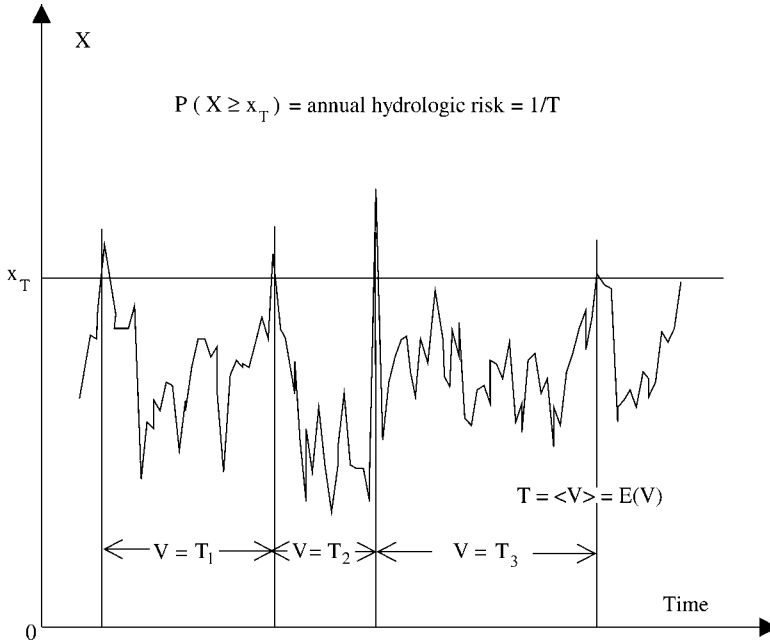


Figure 3.9 Definition of the T -year event.

Let A be the exceedance event, that is

$$A = X \geq x_T \quad (3.29)$$

and

$$\bar{A} = X < x_T \quad (3.30)$$

the event of non-exceedance. A and \bar{A} are two mutually exclusive and complementary events with the sum of probabilities equal to 1. If $k - 1$ years of non-exceedance occur before a year of exceedance is realised, then from Equations 3.29 and 3.30 we have

$$P(V = k) = P(\underbrace{\bar{A} \bar{A} \dots \bar{A}}_{k-1} A) = (1 - p_F)^{k-1} \cdot p_F \quad (3.31)$$

The discrete probability distribution law is known as *geometric law*. By use of Equation 3.31, the mean value of V (Papoulis, 1965) is

$$E(V) = \sum_{k=1}^{\infty} P(V = k) \cdot k = \left\{ \sum_{k=1}^{\infty} (1 - p_F)^{k-1} \cdot k \right\} p_F = \frac{1}{p_F} \quad (3.32)$$

Because, by definition, $T = E(V)$, the result (Equation 3.32) is equivalent to that expressed by Equation 3.28.

In fact, the occurrence of A and \bar{A} follows a Bernoulli sequence. This means that the probability that A may occur k times in n future years follows a *binomial distribution law* $B(n, k, p_F)$, where

$$\begin{aligned} P(A \text{ occurs } k \text{ times over } n \text{ years}) &= \binom{n}{k} p_F^k (1-p_F)^{n-k} = \frac{n!}{k!(n-k)!} p_F^k (1-p_F)^{n-k} \\ &= B(n, k, p_F) \end{aligned} \quad (3.33)$$

For risk and reliability computations it is necessary to know the behaviour of the system over an n -year period. This period should be the life-time of a hydraulic structure.

The *reliability* of the system over n years is the probability that the T -year event is not exceeded in that period. This is

$$R_{e,n} = P(X < x_T)_n \quad (3.34)$$

This means that x_T should not be exceeded every year. Because of probabilistic independence, we have

$$R_{e,n} = P(X < x_T)_n = (P(X < x_T))^n = (1 - P(X \geq x_T))^n \quad (3.35)$$

The n -year risk $p_{F,n}$ is equal to $1 - R_{e,n}$. From Equation 3.35 and using Equation 3.28 we obtain

$$p_{F,n} = 1 - R_{e,n} = 1 - (1 - P(X \geq x_T))^n = 1 - \left(1 - \frac{1}{T}\right)^n \quad (3.36)$$

More precisely, $p_{F,n}$ is the risk of exceedance or the risk of x_T occurring at least once in n years.

Table 3.3 gives the risk of exceedance of the T -year flood in a particular period of n years.

Table 3.3 Risk of exceedance (%) of the T -year flood in n years.

n	Return Period = T years						
	5	10	20	50	100	500	1000
1	20	10	5	2	1	0.1	0.1
2	33	19	10	4	2	0.4	0.2
5	63	41	22	10	5	1	0.5
10	87	65	40	18	9	2	1
20	98	88	64	33	17	4	2
100	> 99.9	> 99.9	99.4	87	60	18	9
500	> 99.9	> 99.9	> 99.9	> 99.9	99.6	63	39
1000	> 99.9	> 99.9	> 99.9	> 99.9	> 99.9	96	63

Example 3.4

Find the risk of occurrence in a period of 5 years of a flood with a return period of 20 years.

From Equation 3.36 we have

$$p_{F,5} = 1 - \left(1 - \frac{1}{20}\right)^5 = 22\%.$$

Example 3.5

The maximum daily concentration of nitrates in a river has been found to follow a probability density distribution of an exponential type. The mean daily concentration is 6.08 mg/l and the maximum allowed is 14 mg/l. It will be assumed that values of daily concentrations are statistically independent of each other. Find

- (a) the risk of pollution in a single day,
- (b) the return period in days,
- (c) the risk of pollution in the next 5 days,
- (d) the risk of pollution for the first time on the fifth day,
- (e) the risk of having exactly one exceedance in 5 days.

The general form of an exponential probability density distribution is

$$f_C(c) = \lambda e^{-\lambda c}$$

where C is the maximum daily concentration in mg/l and λ a constant equal to $1/E(C)$. $E(C)$ is the mean value of C , equal to 6.08 mg/l. We have, $\lambda = 1/6.08$ and

- (a) the risk of pollution in a single day is the probability that C exceeds the critical value $C_0 = 14$ mg/l. In this example, the probability distribution is known, so that by use of Equation 3.27 we obtain

$$p_F = P(C \geq C_0) = P(C \geq 14) = \int_{14}^{\infty} \lambda e^{-\lambda c} dc = e^{-14\lambda} = e^{-\frac{14}{6.08}} = 0.10$$

- (b) according to Equation 3.28 the return period T in days is given by

$$T = \frac{1}{p_F} = \frac{1}{0.10} = 10 \text{ days}$$

- (c) the risk of pollution in five consecutive days is given by Equation 3.36 or by Table 3.3 and has the following value

$$p_{F,n} = 1 - \left(1 - \frac{1}{T}\right)^n = 1 - (1 - 0.10)^5 = 1 - 0.90^5 = 1 - 0.59 = 0.41$$

This is the probability of having at least one 10-day pollution event in 5 days. We can see that the same result may be obtained if we use Equation 3.31 to compute the probability of time V having an exceedance of less or equal to 5 days, that is

$$\begin{aligned}
p_{F,n} &= P(V \leq 5) = \sum_{k=1}^5 (0.10)(0.90)^{k-1} \\
&= 0.10 + (0.10)(0.90) + (0.10)(0.90)^2 + (0.10)(0.90)^3 + (0.10)(0.90)^4 \\
&= 0.10(1 + 0.90 + 0.81 + 0.729 + 0.656) = (0.10)(4.095) \cong 0.41
\end{aligned}$$

(d) the probability of having exceedance on the fifth day is

$$P(V = 5) = (0.10)(0.90)^4 = 0.0656$$

(e) by use of the binomial distribution law (Equation 3.32) we have

$$\binom{5}{1} (0.10)(0.90)^{5-1} = \frac{5!}{1!(5-1)!} (0.10)(0.90)^4 = 5(0.10)(0.656) = 0.328$$

3.1.3.1 Probability Distribution of Extremes

Several theoretical laws have been proposed to predict the probability distribution of extreme values. By use of *extreme-value theory*, Gumbel has shown that in a series of extreme values X_1, X_2, \dots, X_N , the probability that X will be less than the T -year value x_T is given by

$$P(X < x_T) = e^{-e^{-y}} = 1 - \frac{1}{T} \quad (3.37)$$

where

$$y = a(x_T - X_0) \quad (3.38)$$

$$a = \frac{\pi}{(\sqrt{6}\sigma_X)} \quad (3.39)$$

$$x_0 = E(X) - 0.45\sigma_X \quad (3.40)$$

$$E(X) = (\text{mean value of } X_i) = \sum_{i=1}^N \frac{X_i}{N} \quad (3.41)$$

$$\sigma_X = \text{standard deviation given by } \sigma_X = \sqrt{\frac{\sum (X - X_i)^2}{N-1}} \quad (3.42)$$

Combining Equations 3.37 and 3.38 we have:

$$x_T = x_0 - \frac{\ln\left\{\ln\left(\frac{1}{1-(1/T)}\right)\right\}}{a} \quad (3.43)$$

For a given sample of values X_1, X_2, \dots, X_n , first the parameters $E(X)$ and σ_x can be estimated using Equations 3.41 and 3.42 and then the values of x_0 , a and x_T for a given return period T are calculated, as given by Equations 3.40, 3.39 and 3.43.

3.1.3.2 Analysis of Frequency

If N observations are given, from which a certain hydrologic event is found to be exceeded m times, then the frequency

$$f = \frac{m}{N}$$

may be considered to be very close to the exceedance probability. The return period T may be estimated by Equation 3.36 as the inverse of the exceedance probability, that is

$$T = \frac{1}{f} = \frac{N}{m} \quad (3.44)$$

In practice, data may be arranged in descending order, starting from the highest maximum annual value. If r is the rank of the considered event and N the total number of observations, then this event has been exceeded r times over N observations. According to Equation 3.44 the return period T should be

$$T = \frac{N}{r} \quad (3.45)$$

Example 3.6

Data concerning rainfalls are available from a meteorological station located on a Mediterranean island. Rainfall is characterised by its duration in minutes and height in millimetres. Although the data extend over a limited period of time, which is only 7 years, find the rainfall heights of return periods of 10, 20 and 50 years for rainfall durations of 15 min and 1 h respectively.

Table 3.4 summarises in descending order the data for maximum annual rainfall heights for given rainfall durations.

Table 3.4 Ranking in descending order of observed maximum annual rainfall heights $h(t)$ for rainfall durations of 5, 10, 15, 30 and 60 min.

Rank (r)	Rainfall duration (min)				
	5'	10'	15'	30'	60'
1	10	16.4	20.3	35.0	50.7
2	9.5	14.9	18.9	24.3	31.0
3	8.7	14.3	16.7	22.5	28.2
4	7.6	13.6	16.1	20.8	27.9
5	7.4	11.6	14.6	20.5	26.7
6	7.3	11.4	13.3	18.3	25.4
7	6.9	10.7	12.6	17.9	25.4

Table 3.5 Observed maximum annual rainfall heights $h(t)$ as functions of the return period T (years) for rainfall durations of 5, 10, 15, 30 and 60 min.

Rainfall duration $t(\text{min})$	T years						
	1	1.17	1.4	1.75	2.3	3.5	7
5	6.9	7.3	7.4	7.6	8.7	9.5	10.0
10	10.7	11.4	11.6	13.6	14.3	14.9	16.4
15	12.6	13.3	14.6	16.1	16.7	18.9	20.3
30	17.9	18.3	20.5	20.8	22.5	24.3	35.0
60	25.4	25.4	26.7	27.9	28.2	31.0	50.7

By use of Equation 3.45, with $N=7$ and r the rank of rainfall, for each maximum annual height, the corresponding return period T can be defined. Table 3.5 shows the results for each group of rainfall having the same duration.

Using the values given in Table 3.5 the rainfall height h_T may be plotted as a function of the return period T , for rainfalls of given duration.

Another possibility for establishing the relationship between h_T and T is the use of a probability distribution function of extreme values, such as Gumbel's law, given by Equation 3.37. The necessary steps for this may be summarised as follows:

- (1) Using the available data X_i , which, for this example, are the maximum annual rainfall height h_i , as given in Table 3.4, the mean value $E(h)$ and the variance σ_h can be computed by application of Equations 3.41 and 3.42.
- (2) The parameters a and h_0 can be estimated by using Equations 3.39 and 3.40.
- (3) For every value during the return period T the corresponding T -year value of the rainfall height h_T is computed from Equation 3.43.

Results are shown in Figure 3.10 for rainfalls of duration of 15 min, where the return period T in the x -axis is expressed in logarithmic coordinates and the best least-squares logarithmic line is found to fit the data.

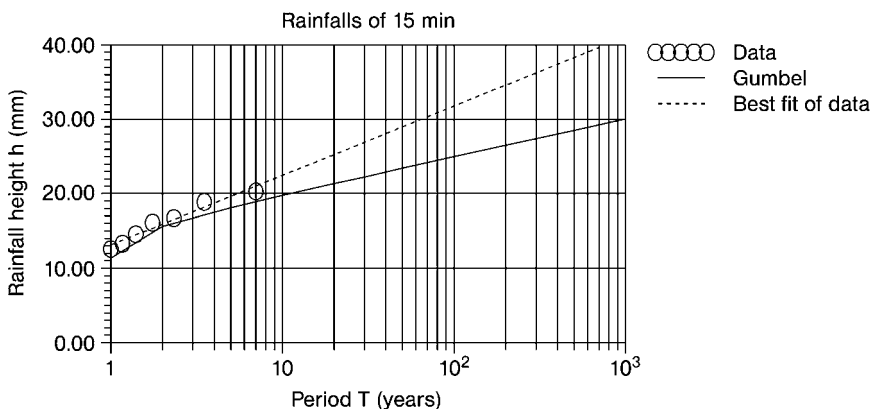


Figure 3.10 Rainfall height (mm) versus the return period T (years) for 15-min rainfall duration.

Table 3.6 Ranking in descending order of observed maximum annual rainfall intensity i (mm/h) for rainfall durations of 5, 10, 15, 30 and 60 min.

Rank	Duration				
	5'	10'	15'	30'	60'
1	120.0	98.4	81.2	70.0	50.7
2	114.0	89.4	75.6	48.6	31
3	104.4	85.8	66.8	45.0	28.2
4	91.2	81.6	64.4	41.6	27.9
5	88.8	69.6	58.4	41.0	26.7
6	87.6	68.4	53.2	36.6	25.4
7	82.8	64.2	50.4	35.8	25.4

The maximum height for return periods $T=5, 10$ and 50 years, which is useful for the design of hydraulic structures, may be estimated from Figure 3.10. It can be seen that Gumbel's distribution law tends to underestimate, at least in this case, the rainfall height for a given return period.

Another way of making estimations of the rainfall height for given return periods is to use as a variable the rainfall intensity i (mm/h), defined as the ratio between the rainfall height h (mm) and the time t (h), that is

$$i \text{ (mm/h)} = \frac{h \text{ (mm)}}{t \text{ (h)}} \quad (3.46)$$

Data on rainfall height given in Table 3.4 have been transformed into rainfall intensity by means of Equation 3.46 and the results are reported in descending order in Table 3.6. By means of Equation 3.45, rainfall intensities given in Table 3.6 are expressed in terms of the return period and the results are given in Table 3.7.

Rainfall intensity–duration curves are given in Figure 3.11 in natural (a) and log–log (b) coordinates.

Table 3.7 Observed maximum annual rainfall intensity i (mm/h) as a function of the return period T (years) for rainfall durations of 5, 10, 15, 30 and 60 min.

t (min)	T years						
	1	1.17	1.4	1.75	2.3	3.5	7
5	82.8	87.6	88.8	91.2	104.4	114.0	120.0
10	64.2	68.4	69.6	81.6	85.8	89.4	98.4
15	50.4	53.2	58.4	64.4	66.8	75.6	81.2
30	35.8	36.6	41.0	41.6	45.0	48.6	70.0
60	25.4	25.4	26.7	27.9	28.2	31.0	50.7

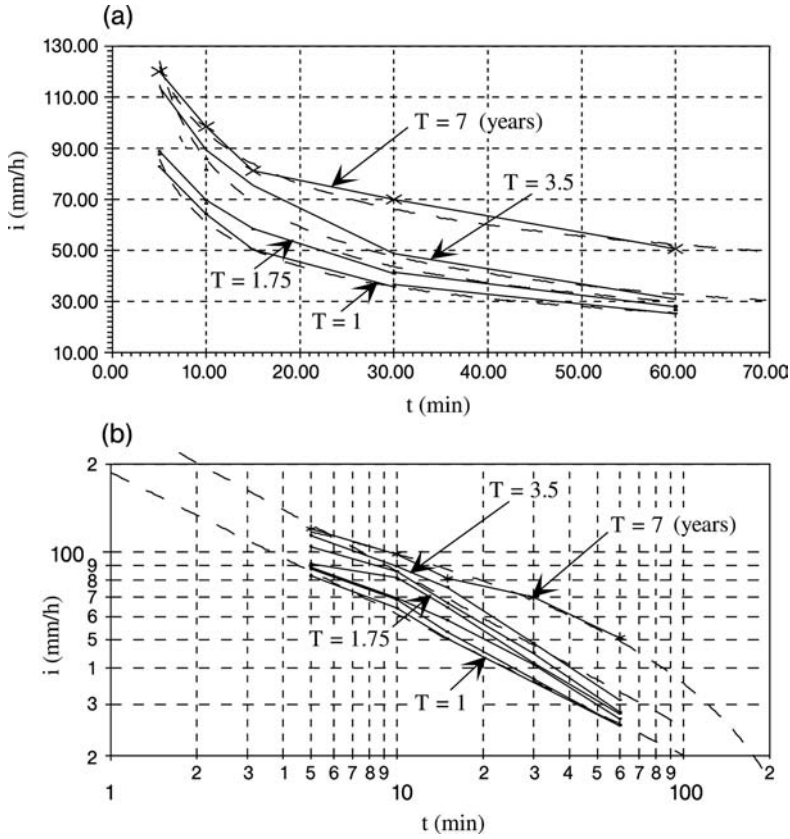


Figure 3.11 Rainfall intensity (mm) versus duration t (min) in natural (a) and log-log (b) coordinates.

The best fit for rainfall curves shown in Figure 3.11 was achieved using the relationship

$$i(t, T) = A(T)t^{-B(T)} \quad (3.47)$$

where i is the rainfall intensity (mm/h), t the time (min) and T the return period (years). The best fit for functions $A(T)$ and $B(T)$ was achieved by the least squares method, and the results are shown in Figure 3.12.

Comparison between the Gumbel distribution, best fit of A and B and best fit of data is shown in Figure 3.13 for rainfalls of 15-min and 1-h duration.

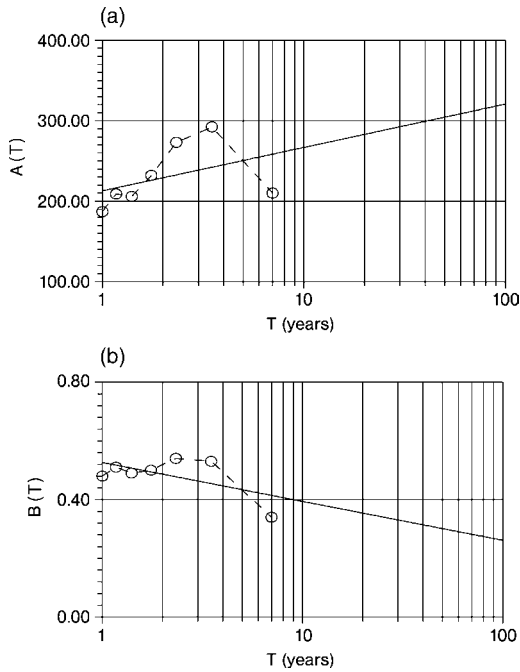


Figure 3.12 Best fitting of $A(T)$ (a) and $B(T)$ (b) coefficients.

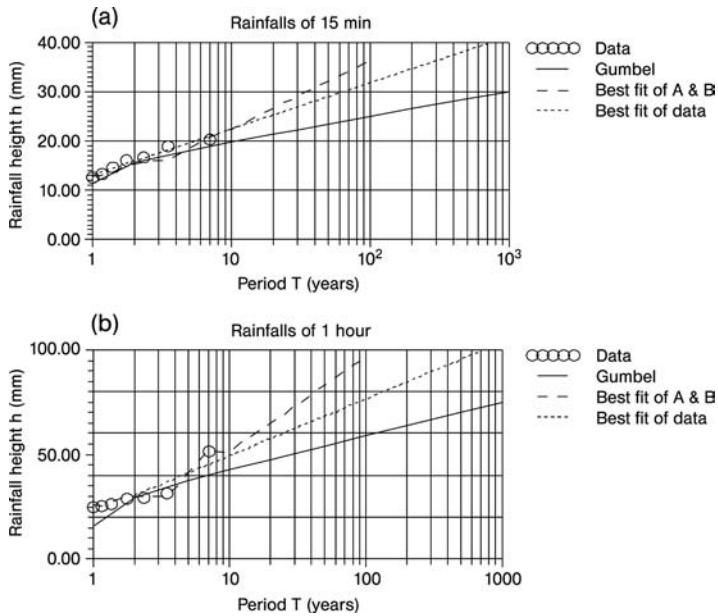


Figure 3.13 Rainfall height (mm) versus the return period T (years) for 15-min and 1-h durations.

The above example may be applied in areas where flooding due to intensive precipitation may occur. Coastal cities with relatively steep topography above a developed area are very vulnerable. Increase of population and extension of residential areas lead to the evacuation of larger quantities of storm water. Existing storm sewer systems are not sufficient to receive the high rates of discharge from storm water. As a consequence, the lower parts of the cities become flooded, and public and private properties are damaged.

Two different approaches have been developed to protect urban areas from storm waters. The first is based on the principle that storm water should be kept at a distance from the city and be evacuated as quickly as possible. According to this approach, dams and reservoirs may be designed upstream of urban areas to contain the volume of floodwater. Also streams may be deviated from urban areas by channels and tunnels, which divert stormwater far from the city.

The second approach provides for storm water to be stored in places within the urban area. The volume of water is then evacuated slowly by the storm-sewer system of the city. The basins are integrated into the activities of the city and can be used as gardens, picnic areas, children's playgrounds, or stadiums with sport facilities. For designing such facilities it is necessary to estimate rainfall intensities and heights for return periods of 20 or 50 years. As has been shown in the Example 3.5, the available rainfall data may be taken into account in two different ways: (i) using a statistical analysis to quantify the maximum intensity of rainfall of a given duration and return period, and (ii) fitting the data by a probability distribution law of extremes, such as Gumbel's distribution.

3.1.4

Stochastic Modelling

Stochastic modelling is a general methodology that allows the introduction of probabilities in order to simulate systems which are subject to uncertainties. Such systems may be physical hydrological entities, such as those studied in this book (coastal areas, rivers, aquifers) or technological engineering systems, such as dams, water distribution systems or wastewater treatment plants. In reliability engineering, the aim of using stochastic modelling is to quantify uncertainties in order to assess the risk which is associated with the use or operation of the system.

In complex technological systems, such as nuclear reactors or chemical plants, the utilisation of probabilities in order to assess the safety of such systems is known as *Probabilistic Risk Assessment* (PRA) or *Probabilistic Safety Assessment* (PSA) (Apostolakis, 1990). In these complicated situations, experts' opinions and subjective probabilities play a predominant role, because it is difficult to validate or obtain model assumptions and model parameters 'objectively'.

In stochastic modelling for reliability analysis of water quality, hydrological and water quality variables or parameters are considered as random or stochastic variables. To understand better the methodology, the stochastic approach is compared to the classical deterministic engineering modelling method. Connection between the two approaches has been analysed by Ganoulis and Morel-Seytoux

(1985) for the case of aquifer systems. This is summarised here for more general cases, such as physical hydrological systems.

3.1.4.1 Deterministic Modelling

A physical hydrological system may be characterised by a set of physical parameters, noted by the vector $\mathbf{a} = \{a_1, a_2, \dots, a_k\}$. The system receives some *inputs* or *excitations* defined by a set of independent variables and represented by the vector $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$. Its *response* or *reaction* is characterised by the vector of *output* or *dependent variables* $\mathbf{y} = \{y_1, y_2, \dots, y_m\}$.

According to the deterministic approach, there is a unique relationship between the deterministic output y_d of the system, the input \mathbf{x} and the vector of system parameters \mathbf{a} . This means that a law or an equation may be found, usually as a result of integration of partial differential equations, having the following functional form

$$y_d = f(\mathbf{x}, \mathbf{a}) \quad (3.48)$$

More precisely y_d is the *model conditional deterministic solution*. This solution is subject to two main conditions:

- (a) the model assumptions, which may be satisfactory for a given set of objectives,
- (b) the current state-of-knowledge in the field.

The *actual* or *unconditional* or *true* value \mathbf{y} may be written in the form

$$\mathbf{y} = \mathbf{y}_d + \boldsymbol{\varepsilon}_d \quad (3.49)$$

where $\boldsymbol{\varepsilon}_d$ is the *deterministic deviation* or *error*, which, as stated in Chapter 2, Section 2.3, is due to two main causes:

- (a) the intrinsic randomness of the system (*aleatory uncertainties*),
- (b) the *epistemic* or *man-induced uncertainties*.

Epistemic uncertainties include parameter uncertainties due to parameter definition and measurement (physical meaning of parameters, precision of instruments and human errors in order to obtain the value of coefficients \mathbf{a}) and numerical errors, when the relationship shown in Equation 3.48 is computed, usually by integration of partial differential equations.

As a typical example, let us consider the flow in a two-dimensional confined aquifer shown in Figure 3.14. In this example, the hydrological system is characterised by a single parameter, which is the transmissivity $T(\text{m}^2/\text{s})$. Inputs are the pumping and recharge flow rates q_p ($\text{m}^3/\text{s}/\text{m}^2$) that we can see in Figure 3.14, and the boundary conditions (values of piezometric height $h(\text{m})$ or its normal derivative $\partial h/\partial n$ along parts of the boundary S). The output vector may be composed by the groundwater flow velocity and the piezometric height h , or by the latter only.

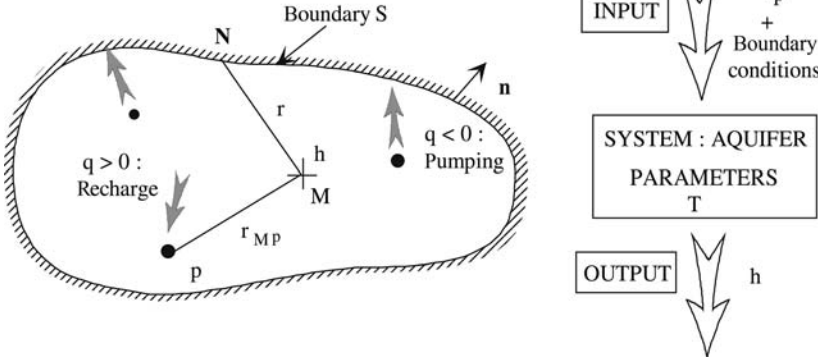


Figure 3.14 Flow in a two-dimensional confined aquifer.

The deterministic model of the problem, based on a set of assumptions and the actual state-of-knowledge, is composed by Darcy's law and the mass continuity equation. This model will be explained further in Chapter 5. Combining these two equations, a Poisson-type partial differential equation is obtained in the form

$$T\nabla^2 h + \sum_p q_p \delta_p = 0$$

where δ_p is the Dirac's delta function. Introducing Green's function $h^* = (1/2\pi) \ln(1/r)$, we obtain the following analytical solution for the piezometric height at every point M inside the aquifer

$$h = \oint_S \left(h^* \frac{\partial h}{\partial n} - h \frac{\partial h^*}{\partial n} \right) dS + \sum_p \frac{q_p}{2\pi T} \ln \frac{1}{r_{Mp}}$$

This is the deterministic solution of the model, which is a special case of the Equation 3.48 and having the form

$$h_d = f \left(q_p, h_s, \left(\frac{\partial h}{\partial n} \right)_s, T \right)$$

If in the above solution the integral is computed numerically (Boundary Element Method), then the deterministic solution will contain some numerical errors and any error related to the estimation of the parameter T (epistemic uncertainties). It will also contain errors due to the intrinsic randomness of the system (spatial variability of the transmissivity T and uncertainties on inputs). The deterministic approach is not able to take into account these aleatory uncertainties, which can be handled by the stochastic approach.

3.1.4.2 Stochastic Modelling

In the stochastic approach, the physical parameters of the hydrological system \mathbf{A} and the inputs \mathbf{X} are considered to be *random* or *stochastic variables*, having some probability distribution functions. By use of physical conservation laws (usually in the form of partial differential equations) or empirical statistical analysis of the available data, a model is first formulated in order to describe the system. This model, as in the case of the deterministic approach, is subject to two main conditions:

- (a) the model assumptions, which may be satisfactory for a given set of objectives,
- (b) the current state-of-knowledge in the field.

Then, using this model, the objective of the stochastic approach is to determine the probability distribution law of the dependent variable \mathbf{Y} , in the form

$$P(\mathbf{Y} \leq y) = F(\mathbf{X}, \mathbf{A}, y) \quad (3.50)$$

The stochastic value \mathbf{Y} may be expressed as the sum between the expected value $\langle \mathbf{Y} \rangle$, according to the probability law (Equation 3.50), and the *stochastic deviation* or error ϵ_s , in the form:

$$\mathbf{Y} = \langle \mathbf{Y} \rangle + \epsilon_s \quad (3.51)$$

Except in some specific cases of linear problems, generally we have

$$y_d \neq \langle \mathbf{Y} \rangle \quad (3.52)$$

By comparing the relationships in Equations 3.49 and 3.51 and setting, for a given realisation y of the stochastic variable \mathbf{Y} , the equality $y = \mathbf{Y}$ one obtains

$$\epsilon_d = \langle \mathbf{Y} \rangle - y_d + \epsilon_s \quad (3.53)$$

Excluding all numerical, experimental and model structural errors, the deviations ϵ_d and ϵ_s are due only to the physical uncertainties of the system. In this case, it can be seen from Equation 3.51 that the stochastic approach furnishes a formal procedure for computing ϵ_s , because both \mathbf{Y} and $\langle \mathbf{Y} \rangle$ may be evaluated. Also by use of stochastic techniques some measures of ϵ_s , such as the standard deviation or the confidence interval may be estimated. This can be interpreted as an advantage of the stochastic approach over the deterministic analysis.

Applying the stochastic approach to the example of aquifer flow shown in Figure 3.14, the transmissivity T or the related hydraulic conductivity K may be considered to follow a log-normal probability density distribution of the form

$$f(k) = \frac{1}{k\sqrt{2\pi}\sigma} \exp \left[-\frac{(\ln k - \mu)^2}{2\sigma^2} \right] \quad k > 0, \sigma > 0, -\infty < \mu < \infty$$

Independently if the inputs are considered as deterministic or random variables, because of the stochastic character of the parameter K , the output variable h

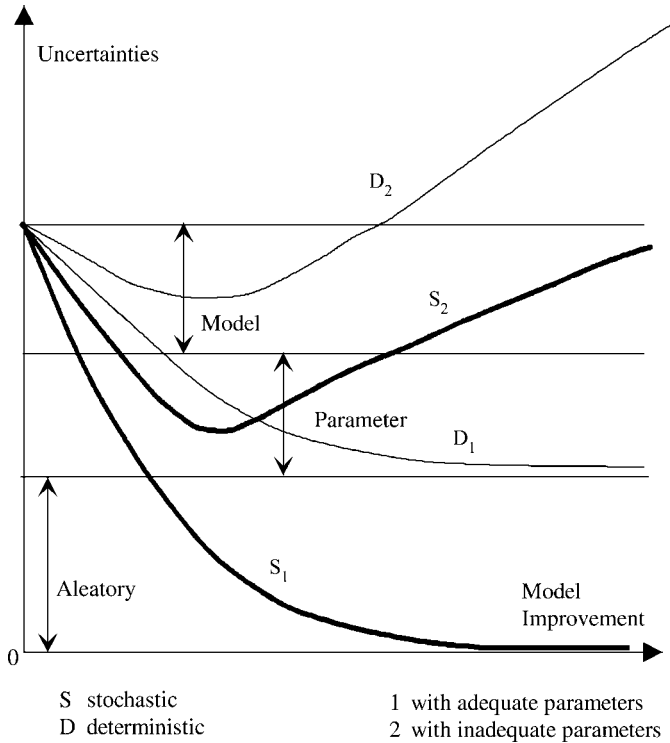


Figure 3.15 Qualitative relationship between modelling improvement and various uncertainties.

should have a probability distribution function. If this is evaluated in the form of Equation 3.50, then the aleatory uncertainties due to the random variation of the aquifer parameter should be evaluated. The state-of-knowledge for stochastic modelling of aquifer flow includes multivariate normal distributions and exponential correlation functions for the hydraulic conductivity random field.

As shown qualitatively in Figure 3.15 stochastic modelling can minimise all uncertainties as long as the model has sufficient parameters and is improved (in terms of complexity and structure). A deterministic model with sufficient or adequate parameters can minimise parameter and model errors, but it is not able to handle aleatory uncertainties. Both kinds of models fail if the parameters they use are inadequate. In such cases, it may be an optimum for a certain degree of the model's complexity. Beyond this optimum uncertainties increase together with model's complexity.

One important question in the stochastic modelling of hydrological systems is the change in the spatial heterogeneity scales (Ganoulis, 1986). Furthermore various methods and tools have been extensively used in the past for stochastic simulation, such as

- time series analysis, filtering, kriging,
- stochastic differential equations,

- spectral analysis,
- Taylor series and perturbation analysis,
- Monte-Carlo simulation.

Example 3.7

As an example the kinetics of fecal bacteria is considered. It is known (Quetin and De Rouville, 1986; UNEP/WHO, 1985) that the number of bacteria N per unit water volume decreases exponentially according to the law

$$N = N_0 \exp(-\lambda t) \quad (3.54)$$

where N_0 is the initial number of bacteria per unit water volume. Introducing the T_{90} , that is, the time required for eliminating 90% of the bacteria, λ is related to T_{90} by the following expression

$$\lambda = \frac{\ln 10}{T_{90}} = \frac{2.3}{T_{90}} \quad (3.55)$$

Various measurements of T_{90} from several locations indicate that T_{90} is not constant, but follows a log-normal distribution (Quetin and De Rouville, 1986). Find the law of bacterial decay as a function of the mean value and the variance of T_{90} .

If T_{90} is a log-normal random variable, then $\ln(T_{90})$ is normal with parameters μ and σ . We have

$$f[\ln(T_{90})] \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-0.5\left(\frac{\ln(T_{90})-\mu}{\sigma}\right)^2\right] \quad (3.56)$$

where μ and σ are the mean value and standard deviation of $\ln(T_{90})$ respectively. Combining the Equations 3.54 and 3.55 we have $N = N_0 \exp(-2.3 t/T_{90})$. The mean value of N may be found as follows

$$\begin{aligned} \langle N \rangle &= \int_0^{\infty} N f(N) dN = \int_0^{\infty} N f(T_{90}) d(T_{90}) = \int_0^{\infty} N f(\ln T_{90}) d(\ln T_{90}) \\ &= N_0 \int_0^{\infty} \exp\left(\frac{-2.3t}{T_{90}}\right) f(\ln T_{90}) d(\ln T_{90}) \end{aligned} \quad (3.57)$$

By successive transformations (Quetin and De Rouville, 1986), we arrive at the following relationship

$$\langle N \rangle = N_0 \int_0^{\infty} \exp\left\{\frac{-2.3t}{T_{90}} \exp(-\sigma x) - x^2/2\right\} dx \quad (3.58)$$

where $x = t/\hat{T}_{90}$ and \hat{T}_{90} is the geometric mean of T_{90} .

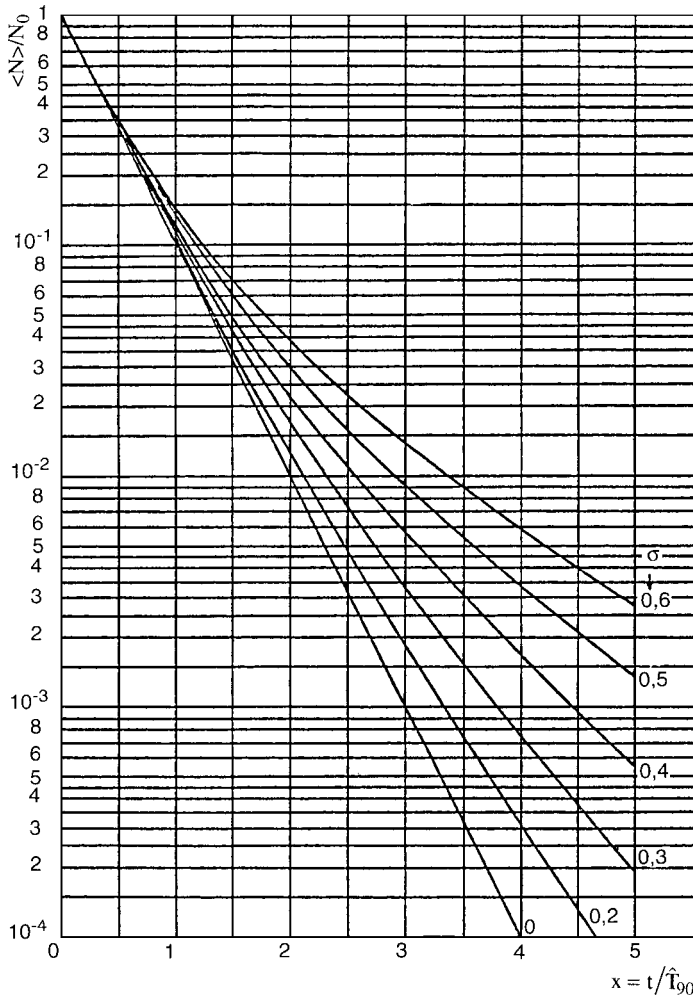


Figure 3.16 Number of bacteria as a function of the statistical properties of T_{90} (Quetin and De Rouville, 1986).

Knowing T_{90} and σ the graph in Figure 3.16 gives the mean mortality ratio $\langle N \rangle / N_0$ of bacteria, as a function of s and $x = t / \hat{T}_{90}$.

3.1.5

Monte Carlo Simulation

This is a general simulation technique, which may be applied when some random variables are related to deterministic functional relationships. Following the Monte Carlo method, several possible realisations of a random variable would be produced, from which the statistical properties of the variable, such as mean value and variance, are obtained.

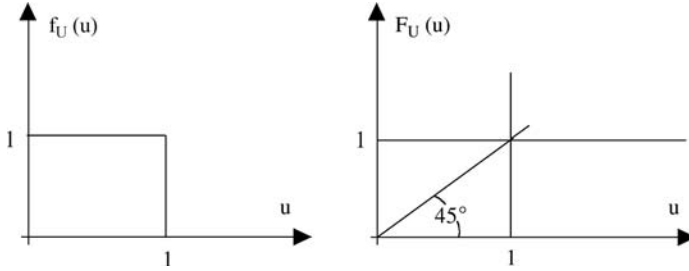


Figure 3.17 The standard uniform random variable U .

The main point of the technique is to generate samples having a prescribed probability distribution function. The easiest way for this is to start with samples of random numbers, which are realisations of the *standard uniform random variable* U . This is a random variable with uniform probability density distribution $f_U(u)$ between 0 and 1 (Figure 3.17).

As shown in Figure 3.17, the cumulative function $F_U(u)$ is the bisecting line in the plane $u - F_U(u)$. We have

$$F_U(u) = P(U \leq u) = \int_0^u dx = u \quad (3.59)$$

Methods for generating random numbers with uniform probability distribution are mainly based on recursive relationships of the form

$$x_{k+1} = (ax_k + b) \pmod{m} \quad (3.60)$$

where a and b and m are non-negative integers. The Equation 3.60 means that residues of modulus m are first computed as

$$x_{k+1} = (ax_k + b) - m \left\{ \text{Int} \left(\frac{ax_k + b}{m} \right) \right\} \quad (3.61)$$

where Int is the integral part of the number. Then random numbers between 0 and 1 are obtained by the ratio

$$u_{k+1} = \frac{x_{k+1}}{m} \quad (3.62)$$

Numbers generated by use of such a procedure are not real random numbers. They have a pattern and are cyclically repeated. For that reason they are called *pseudo-random* numbers. In order to avoid small periods of cycles, the constants a , b and m should take large values. Numbers generated by such a procedure should be tested for statistical independence and uniform distribution. Samples of U , such as (u_1, u_2, \dots, u_n) are nowadays generated on several modern computers by means of appropriate internal functions.

Now, having generated a sample of uniformly distributed random numbers u_k , the corresponding numbers x_k , which belong to a sample of probability

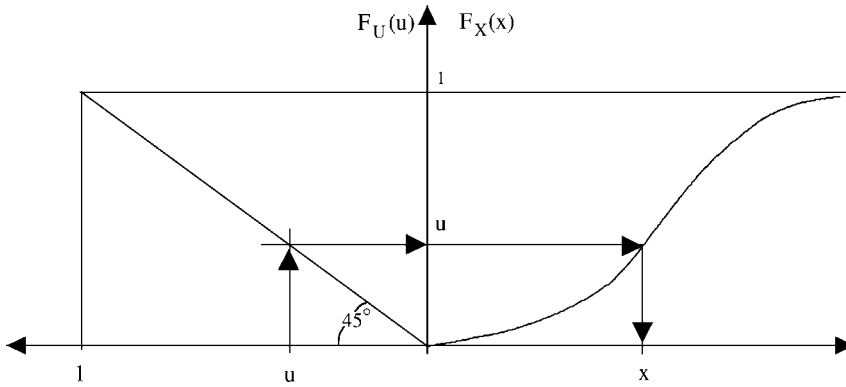


Figure 3.18 Relationship between random variables X and U .

distribution functions $F_X(x)$, may be generated by use of the following relationship (Figure 3.18)

$$\begin{aligned} F_X(x_k) &= F_U(u_k) = u_k \\ x_k &= F_X^{-1}(u_k) \end{aligned} \quad (3.63)$$

For reliability computations, the Monte Carlo simulation technique may proceed in three steps:

- (1) Generation of synthetic samples of random numbers, following specified probability distributions. This may be done for input variables, loads and resistances.
- (2) Simulation of the system by means of a model, where values of generated random variables are taken into account.
- (3) Reliability assessment of the system by counting the number of satisfactory realisations over the total number of realisations. Thus, the probability of success or the system reliability may be estimated.

The Monte Carlo simulation technique is a powerful tool, capable of representing complex systems with non-linear structures. It is equivalent to the experimental methodology, by which testing of the system is performed by repetition of experiments. Therefore, the Monte Carlo simulation technique has the drawbacks of any experimental method: lack of insight regarding the structure of the system and difficulties in making synthesis of the results. Also, for complex systems, a considerable amount of computing time may be necessary and sometimes inconsistent results may be obtained because of sampling variabilities.

Example 3.8

Indicate how samples with (a) exponential (b) normal and (c) log-normal distributions may be generated.

(a) Exponential distribution $F_X(x) = 1 - \exp(-\lambda x)$

By use of the procedure indicated in Figure 3.18 we have

$$F_X(x) = 1 - \exp(-\lambda x) = F_U(u) = u \quad (3.64)$$

Resolving Equation 3.64 above, for x we obtain

$$x = F_X^{-1}(u) = -\frac{1}{\lambda} \ln(1-u) \quad (3.65)$$

Random numbers with exponential distribution are generated by use of the relationship

$$x_k = -\frac{1}{\lambda} \ln(1-u_k) \quad (3.66)$$

where u_k are random numbers uniformly distributed.

(b) Normal distribution

$$F_X(x) = \int_{-\infty}^x \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{0.5\left(\frac{x-\mu}{\sigma}\right)^2\right\} dx$$

If X is a normal variate with parameters μ and σ , it is known that the variate $Z = (X - \mu)/\sigma$ is also normal with $\mu = 0$ and $\sigma = 1$. If a sample of random numbers z_k is generated, then x_k may be obtained by use of the relationship

$$x_k = \mu + \sigma z_k \quad (3.67)$$

Because it is difficult to inverse the normal distribution function, another method may be used to generate samples of z_k . It may be shown that if u_i are uniformly distributed random numbers, then by use of

$$z_k = \frac{\sum_{i=1}^m u_i - \frac{m}{2}}{\left(\frac{m}{12}\right)^{1/12}} \quad (3.68)$$

a sample of $N(0,1)$ may be obtained.

For $m = 12$, Equation 3.68 takes the simple form

$$z_k = \sum_{i=1}^{12} u_i - 6 \quad (3.69)$$

Having obtained from Equation 3.69 z_k numbers with $N(0,1)$ distribution, Equation 3.67 may be used to generate random numbers x_k with $N(\mu, \sigma^2)$.

(c) log-normal distribution

$$F_X(x) = \int_{-\infty}^x \frac{1}{x\sigma\sqrt{2\pi}} \exp\left\{0.5\left(\frac{\ln x - \mu}{\sigma}\right)^2\right\} dx$$

If X is log-normal, then the random variable $\ln X$ is $N(\mu, \sigma^2)$. Noting $Y = \ln X$, first the sample of random numbers y_k is obtained with $N(\mu, \sigma^2)$, by use of the procedure described above. Then random numbers x_k with log-normal distribution are computed from the following relationship

$$x_k = e^{y_k} \quad (3.70)$$

3.2

Fuzzy Set Theory

3.2.1

Fuzzy Regression

Obtaining functional relationships between available data, which are realisations of two or more variables, is very important and useful in engineering. The first advantage is to compile available information by means of analytical expressions, which may be further used in mathematical and numerical computerised models. The second merit of such formulation is the possibility of making predictions using the available data.

In a stochastic framework, statistical regression between two or more variables is a very popular technique. It is a statistical method for smoothing available data by means of linear or non-linear functional relationships between variables. However, the accuracy of the method is subject to two main sources of inaccuracies:

- (a) Errors due to the method for data sampling and analysis (experimental observation errors and errors connected to the laboratory analytical techniques). These errors create uncertainties referred to as the *quality of observations*.
- (b) The number of data available or the extent of data population used to perform the mean-square statistical regression. These are statistical errors related to the *quantity of information*.

Uncertainties related to the number of available data create bias in the form and value of coefficients of the regression functions. This is the most serious limitation of classical regression analysis based on three basic assumptions concerning the deviations or approximation errors, which should

- (a) form a sequence of *independent* random variables,
- (b) have a *zero* mean value,
- (c) have a *constant* variance.

To check the above assumptions a sufficient number of data (usually more than 30) is necessary in order to obtain meaningful statistical parameters, such as the correlation coefficient. If the available sample of observations is small, the estimation

of the regression function coefficients and the analysis of variance are biased and there is no guarantee that the basic assumptions on approximation errors are valid. On the other hand, if the statistical hypothesis for validation of a given regression is rejected on the basis of the analysis of variance, there is no alternative to using the data, even if they are of good quality.

Most of the time, data for some of the required variables are scarce, mainly because of the high cost required to obtain them. Take for example the sea water temperature in coastal areas. This is a dependent variable mainly influenced by the ambient air temperature and also by other factors such as the water currents, water depth and wind conditions. In a first approximation, at a given location and depth, water temperature may be considered as the dependent variable, and a function of the ambient air temperature (independent variable). Usually, because of the high cost of obtaining water temperature at various depths, the available data are limited in number. In this case, an alternative solution to the classical regression analysis is the application of fuzzy regression. First developed by Tanaka *et al.* (1982), fuzzy regression is a technique used to derive functional relationships between observations which are uncertain, without bias due to limited number of data. This technique has been applied in hydrology (Bardossy *et al.*, 1990) and also other fields of engineering.

3.2.1.1 Fuzzy Regression as an Extension of Interval Analysis

Fuzzy regression may be considered as an extension of interval analysis as applied to the regression problem. Interval arithmetics (Moore, 1979) may be interpreted as a special case of fuzzy number calculus (Kaufmann and Gupta, 1985). It is very useful to start with arithmetic operations on intervals and then to generalise on multiple intervals with variable confidence levels. These are in fact fuzzy numbers.

Let x_i be the vector components of i independent observations and y_i the data vector components of the dependent variable, with $i = 1, 2, \dots, n$. The n pairs of observations are given in the following form

$$(x_i, y_i) \quad i = 1, 2, 3, \dots, n \quad (3.71)$$

3.2.1.2 Statistical Regression

As shown schematically in Figure 3.19, in a probabilistic framework (Ang and Tang, 1975), x_i and y_i are considered as realisations of two random variables X and Y . If for a given value of $X=x$ the mean value of Y is noted as $\mu_{y/x}$, then a linear regression function is defined as

$$\mu_{y/x} = A + Bx \quad (3.72)$$

where A and B are constants. These are determined in order to minimise the deviations

$$\varepsilon_i = |\mu_{y/x_i} - y_i| = |A + Bx_i - y_i| \quad (3.73)$$

According to the least-square method, the 'best' line is obtained by minimising the sum of the squared deviations, that is

$$\sum_i (A + Bx_i - y_i)^2 \rightarrow \min \quad (3.74)$$

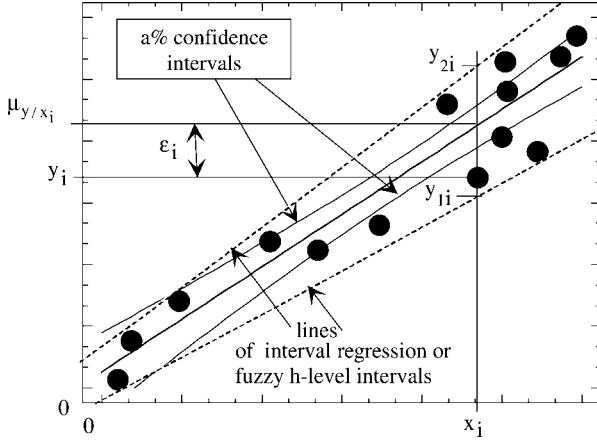


Figure 3.19 Definition of statistical and fuzzy linear regressions.

3.2.1.3 Interval Regression

Instead of considering X and Y as random variables it may be assumed that both or one of them are fuzzy numbers, noted as \tilde{X} and \tilde{Y} . A first approximation is to take X and Y as interval variables \bar{X} and \bar{Y} .

An interval \bar{C} is defined generally by an ordered pair of numbers c_1 and c_2 , where c_1 is the lower and c_2 the upper limit of \bar{C} , that is

$$\bar{C} = [c_1, c_2] = \{c : c_1 \leq c \leq c_2\} \quad (3.75)$$

Interval arithmetics is an extension of number arithmetics to closed intervals (Moore, 1979). For example addition and multiplication by a constant k are defined as

$$\bar{A} + \bar{B} = [a_1, a_2] + [b_1, b_2] = [a_1 + b_1, a_2 + b_2]$$

$$k\bar{A} = k[a_1, a_2] = [ka_1, ka_2]$$

Instead of the lower and upper limits c_1 and c_2 , an interval \bar{C} may be defined by its centre c_k and radius c_r . In this case we use the notation

$$\bar{C} = \langle c_k, c_r \rangle \quad \text{where } c_k = \frac{1}{2}(c_1 + c_2) \text{ and } c_r = \frac{1}{2}(c_2 - c_1) \quad (3.76)$$

Let us consider now the variables, \bar{Y}_i , \bar{A} and \bar{B} as the following intervals

$$\bar{Y}_i = \langle y_{ki}, y_{ri} \rangle, \quad \bar{A} = \langle a_k, a_r \rangle, \quad \bar{B} = \langle b_k, b_r \rangle \quad (3.77)$$

For a given set of observation pairs (y_i, x_i) the following linear regression equation is defined

$$\bar{Y}_i = \bar{A} + \bar{B} x_i \quad (3.78)$$

Introducing the expressions shown in Equation 3.77 into Equation 3.78, we obtain

$$\langle y_{ki}, y_{ri} \rangle = \langle a_k, a_r \rangle + \langle b_k, b_r \rangle x_i \quad (3.79)$$

or

$$\left. \begin{aligned} y_{ki} &= a_k + x_i b_k \\ y_{ri} &= a_r + x_i b_r \end{aligned} \right\} \quad (3.80)$$

The coefficients a_k , a_r and b_k , b_r determine two regression lines, which, as shown in Figure 3.19, are lines of interval regression. They are subject to two conditions:

- (a) the 'total' width between the two lines should be minimal,
- (b) data points must be located between these two lines (Figure 3.19).

The above two conditions are sufficient to formulate mathematically the problem of finding the coefficients a_k , a_r and b_k , b_r . In fact, a linear programming problem may be written as follows

$$\sum_i (y_{2i} - y_{1i}) = 2 \sum_i y_{ri} \rightarrow \min$$

or

$$\sum_i y_{ri} = \sum_i (a_r + x_i b_r) = (n a_r + \left(\sum_i x_i \right) b_r) \rightarrow \min \quad (3.81)$$

where y_{1i} and y_{2i} are the lower and upper coordinates of the regression lines (Figure 3.19). The above condition (i) for minimisation of the total width between the regression lines is mathematically expressed by Equation 3.81.

Conditions (ii) take the following mathematical form

$$y_i \leq y_{2i} = y_{ki} + y_{ri} = (a_k + x_i b_k) + (a_r + x_i b_r) \quad (3.82)$$

$$y_i \geq y_{1i} = y_{ki} - y_{ri} = (a_k + x_i b_k) - (a_r + x_i b_r) \quad (3.83)$$

3.2.1.4 Fuzzy Regression

The interval regression Equation 3.78 is now replaced by the fuzzy linear regression equation

$$\tilde{Y}_i = \tilde{A} + \tilde{B} x_i \quad (3.84)$$

where \tilde{Y}_i , \tilde{A} and \tilde{B} are triangular and symmetrical fuzzy numbers. Introducing the h -level interval (see Figure 3.20), Equation 3.79 is now replaced by the following

$$[y_{ki}, y_{ri}]_h = [a_k, a_r]_h + [b_k, b_r]_h x_i \quad (3.85)$$

It is easy to realise that in order to replace the interval regression lines by the h -level intervals, the fuzzy numbers \tilde{A} (see Figure 3.20) and \tilde{B} should be defined as

$$a'_k = a_k \quad b'_k = b_k \quad (3.86)$$

$$a'_r = \frac{a_r}{1-h} \quad b'_r = \frac{b_r}{1-h} \quad (3.87)$$

where the coefficients a_k , a_r and b_k , b_r are the same with these defined by Equations 3.81, 3.82 and 3.83. For example the value $h = 0.5$ may be chosen for the h -level

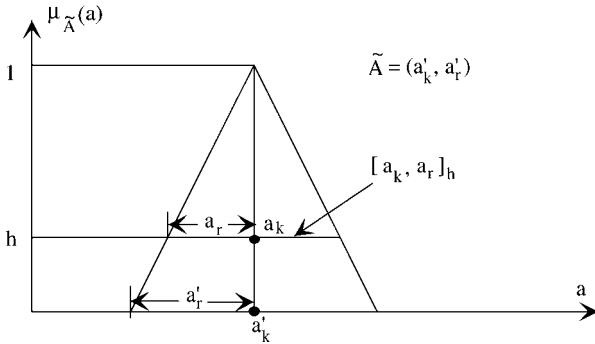


Figure 3.20 Definition of a triangular and symmetrical fuzzy number and h -level interval.

interval corresponding to the regression lines which contain all data points. The fuzzy numbers \tilde{A} and \tilde{B} can be computed from Equations 3.86 and 3.87.

Example 3.9

Temperature data are given in Table 3.8 from five measurements taken at two different depths 1 and 2 in a coastal area. Find a statistical and a fuzzy regression between the two series of data.

Because the number of data is small, application of the statistical regression appears to be problematic. However, using linear regression and minimising the sum of squared deviations given by Equation 3.64, we obtain the results shown in Figures 3.21 and 3.22.

The 'best' fitting line of the form $Y = A + BX$ has the following coefficients

$$A = 3.935 \pm 7.5602 \quad \text{and} \quad B = 0.4125 \pm 0.4368$$

The correlation coefficient is 0.4787 and the statistical hypothesis of linear regression is rejected. This means that for these data the statistical regression is not valid and that no further use of the data looks possible on a statistical framework.

The fuzzy linear regression (3.84) has been applied as an alternative, and the results obtained are shown in Figure 3.23. The regression lines obtained as a solution to the linear programming problem Equations 3.81, 3.82 and 3.83 are taken as 0.5-level confidence intervals. It has been found that

$$[a_k, a_r]_{h=0.5} = [0.0, 0.0] \quad \text{and} \quad [b_k, b_r]_{h=0.5} = [0.579, 0.233]$$

Table 3.8 Temperature data in depths 1 and 2.

	T_1 ($^{\circ}\text{C}$)	T_2 ($^{\circ}\text{C}$)
1	11.2	9.1
2	13.7	10.2
3	15.0	9.3
4	21.4	7.4
5	22.4	18.2

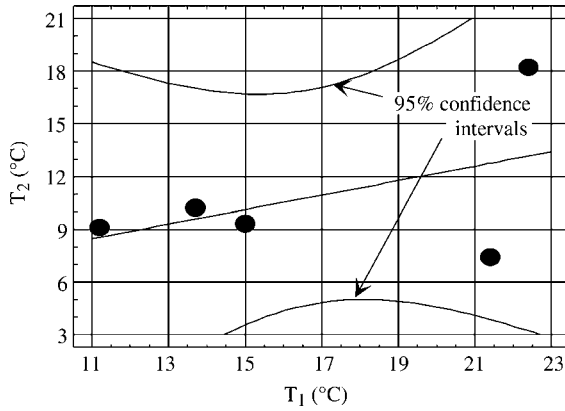


Figure 3.21 Statistical linear regression of a small sample of five data.

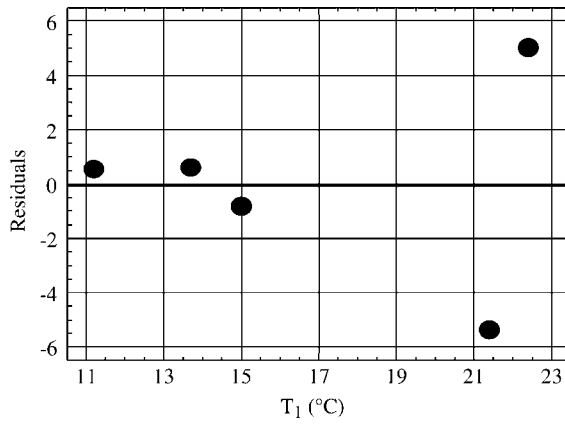


Figure 3.22 Residuals for linear regression of a small sample of five data.

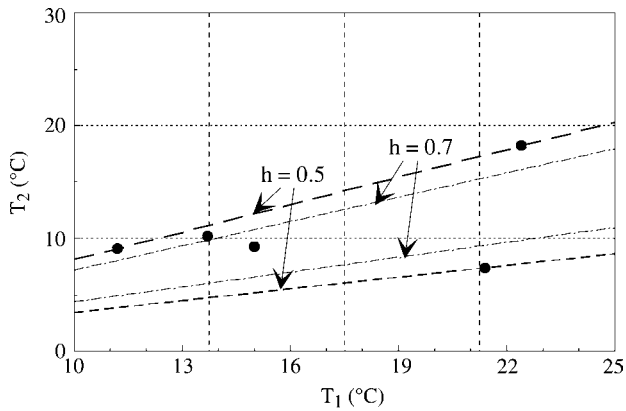


Figure 3.23 Fuzzy linear regression of a small sample of five data.

By use of Equations 3.86 and 3.87 we have

$$\tilde{A} = (a'_k, a'_r) = (0.0, 0.0) \quad \text{and} \quad \tilde{B} = (b'_k, b'_r) = (0.579, 0.466)$$

Example 3.10

Examine whether a statistical or fuzzy linear regression may be applied to temperature data shown in Table 3.9. Data temperature time series are given at three different depths. Also the ambient or atmospheric temperature, which is available

Table 3.9 Temperature (T) data in a coastal area.

t (days)	T_s	T_m	T_b	T_a
15	22.1	17.2	14	16.1
55	25.2	23.1	17.2	26.1
286	8.6	7.4	6.4	12.2
375	23.0	19.2	16.2	19.9
411	23.2	21.4	19.0	26.2
473	22.3	22.4	22.4	27.1
551	14.3	14.1	14.3	15.2
644	10.9	11.2	11.1	7.4
691	16.1	13.7	13.0	7.3
761	22.4	21.6	17.3	22.5
841	25.8	24.1	23.7	26.4
979	9.1	9.3	9.0	5.1
1078	18.4	14.9	13.3	12.6
1119	24.0	19.5	16.7	18.5
1133	25.1	24.5	20.7	20.6
1427	18.1	14.3	13.1	
1622	15.3	16.8	17.2	
1632	13.4	13.4	13.4	
1694	7.3	7.2	7.2	
1764	13.2	11.7	11.4	
1833	24.1	18.8	17.7	
1933	23.7	23.4	21.8	
2024	10.1	10.4	10.6	
2107	10.3	9.6	9.6	
2229	27.3	24.2	20.1	
2339	20.8	20.6	20.6	
2365	15.5	15.8	17.7	
2471	8.2	7.8	7.2	
2545	21.8	16.2	14.1	
2592	26.7	25.6	19.4	
2723	16.6	16.2	16.2	
2868	17.1	15.4	13.6	
3037	25.3	24.8	24.3	
3118	13.2	12.4	10.1	

s , surface; m , mean depth; b , bottom; a , atmospheric.

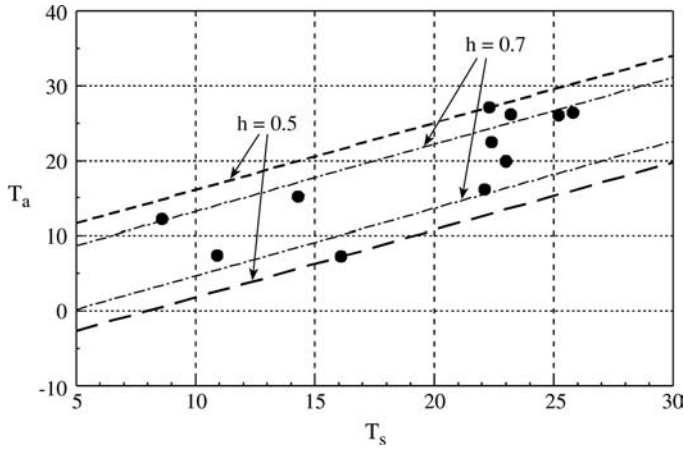


Figure 3.24 Fuzzy linear regression of air versus surface temperature.

for $t < 1427$ days is given. Surface temperature should be considered as the independent variable.

Results of fuzzy linear regression are presented in Figure 3.24. The fuzzy regression coefficients are given by

$$\tilde{A} = (a'_k, a'_r) = (0.0, 14.24) \quad \text{and} \quad \tilde{B} = (b'_k, b'_r) = (0.896, 0.0)$$

There is no restriction in using the above fuzzy regression. It has been established for the 11 first rows of data and verified for the remaining three rows.

Results of statistical regression, which look to be acceptable, are presented in Figures 3.25 and 3.26. The correlation coefficient is equal to 0.833 and the standard error of estimation is 4.418.

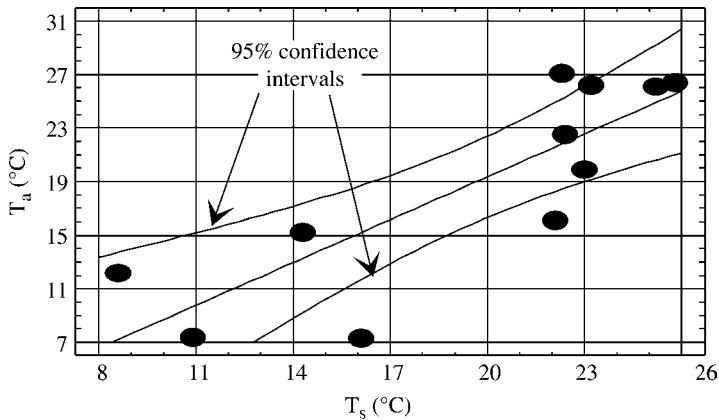


Figure 3.25 Statistical linear regression of air versus surface temperature.

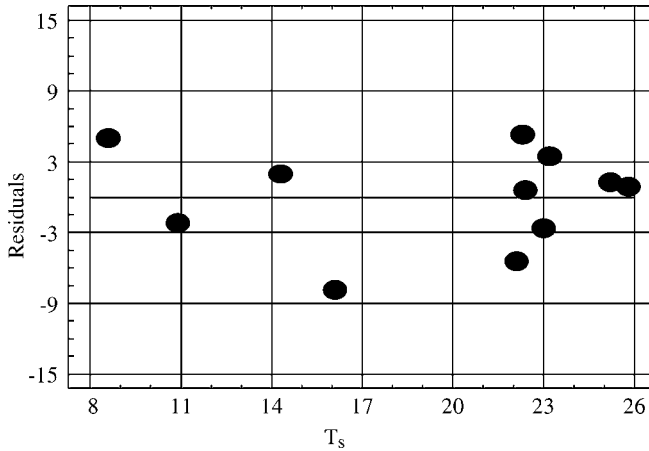


Figure 3.26 Residuals in statistical linear regression of air versus surface temperature.

3.2.2

Fuzzy Modelling

We have seen how fuzzy regression may be useful to express functional relationships between variables, when data are not sufficient in number. In this case statistical analysis fails and it is questionable how to use the data. Fuzzy set theory may provide an alternative methodology for modelling uncertainties and quantifying risk and reliability.

Fuzzy modelling has not yet been extensively developed, although fuzzy numbers and fuzzy relationships have found many applications in control engineering and industrial devices. Especially in Japan, many companies have recently produced a large number of washing machines, air-conditioners, auto-focus cameras and other devices, which are regulated by fuzzy rules. In many cases up to 20% of the energy requirement is saved and control is very smooth and efficient. A very well known example of the application of fuzzy automatic control is the subway in the city of Sendai (Japan). The train operates so smoothly that there is no need to use handrails.

Fuzzy arithmetic, which, as has been seen in Chapter 1, is a generalisation of interval arithmetic, may be applied to deterministic functions relating to fuzzy variables. Take for example the pollutant load in a river, given by the following simple equation

$$M = C Q \quad (3.88)$$

where C is the pollutant concentration (mass/time), Q the river flow rate (volume/time) and M the pollutant mass flow rate (mass/time). If C and Q are considered as fuzzy numbers, then the pollutant load should also be fuzzy in the form

$$\tilde{M} = \tilde{C} \otimes \tilde{Q} \quad (3.89)$$

where the symbol \otimes denotes multiplication between fuzzy numbers as defined in Appendix B.

If instead of the pollutant concentration C a deterministic function of the form $f(C) = aC^n + b$ is valid, then, from Equation 3.88, we have

$$M = (aC^n + b) Q = F(C, Q) \quad (3.90)$$

If the variables C and Q in Equation 3.90 are fuzzy numbers, then application of the extension principle (see Appendix B) may be the tool for computing the membership function of the fuzzy pollutant mass flow rate, that is

$$\mu_{\tilde{M}}(m) = \begin{cases} \max(\min \mu_{\tilde{C}}(c), \mu_{\tilde{Q}}(q)) & \text{if } m = F(c, q) \\ 0 & \text{otherwise} \end{cases} \quad (3.91)$$

Fuzzy arithmetic and the extension principle may be used, when for fuzzy modelling, numerical integration of deterministic partial differential equations having fuzzy values is considered. In this context we should mention that integration and differentiation of functions of real variables, which can take real values, have been defined (Aubin, 1990). This is very important in determining the dynamics of physical hydrologic systems by fuzzy modelling. Although only a few applications of fuzzy modelling are available in water resources engineering and water quality, recent theoretical advances are very promising (Ganoulis *et al.*, 2003; Mpimpas *et al.*, 2008).

3.3

Time Dependence and System Risk

3.3.1

Failure and Reliability Functions

So far we have seen that risk and reliability analysis may be essentially based on the comparison between loads and resistances and on a probabilistic or fuzzy modelling of the system. However, there is another approach, which is formally different from those described earlier. It is based on the observation that a physical system or its component may be considered as a whole entity, operating safely or not, as a function of a single variable. This variable is usually the time t , but it could be another variable describing the state of the system, such as the distance or the number of cycles.

Let T denote the random variable which, when the system fails, takes as an observed value the value t . For example, if the time series of nitrate concentration in a river indicates the degree of pollution, as shown in Figure 3.27, $T = t$ is the time at which the nitrate concentration exceeds the maximum allowed value as given by the water quality standards (failure condition).

To illustrate the fact that t may be another variable other than the time, consider the entrance of a non-wetting fluid 1 (e.g. air) in a porous medium, which is completely saturated by a wetting fluid 2 (e.g. oil) (Figure 3.28).

The entering fluid should be considered as remediating the soil, which has been polluted by fluid 2. Failure occurs when, because of the capillary forces, the entering fluid is blocked at distance $X = x$, where capillary pressure is balanced by the action

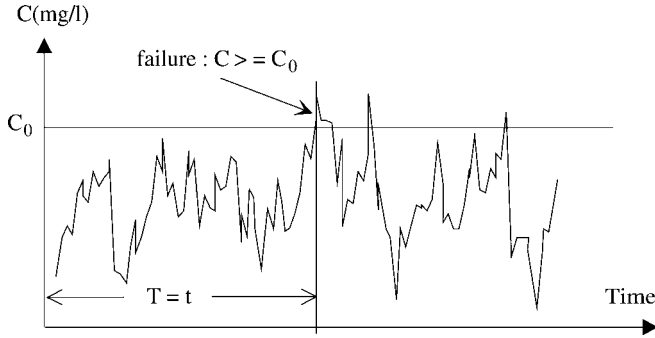


Figure 3.27 Failure time $T = t$ of a time series (concentration of nitrates in mg/l).

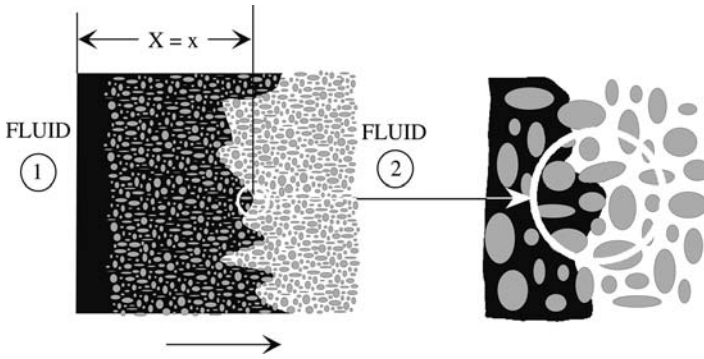


Figure 3.28 Failure distance $X = x$ of intrusion of a non-wetting fluid in a porous medium.

of capillary forces (critical porous openings). This example will be further explained later in this section.

The probability of failure at time t may be expressed by the *failure distribution* $F(t)$ or *unreliability function* $F(t)$, where

$$F(t) = P(T \leq t) \quad t \geq 0 \quad (3.92)$$

The reliability of the system, that is, the probability that the system will operate successfully at time t is given by the following equation

$$R(t) = 1 - F(t) = 1 - P(T \leq t) = P(T > t) \quad (3.93)$$

This is known as the *reliability function* of the system.

If we introduce the probability density distribution function of failure $f(t)$ then we have

$$F(t) = \int_0^t f(t) dt \quad (3.94)$$

and

$$R(t) = 1 - F(t) = 1 - \int_0^t f(t) dt = \int_t^\infty f(t) dt \quad (3.95)$$

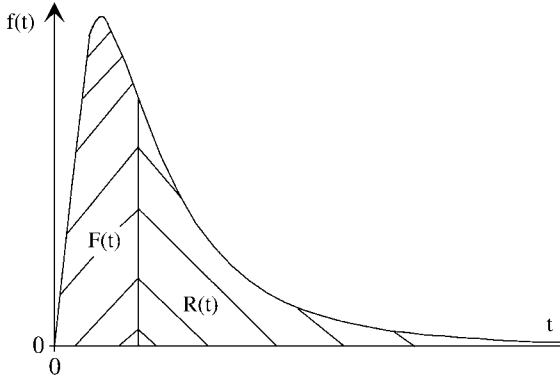


Figure 3.29 Graphical representation of functions $R(t)$ and $F(t)$.

The graphical representation of the reliability and unreliability functions $F(t)$ and $R(t)$ is given in Figure 3.29.

3.3.2

Failure Rate and Hazard Function

The *failure rate* is defined as the probability per unit time that the system will fail in a certain interval $[t_1, t_2]$, given that no failure has occurred prior to time t_1 , where $t_1 < t_2$. The above definition is mathematically expressed by

$$\text{failure rate} = \frac{F(t_2) - F(t_1)}{(t_2 - t_1)R(t_1)} \quad (3.96)$$

The *hazard function* or hazard rate $1(t)$ is defined as the limit of the failure rate, when the time interval tends to zero. From the Equation 3.96 we have

$$\lambda(t) = \lim_{\Delta t \rightarrow 0} \frac{F(t + \Delta t) - F(t)}{\Delta t R(t)} = \frac{f(t)}{R(t)} \quad (3.97)$$

A physical interpretation of the hazard function may be given as

$$\lambda(t) = \frac{\text{failure density } f(t)}{\text{no of components not yet failed}}$$

It is of interest to find the general form of the failure function when the hazard rate is constant. From the Equation 3.97 we have

$$\lambda(1 - F(t)) = \frac{dF(t)}{dt}, \quad \text{or} \quad \frac{dF(t)}{1 - F(t)} = \lambda dt$$

Integrating, we obtain

$$-\ln(1 - F(t)) = \lambda t$$

and

$$\begin{aligned} F(t) &= 1 - e^{-\lambda t} \\ f(t) &= \lambda e^{-\lambda t} \end{aligned} \quad (3.98)$$

The above results indicate that if the hazard rate is constant, the failure probability is of an exponential type. The mean expected time or the Mean-Time-to-Failure (MTTF) is

$$MTTF = \int_0^{\infty} tf(t)dt = \int_0^{\infty} \lambda e^{-\lambda t} dt$$

Integrating by parts we have

$$MTTF = \frac{1}{\lambda} \quad (3.99)$$

3.3.3

Expected Life

This is defined as the average time or the Mean-Time-to-Failure, during which the system or its components operate safely. Thus we have

$$MTTF = \int_0^{\infty} tf(t)dt \quad (3.100)$$

It can be shown that MTTF may be computed in terms of the reliability function as

$$MTTF = \int_0^{\infty} R(t)dt \quad (3.101)$$

Integrating by parts the above equation, we have

$$\begin{aligned} [R(t)]_0^{\infty} - \int_0^{\infty} t dR(t) &= \int_0^{\infty} tf(t)dt = MTTF \\ \text{because if } t \rightarrow \infty \text{ then } \lim[tR(t) \rightarrow 0] \text{ and if } t \rightarrow 0, \quad \lim[tR(t) \rightarrow 0] \end{aligned}$$

Example 3.11

Find the hazard rate for the intrusion of a non-wetting fluid 1, shown in Figure 3.28, in a porous medium saturated by a wetting fluid 2. Assume that all pores may be approximated by a bundle of capillary tubes of constant length ℓ_p with variable radius r .

The fluid 1, which is entering the porous medium is blocked when capillary forces balance the capillary pressure, which is the driving force. As shown in Figure 3.30,

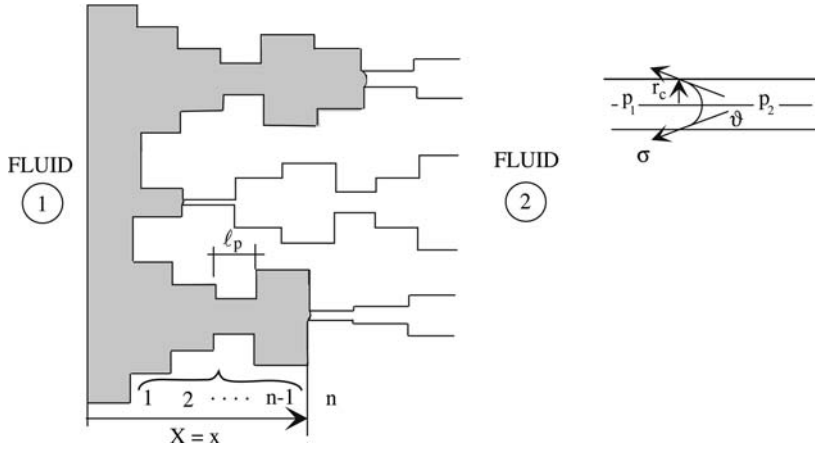


Figure 3.30 Capillary intrusion of a non-wetting fluid in a porous medium.

this happens when a capillary pore has a *critical size*. A capillary tube has a critical radius r_c , when the following equilibrium condition holds

$$\Delta p = p_1 - p_2 = \frac{2\sigma \cos \vartheta}{r_c} \quad (3.102)$$

where

Δp is the capillary pressure,

σ the surface tension,

ϑ the contact angle, and r_c the critical radius.

Thus the failure condition, which means that fluid 1 is physically blocked by the capillary forces, is

$$\text{failure condition: } r \leq r_c \quad \text{subcritical pore} \quad (3.103)$$

On the contrary, there is no failure if the pore is supercritical and the fluid can penetrate, that is, if

$$\text{reliability condition: } r > r_c \quad \text{supercritical pore} \quad (3.104)$$

Now the probability of failure may be expressed in terms of the *pore size density distribution* function $f(r)$, where $f(r) dr$ denotes the proportion of pores having a radius r in the interval $[r, r + dr]$. We will have

$$\text{reliability: } a = P(r > r_c) = \int_{r_c}^{\infty} f(r) dr \quad (3.105)$$

and

$$\text{probability of failure: } \bar{a} = 1 - a = P(r \leq r_c) = \int_0^{r_c} f(r) dr \quad (3.106)$$

As shown in Figure 3.30, fluid 1 penetrates to a distance $X = x$, when there are $n - 1$

supercritical pores ($r > r_c$) before the first subcritical pore ($r \leq r_c$). The intrusion probability is

$$P(N = n) = a^{n-1}(1-a) \quad (3.107)$$

and the failure distribution function

$$F_N(n) = P(N \leq n) = \sum_{i=1}^n a^{i-1}(1-a) = 1-a^n \quad (3.108)$$

Taking into account the length of the pores ℓ_p , the number of pores n between $[0, x]$ is equal to the integer number, which is less or equal to x/ℓ_p . Considering x as a continuous variable, from Equation 3.108 we obtain

$$F(x) = 1 - \exp\left(\ln a \frac{x}{\ell_p}\right) = 1 - \exp(-\lambda x) \quad (3.109)$$

Thus, the failure distribution function $F(x)$ is exponential (see Equation 3.98) with a constant hazard rate, given by

$$\lambda = -\frac{\ln a}{\ell_p} \quad (3.110)$$

The result expressed by Equation 3.110 means that if the pore size distribution is known, the hazard rate may be evaluated by use of Equations 3.105 and 3.110.

3.3.4

System Risk and Reliability

3.3.4.1 Series Systems

A series system is represented schematically in Figure 3.31. When a system is composed of elements connected in series, it is reliable if all the components operate successfully. In fact, the system fails if any one of the components is out of order. This is known as the ‘weakest line’ system.

Probabilistic Risk and Reliability Let E_i denote the failure of component i and E_S the failure of the system. For a series system we have

$$\begin{aligned} E_S &= E_1 \text{ or } E_2 \dots \text{ or } E_n \\ &= E_1 \cup E_2 \dots \cup E_n \end{aligned}$$

The *system risk* or the probability of failure of the system is expressed as

$$p_F = P(E_S) = P(E_1 \cup E_2 \dots \cup E_n) \quad (3.111)$$

If $R_i = 1 - E_i$ denotes the successful operation of the component i , then the proper operation of the series system R_S means that all the components operate properly. This condition takes the form

$$R_S = (R_1 \text{ and } R_2 \dots \text{ and } R_n) = (R_1 \cap R_2 \dots \cap R_n)$$

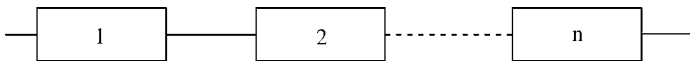


Figure 3.31 Schematic representation of a series system.

For every component we have

$$P(R_i) = 1 - P(E_i) \quad (3.112)$$

and for *independent* operating conditions the *system reliability* is

$$\begin{aligned} P(R_S) &= P(R_1 \cap R_2 \dots \cap R_n) = P(R_1)P(R_2) \dots P(R_n) \\ &= \prod_1^n P(R_i) \end{aligned} \quad (3.113)$$

This is known as the *product rule of reliability*. From this rule we conclude that the system reliability of independent components is bounded as follows

$$0 \leq P(R_S) \leq \min_i P(R_i) \quad (3.114)$$

If a is the probability that a component will fail, then assuming that a is constant for all components, from Equations 3.112 and 3.113 we obtain

$$P(R_S) = (1-a)^n$$

For small values of a , in a first approximation, the system reliability is

$$P(R_S) = 1 - an$$

For example if $a = 10^{-4}$ and $n = 10$ then $P(R_S) = 1 - 10^3 = 0.9990$.

Fuzzy Risk and Reliability Consider now that every component has a resistance and load, both represented by the fuzzy numbers \tilde{R} and \tilde{L} . The safety margin, which has been defined in the case of stochastic variables by Equation 3.8, is now the fuzzy number

$$\tilde{M} = \tilde{R} - \tilde{L} \quad (3.115)$$

The h -level internals of \tilde{R} and \tilde{L} are

$$R(h) = [R1(h), R2(h)] \quad L(h) = [L1(h), L2(h)]$$

For every $h \in [0,1]$, the *safety margin* $M(h)$ is the difference between $R(h)$ and $L(h)$,

$$M(h) = R(h) - L(h) \quad (3.116)$$

As we have seen in Section 2.6 two possible conditions exist (Shrestha *et al.*, 1990)

- (a) Failure : $M(h) \leq 0 \quad \forall h \in [0, 1]$
- (b) Reliability : $M(h) > 0$

A series system will operate successfully if all the components are reliable. If M_i is the safety margin of the i component, then the safety margin of the system M_s is

$$\tilde{M}_S = (M_1 \text{ and } M_2 \dots \text{ and } M_n) = (M_1 \cap M_2 \dots \cap M_n) \quad (3.117)$$

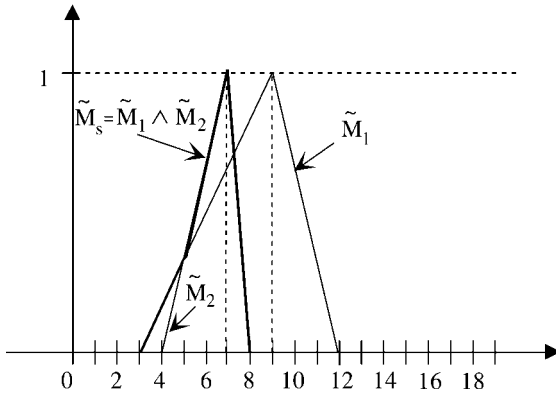


Figure 3.32 Fuzzy reliability of a series system.

According to the fuzzy rule for the intersection of fuzzy numbers (Appendix B), the membership function of the intersection is the minimum of individual memberships. According to the definition of the minimum of fuzzy numbers, we will have (Figure 3.32)

$$\mu_{\tilde{M}_s}(m) = \min(\mu_{\tilde{M}_1}(m), \mu_{\tilde{M}_2}(m), \dots, \mu_{\tilde{M}_n}(m)) \quad (3.118)$$

3.3.4.2 Parallel Systems

As shown in Figure 3.33, the system is composed of parallel sub-systems. It may continue operating properly if any one of the components is still reliable. The failure of such a system requires the failure of all the components.

Probabilistic Risk and Reliability By using the same notations as for a series system, the failure E_S of the system is given by

$$E_S = E_1 \text{ and } E_2 \dots \text{ and } E_n = E_1 \cap E_2 \dots \cap E_n \quad (3.119)$$

and the reliability by

$$R_S = R_1 \text{ or } R_2 \dots \text{ or } R_n = R_1 \cup R_2 \dots \cup R_n \quad (3.120)$$

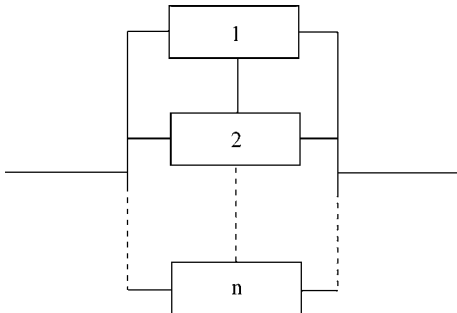


Figure 3.33 Schematic representation of a parallel system.

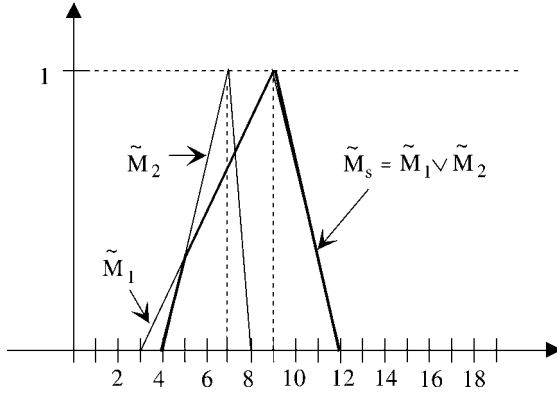


Figure 3.34 Fuzzy reliability of a parallel system.

Assuming independence between individual components, from Equation 3.119 we derive the risk of a parallel system as

$$\begin{aligned} P(E_S) &= P(E_1 \cap E_2 \dots \cap E_n) = P(E_1)P(E_2) \dots P(E_n) \\ &= \prod_{i=1}^n P(E_i) = \prod_{i=1}^n \{1 - P(R_i)\} \end{aligned} \quad (3.121)$$

The system reliability is the complement of the system risk, that is

$$P(R_S) = 1 - \prod_{i=1}^n \{1 - P(R_i)\} \quad (3.122)$$

Fuzzy Risk and Reliability The system is safe if any one of its components operate safely. In terms of the safety margin of the system and its components, we have

$$\tilde{M}_S = (M_1 \text{ or } M_2 \dots \text{ or } M_n) = (M_1 \cup M_2 \dots \cup M_n) \quad (3.123)$$

According to the fuzzy rule for the union of fuzzy numbers (Appendix B), the membership function of the union is the maximum of individual memberships. According to the definition of the maximum of fuzzy numbers, we will have (Figure 3.34)

$$\mu_{\tilde{M}_S}(m) = \max(\mu_{\tilde{M}_1}(m), \mu_{\tilde{M}_2}(m), \dots, \mu_{\tilde{M}_n}(m)) \quad (3.124)$$

3.4

Questions and Problems – Chapter 3

Stochastic Approach

- The demand for urban drinking water and the supply of drinking water are variable and follow probability distributions with mean values and standard deviations given by the following table:

Variable	Mean value	Standard deviation
Demand	3	1
Supply	5	0.75

Calculate the risk and the reliability of the water supply system assuming that both variables above follow

- (a.1) normal distributions,
 - (a.2) log-normal distributions.
- (b) The concentration of a toxic substance causing the death of fish in a lake is called lethal concentration C_λ .
- (b.1) Three industries 1, 2 and 3 release from time to time wastewaters containing this toxic substance. The three corresponding releasing events are E_1 , E_2 and E_3 . If C is the concentration of the toxic substance measured at one station in the lake, give at least two expressions for having lethal risk in the lake.
 - (b.2) The probabilities of E_1 , E_2 and E_3 are 0.25, 0.50 and 0.25. If A is the event of lethal contamination and the conditional probabilities are $P(A/E_1)=0.76$, $P(A/E_2)=0.25$, $P(A/E_3)=0.05$, calculate the probability of A when all three industries are active.
 - (b.3) Given that A has occurred, what is the probability of each industry being responsible?
- (c) A 100-year flood means
- (c.1) a flood that occurs only once every 100 years,
 - (c.2) the probability of such a flood occurring next year is 1:100,
 - (c.3) the flood will occur after 100 years.

Fuzzy Modelling

- (a) The groundwater velocity of pollutants in one-dimensional flow without dispersion is given by the following Darcy's law:

$$V = \frac{Ki}{\eta R} \quad (3.125)$$

where

K is the hydraulic conductivity

i the hydraulic gradient

η the soil porosity

R the retardation factor

Because of various uncertainties, all the above variables have one lower and one upper limit and therefore they are represented by the following intervals:

$$K = [100-1000] \text{ m.yr}^{-1}, i = [10^{-4}-10^{-3}], \eta = [0.1-0.3], R = [5-80].$$

If the point pollutant source is located 100 m away from a groundwater well, calculate how long it may take for the pollutants to reach the well.

How you can generalise the above calculations if K , i , η , and R are fuzzy numbers rather than intervals?

- (b) Calculate the time needed to reach the well if the retardation factor is given by the following expression:

$$R = 1 + (\rho K_d / \eta) \quad (3.126)$$

where

ρ is the soil density = [1500 – 1700] kg/m³

K_d = the soil–water partition coefficient = 10^{-3} m³ kg⁻¹

For the calculation you may use the following equation that is derived by combining the Equations 3.125 and 3.126

$$V = \frac{Ki}{\eta(1 + \rho(K_d/\eta))} \quad (3.127)$$

- (c) In Equation 3.127 the soil porosity occurs twice. Instead of using Equation 3.127 you may first write this equation as

$$V = \frac{Ki}{\eta + \rho K_d} \quad (3.128)$$

Calculate again the time needed to reach the well by using the previous data and Equation 3.127. What can you conclude?

Time Dependence and System Risk

- (a) We know that if the ‘failure rate’ of a pump λ is constant its ‘failure function’ is the exponential function.

- (a.1) two pumps are connected ‘in series’ and have the following failure rates

$$\lambda_1 = 4 \cdot 10^{-4} \text{ hours}^{-1}, \lambda_2 = 6 \cdot 10^{-4} \text{ hours}^{-1}$$

Find the failure rate of the system.

- (a.2) Find the system’s reliability at $t = 2000$ hours.

- (a.3) Calculate the system’s mean time to failure.

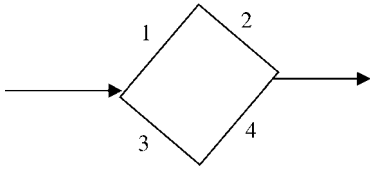
- (b) Cylindrical pipes may be connected ‘in series’ or ‘in parallel’. If the risk of failure of the i pipe is p_i

- (b.1) Find the risk and the reliability of n pipes connected in series.

- (b.2) Find the risk and the reliability of n pipes connected in parallel.

- (b.3) Calculate the above risks and reliabilities for $n = 10$ and if every pipe has equal risk of failure $p = 0.01$.

- (c) In the water supply system shown in the following figure, every pipe has a risk of failure equal to $p = 0.01$.



Find the risk and the reliability of the water supply system.

4

Risk Assessment of Environmental Water Quality

There are many causes leading to pollution in coastal areas, rivers and aquifers. Added to the effluents from *municipal* and *industrial* sources there is pollution created by *natural* causes, such as the carry-over of nutrients and sediments in river deltas. Agricultural activities may overload soils with fertilisers and pesticides. Washing-off of the soil by rainfall produces high concentrations of nitrates, phosphorus and toxic chemicals in rivers, aquifers and coastal waters.

It has been recognised that nowadays water pollution is one of the most crucial environmental problems world-wide and especially in Europe. Apart from local sources discharging wastewaters, pollution in aquifers and surface waters originates mainly from diffuse sources scattered over the entire river basin. The substantial increase in recent years in the quantity of effluents from human activities has also led to serious water pollution. This is particularly so in cases where the water body is enclosed or suffers from weak circulation.

The problem facing engineers is the prediction in space and time of the concentration of a pollutant substance introduced into the water body. The analysis of this problem using mathematical or physical models may assist in the optimal design of wastewater treatment plants, the positioning of wastewater outfalls and the determination of the flow rate and composition of effluents at the discharge outlet. The basic criterion for such an investigation is the need to ensure that a given pollution limit is not exceeded in water areas of particular interest (e.g. areas that are important economically and for tourism). The maximum allowable concentration limits are obviously a matter for legislation and standards, and should take into account the protection of the water ecosystems (phytoplankton, zooplankton, fishing, etc.) and also economic, aesthetic and cultural factors.

In grouping different pollutants we may distinguish between liquid and solid pollutants. The liquid effluents may either dissolve (urban wastewater) or not (crude oil) in the water. Also they may or may not consist of conservative pollutants. Non-conservative pollutants are those whose concentration is increased or reduced locally due to biological, chemical, radioactive or other interactions. On the other hand, solid pollutants, are conservative and may consist of particles which are either fine (of the order of $1\ \mu\text{m}$) or coarse (of the order of $1\ \text{mm}$).

From the engineering point of view, environmental water quality is subject to several types of uncertainty. These are related to the high variability in space and time of the hydrodynamic, chemical and biological processes involved. Quantification of such uncertainties is essential for the performance and safety of engineering projects.

This chapter first examines the case of coastal water pollution, where land-based treatment of wastewaters is considered together with the design of the disposal of effluents in the marine environment. In coastal areas, for example, a submarine outfall can be used at the discharge of the treatment plant. A short outfall is sufficient if tertiary treatment of sewage is carried out, involving not only decantation and biological degradation but also nitrification and denitrification. When only primary treatment is used, a long outfall could be used instead in order to meet the environmental objectives. Various local constraints usually impose limiting factors on the design of effluent disposal. These are related to the regional development of the area, the land uses and the economic capabilities of the responsible sewerage board. The use of advanced engineering tools, such as risk analysis and computerised mathematical modelling techniques, may reduce uncertainties in the design.

4.1

Risk in Coastal Water Pollution

4.1.1

Uncertainties in Coastal Water Quality Processes

The general problem of coastal water quality risk assessment may be stated as follows.

Time series of a given pollutant concentration have been recorded at one characteristic station S near the river mouth (Figure 4.1). The general question is under which circumstances is there a risk of pollution in another characteristic location M shown in Figure 4.1? In the same figure, the pollutant concentration contours are shown. Other available data are the current velocity components u and v , in the form of time series recorded at station R (Figure 4.1).

Environmental quality standards (EC directives) provide the allowable levels of pollution in terms of percentile values C_p of pollutant concentration. These are pollutant levels not to be exceeded by, at least, $p\%$ of the samples. In terms of probability, there is no pollution if

$$P(C_M < C_p) \geq p\% \quad (4.1)$$

where: $P(\cdot)$: is the probability; C_M : the pollutant concentration at the station M; C_p : the percentile of the allowed pollutant concentration; p : a fixed level of confidence, for example 80%.

There is risk of pollution if

$$P(C_M \geq C_p) > (1-p)\% \quad (4.2)$$

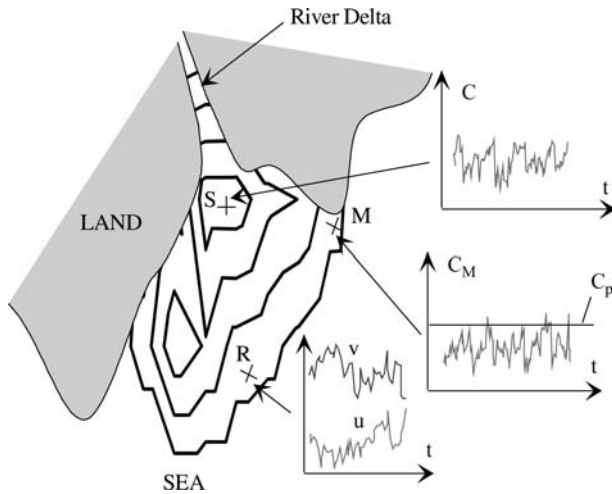


Figure 4.1 Risk of pollution in a coastal area near a river delta.

If $p = 80\%$ the condition represent by Equation 4.2 means that there is pollution if more than 20% of the samples exceed the allowed level of concentration.

In engineering risk and reliability analysis the probability $P(C_M \geq C_p)$ is known as the engineering risk. In this case C_M is taken as a stochastic variable; thus stochastic mathematical modelling may be used to evaluate the probability. Alternatively, C_M may be considered as a fuzzy number; fuzzy calculus may be applied to obtain the engineering risk.

There are two principal tasks in coastal water quality engineering risk analysis:

- assess the risk of non-compliance with the environmental quality standards in the water body to be protected, and
- quantify the risk of exceeding the maximum receiving capacity of the coastal area.

The environmental quality standards are defined on the basis of frequency of occurrence of the most critical pollutants in relation to the water uses. For example, in shellfish-growing coastal waters, bacterial contamination is the most critical type of pollution. EC standards require that 80% of the samples do not exceed a number of fecal coliforms equal to 70 per 100 millilitres (Council of European Communities, 1979).

The maximum receiving capacity of a given pollutant in the marine environment is the maximum quantity of this pollutant which can be eliminated by physical, chemical and biological mechanisms, without any perturbation of the coastal ecosystem (Ganoulis, 1988b, 1989). This part of the design requires a good knowledge of the biological coastal environment, such as phytoplankton, zooplankton and benthic populations.

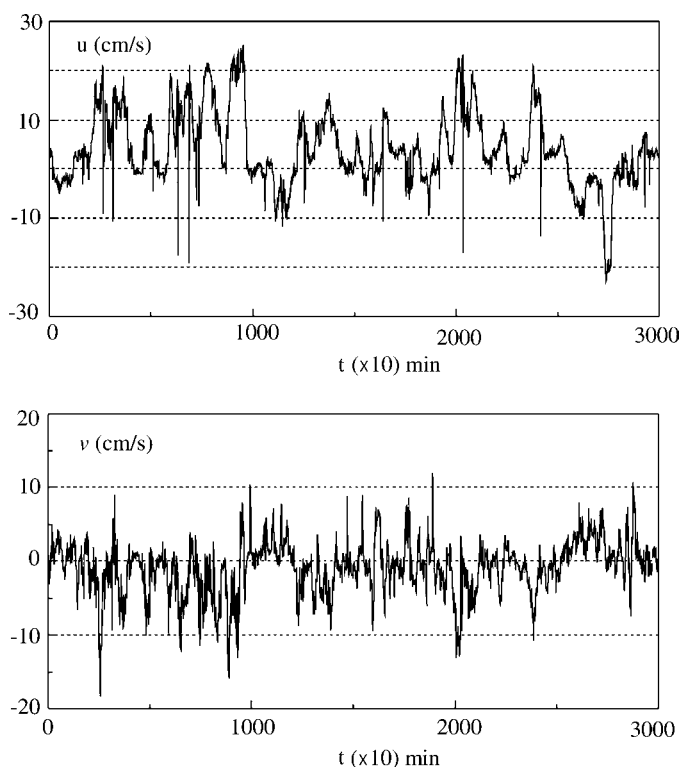


Figure 4.2 Time series of wind generated current components u and v .

The large size of the areas of interest (bays, coastal areas) and the relatively large time scales of interest (days, seasons), generate high variabilities in the processes involved both in time and space. Figure 4.2 shows a typical time history of the intensity of coastal currents recorded at one station.

Time and space scales of observation are very important for analysing the stochastic characteristics of hydrodynamic and water quality phenomena (Ganoulis, 1986). Marine pollution occurs on three scales: macro-, meso- and micro-scale or engineering scale. Some characteristics of these scales are summarised in Table 4.1.

Table 4.1 Space and time scales in coastal dispersion.

Scale	Characteristic horizontal length	Characteristic time	Example
Macro-scale	10^3 km	Season/year	Major ocean currents
Meso-scale	10^2 km	Month/week	Dispersion around an island in the Aegean sea
Microscale or engineering scale	1–10 km	Hour/day	Dispersion in a bay or coastal area

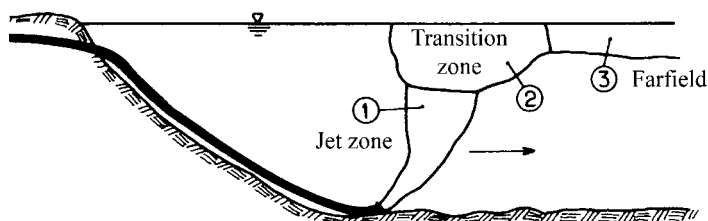


Figure 4.3 Dispersion from a submarine outfall.

Macro-scale dispersion usually refers to quasi-steady global ocean currents associated with synoptic eddies. Meso-scale currents and dispersion are characterised by density stratification and horizontal fronts, generating unsteady eddies. Micro-scale currents and pollutant dispersion are of particular interest in coastal engineering. This is the scale for the design of submarine outfalls, harbours and coastal protection structures.

Figure 4.3 shows a typical example of dispersion of pollutants discharging from a submarine outfall. Dispersion characteristics in coastal areas are highly variable in space and time. The values of the dispersion coefficient D range between 1 and $10 \text{ m}^2/\text{s}$. Direct measurements of D using floating drogues or rhodamine are of limited value, because they usually reflect specific conditions at the experimental site.

The near-field or jet zone (1) is distinguished from the far-field or the dispersion zone (3). In the jet zone, fluid from the surrounding region is entrained by turbulent flow, and dilution occurs by a factor of up to 10^3 as a result of mixing the wastewaters. In the far-field, the wastewaters are transported by currents and mixing is caused by turbulent diffusion. The dilution is one or two orders of magnitude smaller than in the jet zone. For non-conservative pollutants, chemical and biological interactions cause additional dilution to take place (Fischer *et al.*, 1979).

Although good experimental and mathematical information exists for the near-field problem, many questions concerning far-field dilution still remain unanswered. This is important for outfall design, because pollutants can be convected by local currents and reach regions of particular ecological interest. The time scales in this process vary from a few hours to a day, the dilution is slow compared to that in the near-field, but the bacterial decay can be important.

Randomly varying currents, turbulent dispersion and physicochemical interactions generate significant variations in physical and water quality parameters. As an example (Ganoulis, 1988a, 1988b, 1990), the time series of the water temperature and dissolved oxygen measured at two different depths (surface, bottom) at the same station is shown in Figure 4.4. The frequency of sampling in this case is seasonal (approximately 3 months), with irregular time intervals between samplings.

4.1.2

Mathematical Modelling

To evaluate the risk of pollution in coastal waters it is necessary to know the evolution in time and space of the pollutant concentration. In the general case, we

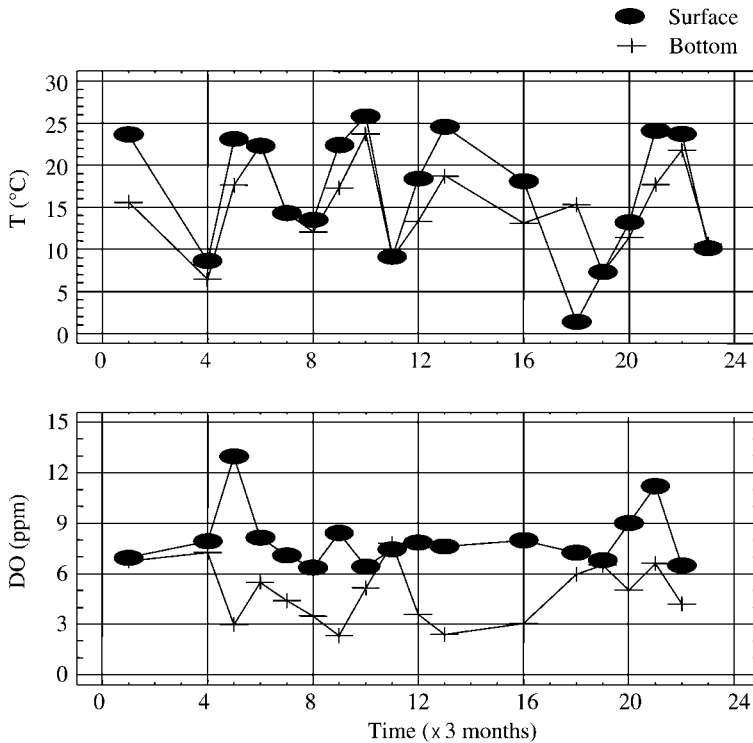


Figure 4.4 Time series of water temperature (T) and dissolved oxygen (DO) in a coastal area.

have three-dimensional variations in space of the pollutant concentration. In most cases of practical interest and far from the pollutant source, it is sufficient to know the two-dimensional pollutant concentration field, which is usually extended in a thin layer near the free surface of the sea.

The fate of a conservative pollutant concentration is formulated by means of the convective–diffusion equation. This is a partial differential equation expressing

- (a) the transport of pollutants by coastal currents, and
- (b) the turbulent diffusion process of pollutants in the sea.

To obtain the final form of the mathematical model, the mass conservation of the pollutant is taken into consideration. For non-conservative pollutants supplementary terms should be added to the convective–diffusion equation representing

- (c) the biochemical interactions between different pollutant constituents.

4.1.2.1 Molecular Diffusion

Let us derive the convective–diffusion equation step-by-step. For the time being, the current velocities are assumed to be zero. Even in this special case of zero transportation, the pollutant concentration varies because of the *diffusion process*. This is due to the non-uniform spatial distribution of the pollutant concentration (Figure 4.5).

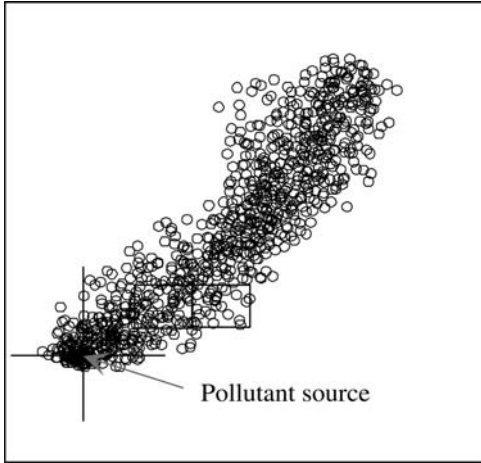


Figure 4.5 Two-dimensional diffusion of pollutant particles.

Diffusion occurs when the molecules of two fluids spontaneously mix together without any resulting chemical reaction. When both fluids are in hydrodynamic and thermal equilibrium mixing is based solely on molecular interactions. In this case the phenomenon is called *molecular diffusion*. If, simultaneously, the two fluids move and the flow is turbulent, the mixing process is more complicated and is called *turbulent diffusion*. For environmental engineering applications, clearly the main interest is in the mathematical and physical description of turbulent diffusion. However, for a better understanding of this phenomenon, it is useful to study molecular diffusion first.

This situation is shown in Figure 4.5, in the form of particles representing the pollutant mass. The number of these particles per box is equivalent to the pollutant concentration. If \vec{q} is the flux of pollutant, that is the mass flow rate per unit area, then according to Fick's law, \vec{q} is proportional to the gradient in space ∇C of the concentration C . Let us explain this law, which is phenomenological and of course non-universal, in one dimension. From the two-dimensional distribution of pollutant particles shown in Figure 4.5, two boxes containing N_1 and N_2 particles are considered. The rate of the number of particles crossing the boundary between the two adjacent boxes of dimensions Δx and Δy is computed as follows (Figure 4.6).

According to Fick's law, the pollutant flux q_x in the x direction is proportional to the difference in the number of particles per unit length that is, to the expression $(N_1 - N_2)/\Delta x$. This means that the component q_x of the flux \vec{q} is given by

$$\begin{aligned}
 q_x &= k \left(\frac{N_1 - N_2}{\Delta x} \right) = - \frac{k}{(\Delta x)(\Delta y)} \left(\frac{C_2 - C_1}{\Delta x} \right) \\
 &\cong -D \left(\frac{\partial C}{\partial x} \right)
 \end{aligned}
 \tag{4.3}$$

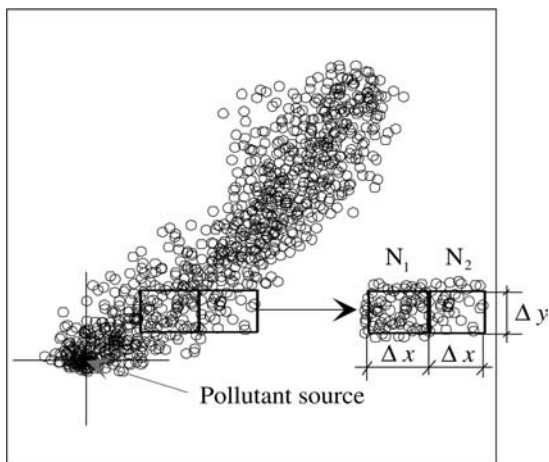


Figure 4.6 Longitudinal diffusion flux of pollutant particles.

where D is a coefficient of proportionality with dimensions $[L^2/T]$, called *diffusion coefficient* or *molecular diffusivity*. The minus sign in the Equation 4.3 is due to the fact that particles move from higher to lower concentrations. This means that, for a decreasing function $C = C(x)$, the derivative $\partial C / \partial x$ is negative and q_x positive. Using the same definition as in Equation 4.3 in the y -direction, the y component of the flux is written as

$$q_y = -D \left(\frac{\partial C}{\partial y} \right) \quad (4.4)$$

The expressions shown in Equations 4.3 and 4.4 lead to Fick's law in vector form

$$\vec{q} = -D(\nabla C) \quad (4.5)$$

which is the general expression in the three-dimensional space of the pollutant flux $\vec{q} = (q_x, q_y, q_z)$.

In the elementary volume of unit cross-section shown in Figure 4.7, the pollutant mass is equal to $C(x, t) (1) \Delta x$.

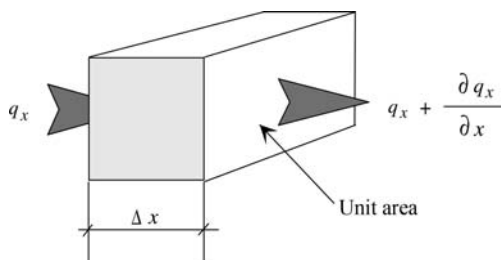


Figure 4.7 Longitudinal flux of pollutant in an elementary volume.

The time change rate of this mass is

$$\left(\frac{\partial C}{\partial t}\right)\Delta x \quad (4.6)$$

Mass conservation dictates that the above mass time rate should equal the mass flow rate crossing the unit surface (Figure 4.7), given by

$$q_x(x, t) - \left\{ q_x(x, t) + \frac{\partial q_x(x, t)}{\partial x} \Delta x \right\} = - \frac{\partial q_x(x, t)}{\partial x} \Delta x \quad (4.7)$$

Equating (4.6) and (4.7) yields the mass conservation equation

$$\frac{\partial C}{\partial t} \Delta x = - \left(\frac{\partial q_x}{\partial x} \right) \Delta x \quad (4.8)$$

Using Fick's law (Equation 4.3) we obtain

$$\frac{\partial C}{\partial t} = - \frac{\partial}{\partial x} \left(-D \frac{\partial C}{\partial x} \right) = D \frac{\partial^2 C}{\partial x^2}$$

This is the one-dimensional diffusion equation. In three dimensions, the second part may be completed with similar terms along the y and z directions, to yield

$$\frac{\partial C}{\partial t} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) \quad (4.9)$$

Now suppose that u is the velocity of currents in the x direction and that the flow is one-dimensional. Consider a unit area perpendicular to the x -axis as shown in Figure 4.7. The product $u C$ is the flux of pollutant mass in the x -direction.

This is the convective flux to be added to the diffusive flux in order to obtain the total flux q_x . Adding the two fluxes we obtain

$$q_x = uC - D \left(\frac{\partial C}{\partial x} \right) \quad (4.10)$$

In the three-dimensional space the above equation takes the form

$$\vec{q} = \vec{V} C - D(\nabla C) \quad (4.11)$$

where $\vec{V} : (u, v, w)$ is the velocity vector.

Applying the mass conservation Equation 4.8 to Equation 4.10 we obtain

$$\frac{\partial C}{\partial t} = - \frac{\partial}{\partial x} (uC) + D \frac{\partial^2 C}{\partial x^2} \quad (4.12)$$

In three dimensions the above equation is written as

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x} (uC) + \frac{\partial}{\partial y} (vC) + \frac{\partial}{\partial z} (wC) = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right)$$

For incompressible fluid we have $\text{div} \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$. Using this condition, from Equation 4.12 we obtain the general form of the convective–diffusion equation

as

$$\frac{\partial C}{\partial t} + \frac{u\partial C}{\partial x} + \frac{v\partial C}{\partial y} + \frac{w\partial C}{\partial z} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) \quad (4.13)$$

In Equation 4.13 the molecular diffusion coefficient is a numerical constant. We will see below how this differs in the case of turbulent flows.

4.1.2.2 Turbulent Diffusion

The main attribute of turbulent flow is the random, stochastic variations of all flow characteristics both temporally and spatially. This applies to the basic variables which describe the motion, such as velocities and pressures, but also includes the concentration of pollutants carried by the flow. For the instantaneous variables C and \bar{V} , Equation 4.13 also applies to turbulent flow; and it can be further transformed into

$$\frac{\partial C}{\partial t} + V_i \frac{\partial C}{\partial x_i} = D_M \frac{\partial^2 C}{\partial x_i \partial x_i} \quad (4.14)$$

where D_M is the molecular diffusion coefficient and the repeated indices mean addition, that is

$$V_i \frac{\partial C}{\partial x_i} = \sum_{i=1}^3 V_i \frac{\partial C}{\partial x_i} \quad \text{and} \quad \frac{\partial^2 C}{\partial x_i \partial x_i} = \sum_{i=1}^3 \frac{\partial^2 C}{\partial x_i^2}$$

What matters most in turbulent flow is the mean temporal values \bar{C} and \bar{V}_i of the variables at each point. We have

$$C = \bar{C} + C' V_i = \bar{V}_i + V'_i$$

Substituting in Equation 4.14 we obtain

$$\frac{\partial}{\partial t} (\bar{C} + C') + (\bar{V}_i + V'_i) \frac{\partial (\bar{C} + C')}{\partial x_i} = D_M \frac{\partial^2 (\bar{C} + C')}{\partial x_i \partial x_i}$$

Taking the time averages of all terms, and considering that $\bar{V}'_i = 0$, $\bar{C}'_i = 0$, we obtain

$$\frac{\partial \bar{C}}{\partial t} + \bar{V}_i \frac{\partial \bar{C}}{\partial x_i} + \overline{V'_i \frac{\partial C'}{\partial x_i}} = D_M \frac{\partial^2 \bar{C}}{\partial x_i \partial x_i} \quad (4.15)$$

As the fluid is non-compressible, the continuity equation gives $\frac{\partial V'_i}{\partial x_i} = 0$ so that

$$\overline{V'_i \frac{\partial C'}{\partial x_i}} = \frac{\partial}{\partial x_i} (\overline{C' V'_i})$$

and Equation 4.15 yields

$$\frac{\partial \bar{C}}{\partial t} + \bar{V} \frac{\partial \bar{C}}{\partial x_i} = - \frac{\partial}{\partial x_i} (\overline{C' V'_i}) + D_M \frac{\partial^2 \bar{C}}{\partial x_i \partial x_i} \quad (4.16)$$

We see that this equation is analogous to the advective diffusion Equation 4.13 though it also includes an additional term $-\partial/\partial x_i \overline{C' V_i'}$ which represents the influence of the turbulent flow. More specifically, the term $\overline{C' V_i'}$ is the mean value of the advective diffusion in turbulent flow of the quantity having concentration C' at a unit surface perpendicular to velocity V_i' . Assuming also that Fick's law applies similarly, we obtain

$$\overline{C' V_i'} = -D_{ij} \frac{\partial \bar{C}}{\partial x_j} \quad (4.17)$$

with D_{ij} a matrix representing *turbulent diffusion*. The most significant difficulty in determining the coefficients of turbulent diffusion D_{ij} is that they do not remain constant throughout the whole flow field but depend upon the local characteristics of the flow. Replacing Equation 4.17 in Equation 4.16 we obtain

$$\frac{\partial \bar{C}}{\partial t} + \bar{V}_i \frac{\partial \bar{C}}{\partial x_i} = \frac{\partial}{\partial x_i} \left(D_{ij} \frac{\partial \bar{C}}{\partial x_j} \right) + D_M \frac{\partial^2 \bar{C}}{\partial x_i \partial x_i}$$

or

$$\frac{\partial \bar{C}}{\partial t} + \bar{V}_i \frac{\partial \bar{C}}{\partial x_i} = \frac{\partial}{\partial x_i} \left(D_{ij} \frac{\partial \bar{C}}{\partial x_j} + D_M \frac{\partial \bar{C}}{\partial x_i} \right) \quad (4.18)$$

We now assume that the matrix of turbulent diffusion coefficients D_{ij} is diagonal, that is,

$$D_{ij} = \begin{bmatrix} D_{xx} & 0 & 0 \\ 0 & D_{yy} & 0 \\ 0 & 0 & D_{zz} \end{bmatrix}$$

Equation 4.17 can be written in the form

$$\overline{C' V_i'} = -D_{(ii)} \frac{\partial \bar{C}}{\partial x_i} \quad (4.19)$$

where the parentheses around the indices indicate no addition. Under this assumption, Equation 4.18 may be written as

$$\frac{\partial \bar{C}}{\partial t} + \bar{V}_i \frac{\partial \bar{C}}{\partial x_i} = \frac{\partial}{\partial x_i} \left((D_{(ii)} + D_M) \frac{\partial \bar{C}}{\partial x_i} \right) \quad (4.20)$$

As the scale of turbulent flow is much larger than the scale of molecular motion, the turbulent dispersion of mass is substantially larger than that due to molecular oscillations. Thus, $D_{(ii)} \gg D_M$ and Equation 4.20 may take the form

$$\frac{\partial \bar{C}}{\partial t} + \bar{V}_i \frac{\partial \bar{C}}{\partial x_i} = \frac{\partial}{\partial x_i} \left(D_{(ii)} \frac{\partial \bar{C}}{\partial x_i} \right) \quad (4.21)$$

If turbulent diffusion is isotropic then all coefficients $D_{(ii)}$ take the same value D_T and Equation 4.21 can be written in the form

$$\frac{\partial \bar{C}}{\partial t} + \bar{V} \frac{\partial \bar{C}}{\partial x_i} = D_T \frac{\partial^2 \bar{C}}{\partial x_i \partial x_i} \quad (4.22)$$

In the special case of one-dimensional flow with constant velocity U , this formula is simplified to

$$\frac{\partial \bar{C}}{\partial t} + U \frac{\partial \bar{C}}{\partial x} = D_T \frac{\partial^2 \bar{C}}{\partial x^2} \quad (4.23)$$

If M is the amount of mass added into the flow field at position $x = 0$ and time $t = 0$ then, the solution of the above equation is

$$\bar{C}(x, t) = \frac{M}{\sqrt{4\pi D_T t}} \exp \left\{ -\frac{(x - Ut)^2}{4D_T t} \right\}$$

Analytical solutions may also be found for turbulent diffusion in two-dimensional space, but the main difficulty of the problem is not so much the mathematical solution but rather the physical and mathematical description of the coefficient of turbulent diffusion D_T . In fact, on the basis of the definition of this coefficient as given in Equation 4.19, the value of D_T depends upon the physical characteristics of the flow field. A basic question which arises is to discover how the coefficient of turbulent diffusion is related to the physical characteristics of the flow field and in particular to the scale of the turbulent flow. It is obvious that the value of D_T becomes larger as the characteristic length l of turbulent vortices increases, but the correlation between these two variables differs depending upon whether it refers to *free* turbulent flow or to *shear* turbulent flow as influenced by solid walls. For a *free* and *homogeneous* turbulent flow, Batchelor has used the spectral theory of Colmogoroff to determine that

$$D_T = (\text{const.}) \epsilon^{1/3} l^{4/3} \quad (4.24)$$

where ϵ is the mean value of energy losses due to viscosity per unit mass, and l is the scale of turbulent vorticity involved in diffusion. The relationship shown in Equation 4.24 has been corroborated experimentally by Orlob.

To define the coefficient of turbulent diffusion a number of semi-empirical theories have been developed in the past, such as the Prandtl characteristic length or theories which are based on the method of Lagrange, that is, the monitoring of the motion of one or two fluid particles (Taylor, 1921).

The study of turbulent diffusion in cylindrical or prismatic channels and in one- or two-dimensional flows with free surfaces (rivers, coastal areas, etc.) has been advanced substantially with the hydrodynamic dispersion approach explained below.

4.1.2.3 Turbulent Dispersion

We have seen earlier that the description of turbulent diffusion is based on a change of scale. At the microscopic scale of molecules, molecular diffusion predominates. The superposition of stochastic motions due to turbulence leads to turbulent

diffusion, which develops at a greater scale and is based on the temporal mean turbulent variables at every point. If we now consider a larger scale and treat the phenomenon on the basis of mean velocities in a cross-section perpendicular to the direction of the flow, then we introduce the definition of convective dispersion by means of dispersion coefficients D_x , D_y which correspond to the turbulent dispersion coefficients D_{Tx} , D_{Ty} respectively.

A coastal region or a water body frequently has a geometry which does not allow the modelling of the circulation of the fluid in one dimension. We must therefore consider both velocity components parallel to directions x and y . These velocities are caused by tides, wind currents or other reasons and may be described with the equations of motion and continuity. If H is the depth of the flow, we introduce the mean velocities along the depth U and V from

$$U = \frac{1}{H} \int_0^H u dz \quad V = \frac{1}{H} \int_0^H v dz \quad (4.25)$$

Let C be the mean pollutant concentration along the depth H , then the convective diffusion equation for two-dimensional flow yields

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} = \frac{\partial}{\partial x} \left(D_x \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left(D_y \frac{\partial C}{\partial y} \right) \quad (4.26)$$

The dispersion coefficients D_x and D_y depend upon the flow characteristics and vary according to the velocities U and V . It must be stated that, if the turbulent mixing of the pollutant occurs at the surface of the flow (surface diffusion), we may still use Equation 4.26 by integrating the velocities and the concentrations at a given depth below the surface.

The dispersion of pollutants in a two-dimensional and homogeneous flow of infinite width has been studied by Elder (1959). If q_x and q_y are the flow rates per unit width parallel to directions x and y , ϵ is the frictional loss coefficient and y_0 the depth of the flow stream, the dispersion coefficients are given by

$$D_s = 5,9 \sqrt{\lambda/2} \frac{\sqrt{q_x^2 + q_y^2}}{y_0}$$

$$D_n = 0,2 \sqrt{\lambda/2} \frac{\sqrt{q_x^2 + q_y^2}}{y_0}$$

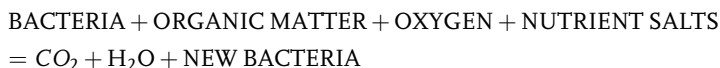
where s and n indicate the direction of the flow and that perpendicular to it, respectively. To describe specific cases of pollution Equation 4.26 may be integrated numerically. For the dispersion coefficients one may use any theoretical or empirical formula, as long as the numerical results satisfactorily describe *in situ* measurements.

4.1.2.4 Growth Kinetics

Wastewaters contain various microbial organisms in the form of dispersions or floculates. The main types are:

- Bacteria: these constitute the major group of micro-organisms (total coliforms, *E. coli*).
- Protozoa: single-cell animal organisms feeding on bacteria.
- Algae: single-cell plant organisms.

In aerobic digestion conditions the fundamental reaction which occurs is



The organic matter consists of carbon compounds such as proteins, carbohydrates, oils, fats, and so on. Since their exact chemical composition may not be determined with ease, these are treated quantitatively in combination through the parameters BOD (biological oxygen demand), COD (chemical oxygen demand) or TOC (total organic carbon).

BOD is the amount of oxygen required for aerobic biological digestion of the organic effluents. This parameter was first introduced in England. Its measurement has been specified to be carried out at 20 °C at the end of 5 days (BOD₅). This was deemed necessary to simulate the water temperature in English rivers, given that these are of relatively short lengths. After 5 days the wastewaters reach the sea, where dilution becomes so large that the occurrence of septic or anaerobic conditions is prevented. As shown schematically in Figure 4.8, at 20 °C, all available organic matter is oxidised after approximately 6–10 days. Subsequently, only biological oxidation of ammoniac nitrogen into nitrates occurs.

At higher temperatures the oxidation of ammoniac nitrogen may proceed faster. In Figure 4.8 the dashed curve shows the total oxygen demand with no nitrification. In this case, an asymptotic value (BOD)_u is reached. As a first approximation, the exponential relationship

$$\text{BOD} = (\text{BOD})_u \{1 - \exp(-kt)\}$$

applies. Then, BOD₅ is approximately equivalent to 65% of (BOD)_u.

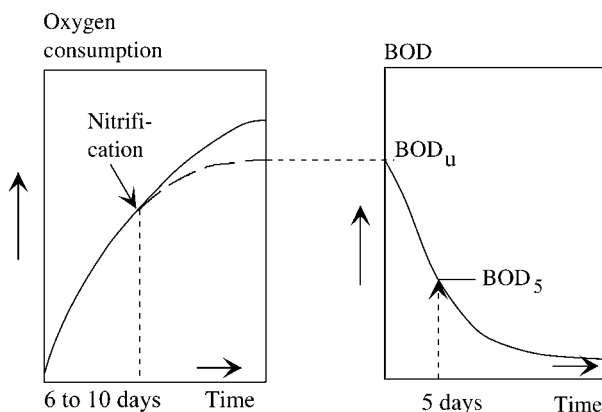


Figure 4.8 Oxygen demand and residual (BOD)_u.

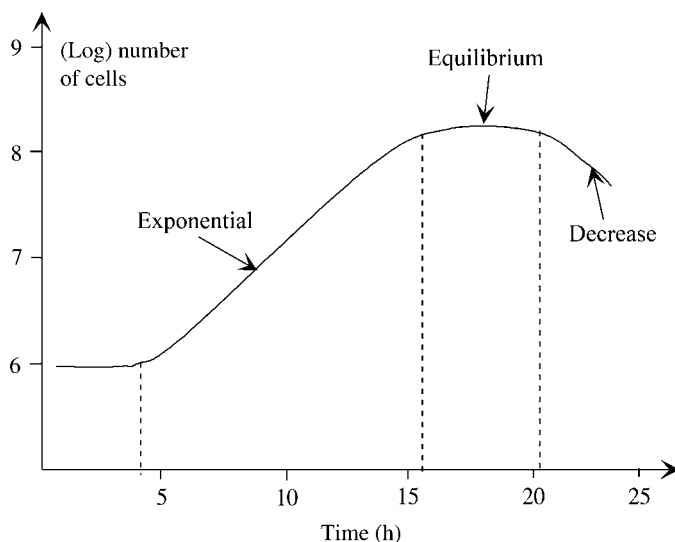


Figure 4.9 Growth curve for bacterial load.

COD is the amount of oxygen required for complete chemical oxidation of the organic content. Bacteria, being living organisms, need special conditions of temperature, nutrients, and so on, to grow. Vitamins and metabolic compounds may catalyse growth, while poisons delay the process.

A typical growth curve for bacteria is shown in Figure 4.9; the time scale is only indicative. Introducing the bacterial load into a solution containing organic matter, the growth of microorganisms is initially very slow (adjustment period). This is followed by exponential growth during which the consumption of organic nutrients is substantial. When food is diminished, an equilibrium condition is reached, followed by a reduction in the cellular organisms (endogenous stage).

The biochemical kinetics of various compounds reacting with each other (bacteria, oxygen, organic matter and nutrients) may be described quantitatively with various formulations. These are based on different modelling of the underlying molecular kinetics.

Let us define C as the concentration of organic compounds (in ppm or g/l or mol/l). For biochemical kinetics the most important parameter is the *biological decay* or *growth* rate dC/dt . This rate increases with the increasing probability of the various reacting compounds coming in contact with each other. For example, the number of possible collisions between the two black and three white spheres in Figure 4.10 is proportional to the product of the number of black and white spheres. Indeed, sphere M_1 , as well as M_2 , may collide with any one of the three spheres A_1 , A_2 or A_3 . The total number of collisions per unit time is proportional to the product between the numbers of white and black spheres (2×3).

Let us define X_a as the number of bacteria per litre, O as the concentration of oxygen and N as the concentration of nutrients. Then, the biological decay rate can be

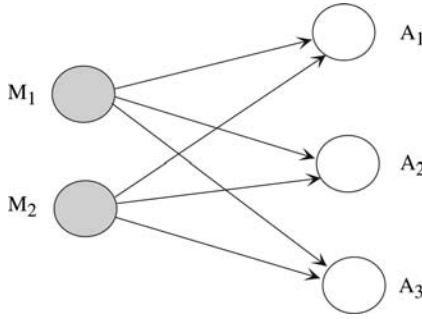


Figure 4.10 Influence of particle concentration on the collision rate.

expressed as

$$\frac{dC}{dt} = -k C X_a O N \quad (4.27)$$

We now assume that the number of white spheres in Figure 4.10 is very large. Since white spheres are present everywhere, the frequency of collisions depends only upon the number of black spheres.

For the case of a constant microbial concentration \bar{X}_a in excess of oxygen and nutrients, Equation 4.27 may be rewritten as

$$\frac{dC}{dt} = -k \bar{X}_a C \quad (4.28)$$

Equation 4.28 describes the biochemical kinetics only for small values of concentration C . When concentration C increases saturation occurs, so that the growth rate becomes independent of concentration (Figure 4.11).

Equation 4.28 may be generalised as follows

$$\frac{dC}{dt} = \frac{-k \bar{X}_a C}{\left(\frac{k_m}{C} + 1\right)} \quad (4.29)$$

When $C \rightarrow \infty$ then $dC/dt \rightarrow \text{constant}$.

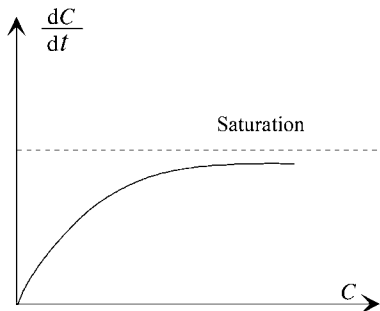


Figure 4.11 Growth rate curve of organic load.

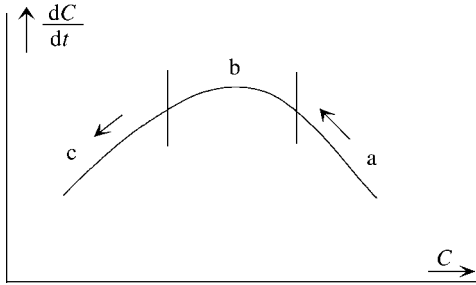


Figure 4.12 Curve of autocatalytic growth of organic matter.

In fact, the digestion rate is influenced by the autocatalytic action of bacteria, which grows during the reaction. As shown in Figure 4.12, starting from point *a* (high concentration *C*), the rate increases with the growth of new bacteria. At the same time, the concentration of organic load drops and we reach an equilibrium region (optimal region, point *b*). Past point *b* the reaction rate is reduced, since the concentration of the organic load drops asymptotically to zero. Equation 4.29 represents the fact that the biological digestion rate of organic matter depends upon the microbial concentration and the concentration of organics.

4.1.2.5 Coastal Circulation

The coastal region in which hydrodynamic circulation occurs is delineated by the coastline and one open sea boundary. The flow is usually unsteady and turbulent with insignificant vertical velocity components. From the sea floor topography and with the horizontal reference plane *Oxy* placed at position *h* (*x*, *y*), Figure 4.13, for three-dimensional coastal circulation the unknown variables are the horizontal velocity components $u = u(x, y, z, t)$, $v = v(x, y, z, t)$ and the free surface elevation $n = n(x, y, t)$.

These variables must satisfy the equations of momentum conservation with the Coriolis forces also taken into consideration. At the right hand side of these equations, apart from pressure, we also have viscous stresses and vorticity. In the case of coastal

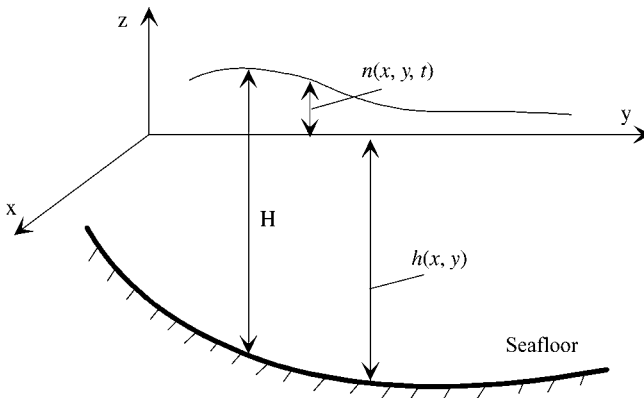


Figure 4.13 Three-dimensional coastal circulation field.

currents, the molecular viscosity stresses may be neglected in comparison to vorticity. The latter, using Boussinesq's approximation, takes the form

$$\tau_{ij} = -\rho \overline{u'_i u'_j} = \nu_T \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (4.30)$$

where ν_T is the eddy viscosity coefficient. For three-dimensional circulation it is necessary to distinguish between the vertical ν_{Tv} and the horizontal ν_{Th} eddy viscosity coefficients. Then, Equation 4.30 becomes

$$\begin{aligned} \tau_{xx} &= \nu_{Th} \frac{\partial u}{\partial y} & \tau_{xy} &= \nu_{Th} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \tau_{xz} &= \nu_{Tv} \frac{\partial u}{\partial z} + \nu_{Th} \frac{\partial w}{\partial z} \\ \tau_{yy} &= 2\nu_{Th} \frac{\partial v}{\partial y} & \tau_{yz} &= \nu_{Tv} \frac{\partial v}{\partial z} + \nu_{Th} \frac{\partial w}{\partial z} & \tau_{zz} &= 2\nu_{Tv} \frac{\partial w}{\partial z} \end{aligned}$$

With the assumptions above and ignoring the vertical velocity components, Equation 4.30 may be combined with the continuity equation to yield

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - f v &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu_{Th} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \nu_{Tv} \frac{\partial^2 u}{\partial z^2} \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + f u &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu_{Th} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \nu_{Tv} \frac{\partial^2 v}{\partial z^2} \end{aligned} \quad (4.31)$$

In these equations the terms $\nu_{Th} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$ and $\nu_{Th} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$ represent the horizontal dispersion of momentum due to turbulence and are usually negligible, compared with the terms $\nu_{Tv} \frac{\partial^2 u}{\partial z^2}$ and $\nu_{Th} \frac{\partial^2 v}{\partial z^2}$ which represent the respective momentum gradients in the vertical direction. Furthermore, we may assume as a first approximation that the vertical pressure distribution is hydrostatic. We may then write

$$p = p_a + \rho g(n - z)$$

where p_a is the atmospheric pressure. The final form of Equations 4.31 becomes

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - f v &= -g \frac{\partial n}{\partial x} + \frac{\partial}{\partial z} \left(\nu_{Tz} \frac{\partial u}{\partial z} \right) \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + f u &= -g \frac{\partial n}{\partial y} + \frac{\partial}{\partial z} \left(\nu_{Tz} \frac{\partial v}{\partial z} \right) \end{aligned} \quad (4.32)$$

In summary it may be said that Equations 4.32 are based on the assumptions of

- incompressible fluid,
- horizontal flow,
- negligible horizontal dispersion of momentum,
- hydrostatic pressure distribution.

To determine the functions $u(x, y, z, t)$, $v(x, y, z, t)$ and $n(x, y, t)$ another equation should be added; this represents the mass concentration in integral form. Thus, the

excess mass per unit volume during dt equals

$$-\rho \left(\frac{\partial}{\partial x} \int_{-h}^n u dz + \frac{\partial}{\partial y} \int_{-h}^n v dz \right) dt \quad (4.33)$$

This mass equals the change in the volume of the element due to changes in free surface elevation. Per unit volume and in time dt we have

$$\rho \frac{\partial H}{\partial t} dt = \rho \frac{\partial n}{\partial t} dt \quad (4.34)$$

Equating the two expressions represented by Equations 4.33 and 4.34, we obtain the integral continuity equation

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} \int_{-h}^n u dz + \frac{\partial}{\partial y} \int_{-h}^n v dz = 0 \quad (4.35)$$

Introducing the mean velocities along the depth U and V , Equation 4.35 becomes

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} (HU) + \frac{\partial}{\partial y} (HV) = 0 \quad (4.36)$$

where

$$U = \frac{1}{H} \int_{-h}^n u dz \quad V = \frac{1}{H} \int_{-h}^n v dz \quad (4.37)$$

In the deterministic approach, the velocity field is obtained by using hydrodynamic models. The development of such models both in 2-D and 3-D space has been used to predict the current fields generated by tides and winds (Baines and Knapp, 1965; Fischer *et al.*, 1979; Churchill and Csanady, 1983; Ganoulis and Krestenitis, 1982; 1984). Numerical algorithms based on finite differences or finite elements have been introduced for the numerical integration of the hydrodynamic equations. Some of these models use coordinate transformations in the 3-D space (Krestenitis, 1987; Krestenitis and Ganoulis, 1987).

However, significant errors are induced in all numerical simulations. These are due to the fact that only a limited number of terms in the Taylor series expansions are taken into account. Explicit algorithms suffer from so-called numerical diffusion. This is an artificial diffusion related to truncation errors. It is superimposed on physical diffusion and leads to an excessive attenuation of the input signals. Implicit finite difference algorithms introduce trailing effects, because the initial signals propagate at greater speeds than physical signals.

Reliable results for pollutant dispersion and transport may be only achieved using time series of recorded water currents particularly in wind-generated coastal circulation.

4.1.3

Random Walk Simulation

Let us consider the one-dimensional diffusion of a mass M introduced at time $t = 0$ in an infinitesimal distance around $x = 0$ (Figure 4.14). Mathematically, this initial condition can be written as

$$C_0 = C(x, 0) = M \delta(x) \quad (4.38)$$

where $\delta(x)$ is the Dirac delta function.

Assuming that the mass M is diffusing without transport, the concentration $C(x, t)$ is the solution of the one-dimensional diffusion equation

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} \quad (4.39)$$

The well-known solution of Equation 4.39 with the initial condition shown in Equation 4.38 is

$$C(x, t) = \frac{C_0}{\sqrt{4\pi Dt}} \exp\left\{-\frac{x^2}{4Dt}\right\} \quad (4.40)$$

If $\sigma^2 = 2Dt$ is substituted into Equation 4.40, the Gaussian distribution

$$\frac{C(x, t)}{C_0} = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{x^2}{2\sigma^2}\right\} \quad (4.41)$$

with zero mean and variance σ^2 is obtained.

Suppose now that a particle located at $x = 0$ oscillates randomly between maximum distances either $+\Delta x$ or $-\Delta x$, with equal probability. For homogeneous probability

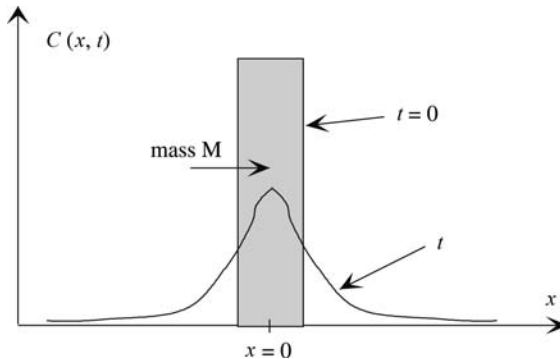


Figure 4.14 Diffusion of mass M introduced at time $t = 0$ at $x = 0$.

distribution function $p(x)$ we will have

$$\begin{aligned} p(x) &= 0 & \text{if } x < -\Delta x \\ p(x) &= \frac{1}{2\Delta x} & \text{if } -\Delta x < x < +\Delta x \\ p(x) &= 0 & \text{if } x > +\Delta x \end{aligned} \quad (4.42)$$

The mean value $m = E(x)$ and the variance $E(x - m)^2$ of this movement are

$$m = E(x) = \int_{-\Delta x}^{+\Delta x} xp(x)dx = 0 \quad (4.43)$$

$$\sigma_s^2 = E(x - m)^2 = \int_{-\Delta x}^{+\Delta x} x^2 p(x)dx = \frac{\Delta x^2}{3} \quad (4.44)$$

According to the central limit theorem, after n steps, the probability density distribution function $P(x, t)$ is Gaussian, with mean value

$$nm = 0 \quad (4.45)$$

and variance

$$S^2 = n\sigma_s^2$$

This means that

$$P(x, t) = \frac{1}{\sqrt{2\pi(n\sigma_s^2)}} \exp\left\{-\frac{x^2}{2(n\sigma_s^2)}\right\} \quad (4.46)$$

Comparison of Equations 4.46 and 4.40 or 4.41 indicates that the two solutions become identical if

$$n\sigma_s^2 = 2Dt \quad \text{or} \quad \sigma_s^2 = 2D\left(\frac{t}{n}\right) = 2D\Delta t \quad (4.47)$$

From Equations 4.47 and 4.44 we can evaluate Δx as

$$\sigma_s^2 = \frac{\Delta x^2}{3} = 2D\Delta t \quad \text{or} \quad \Delta x = \pm\sqrt{6D\Delta t} \quad (4.48)$$

If we introduce a random variable $\text{rnd}(-1, +1)$, which is distributed uniformly between -1 and $+1$, then Equation 4.48 takes the form

$$\Delta x = \sqrt{6D\Delta t} \text{rnd}(-1, +1) \quad (4.49)$$

A random walk simulation of the one-dimensional diffusion equation (Equation 4.39), subject to the initial condition (Equation 4.38), should be performed according to the following steps (Ganoulis, 1977):

- (1) A large number N of particles is introduced at $x = 0, t = 0$.
- (2) Particles move by time increments Δt . If $x_{n,p}$ is the position of the particle p at time $n\Delta t$, then its position $x_{n+1,p}$ at time $(n+1)\Delta t$ should be

$$x_{n+1,p} = x_{n,p} + \sqrt{6D\Delta t} \text{rnd}(-1, +1) \quad (4.50)$$

- (3) Counting the number of particles located between $x - \Delta x/2$ and $x + \Delta x/2$ and dividing by the total number N of particles, a numerical approximation of Equation 4.41 or Equation 4.46 may be obtained.

We may now extend the above for the case in which the fluid moves in the three-dimensional space. If $\vec{V}: (u, v, w)$ is the velocity vector, considering N particles located at time $t = \Delta t$ at positions

$$\vec{r}_{n,p} = (x_{n,p}, y_{n,p}, z_{n,p}) \quad p = 1, 2, \dots, N \quad (4.51)$$

According to the random walk principle, the probability of finding a particle at a given position after time Δt follows a Gaussian distribution with mean value 0 and variance $s^2 = 2\Delta t D$, where D is the dispersion coefficient. Now the particles are moving from time $t = \Delta t$ to time $t + \Delta t = (n+1)\Delta t$ according to the relationships

$$x_{n+1,p} = x_{n,p} + u\Delta t + x_1 \quad (4.52)$$

$$y_{n+1,p} = y_{n,p} + v\Delta t + x_2 \quad (4.53)$$

$$z_{n+1,p} = z_{n,p} + w\Delta t + x_3 \quad (4.54)$$

where u, v, w are the velocity components of the current and x_1, x_2, x_3 random variables following a normal distribution with mean value 0 and variance $s^2 = 2\Delta t D$.

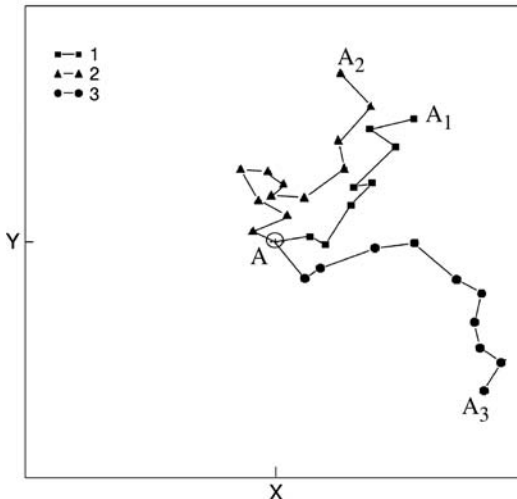


Figure 4.15 Random walk of three particles after 10 time steps.

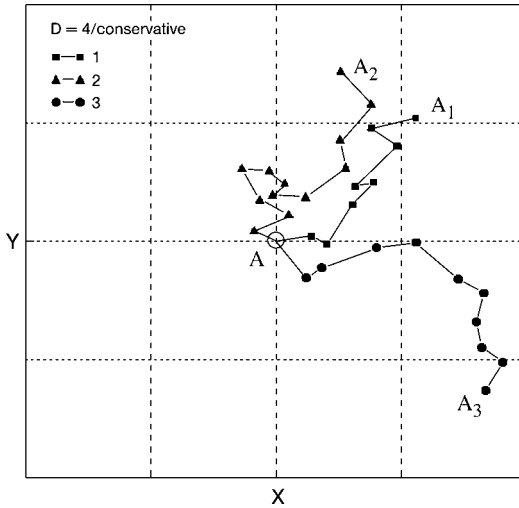


Figure 4.16 Grid overlay in the random walk of three particles.

The procedure is illustrated in Figure 4.15 for three particles initially located at the same point A. Each particle moves according to the relationships shown in Equations 4.52 and 4.53. After 10 time-steps the particles occupy three different positions A_1 , A_2 and A_3 .

To evaluate probabilities and concentrations of the particles, the area is covered by a regular grid (Figure 4.16). Knowing the velocity components u , v at the grid points, particle velocities are computed by linear interpolation. The probability for reaching a given grid cell and consequently the particle concentrations are evaluated by counting the number of particles which fall within the grid square.

If instead of the initial condition (Equation 4.38) a continuous mass concentration is introduced at $x = 0$ as

$$C(x = 0, t) = C_0 \quad (4.55)$$

Then, the analytical solution of the diffusion equation (Equation 4.39) with advection velocity U is given by

$$C = C_0 \left\{ 1 - \operatorname{erf} \left(\frac{x - Ut}{\sqrt{4Dt}} \right) \right\}, \quad x > 0 \quad (4.56)$$

The validation of this random walk simulation is given in Figures 4.17 and 4.18 for $D = 1$ and $D = 0.01 \text{ m}^2/\text{s}$ respectively. In all cases we have $U = 1 \text{ m/s}$ and $\Delta x = 1 \text{ m}$; this means that Peclet numbers based on Δx take the values 1 and 100.

Even if we introduce 10 times more particles (Figure 4.17) oscillations of the random walk simulation persist, although the front of the wave is well described (Figure 4.18) at high Peclet numbers (Ganoulis, 1977).

An example of a two-dimensional random walk simulation is given in Figures 4.19 and 4.20.

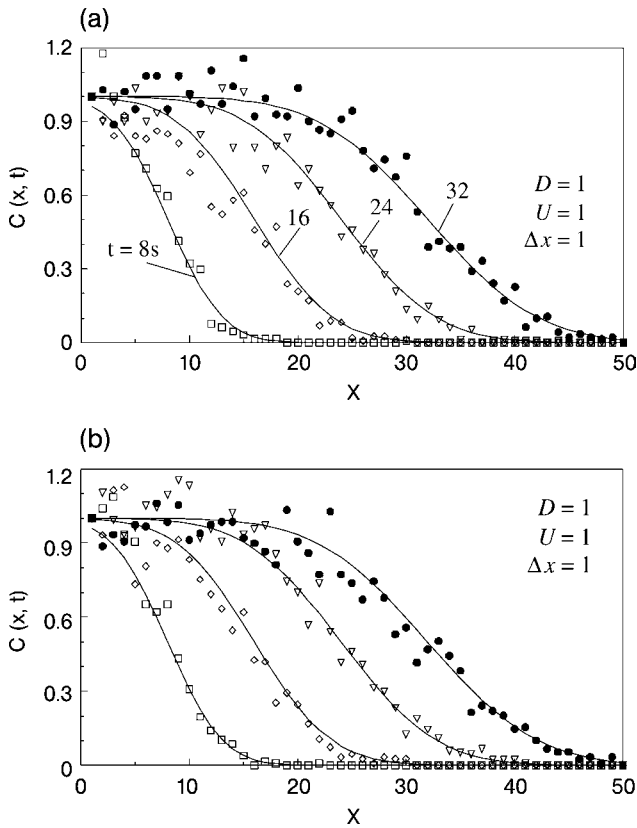


Figure 4.17 Comparison between the analytical solution (Equation 4.56) and random walk simulation for (a) $N = 1000$ and (b) $N = 10000$ (Low Peclet number).

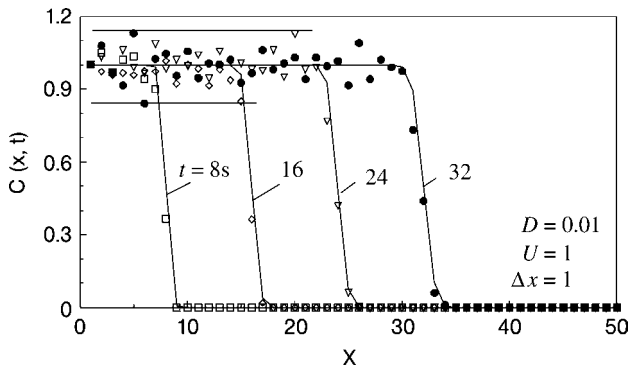


Figure 4.18 Comparison between random walk simulation and the analytical solution (Equation 4.56) for a high Peclet number.

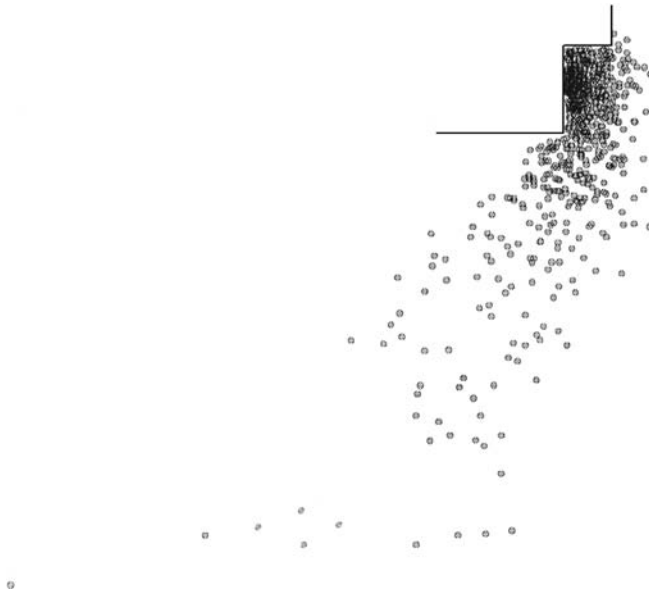


Figure 4.19 Two-dimensional random walk simulation.

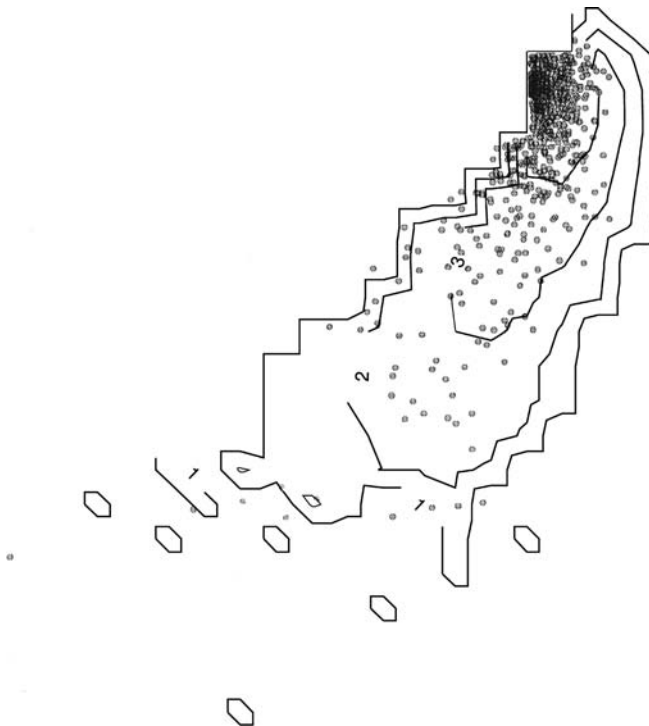


Figure 4.20 Contours showing the impact probabilities from a local source emitting a pollutant at constant concentration $C_0 = 10^5$ (probability is in log coordinates).

However, this method suffers from some drawbacks: first, to obtain statistically meaningful results a large number of particles, at least 10^3 , has to be used. When continuous emissions of pollutant sources take place the necessary number of particles becomes very large. Second, because of the statistical origin of the method, the concentration field shows oscillations and averaging in time may be necessary to obtain smooth results.

When a deterministic current velocity field is used in Equations 4.52 and 4.53, the solution obtained approaches that of the two-dimensional convective–dispersion equation.

4.1.4

Dispersion by Wind-generated Currents

Impact risk from wastewater discharges in the far field is more realistically assessed by using the time data recordings of currents. The time series of wind-generated current velocities which are measured over one whole season are usually *stationary*. Thus all *statistical properties* of the *random variables*, such as the *current velocity and direction* (see Figure 4.2 for example) *are independent of the time origin*.

Now consider a large number of particles (Csanady, 1983; Ganoulis, 1991d; Roberts, 1989) initially located at the same point (point source), but departing at different times $t = n \Delta t$. Each particle moves during a given travel time $T > t$, where $T = t + m \Delta t = (n + m) \Delta t$. The final position $\vec{r}(t + T)$ of the particle after time T may be evaluated using the randomly varying current velocity field $\vec{V}(t)$

$$\vec{r}(t + T) = \int_t^{t+T} \vec{V}(t) dt \quad (4.57)$$

It is obvious that the final position of each particle depends on the initial departing time t . By allocating different values of t to each particle, different final positions of the particles after time T are found. Because of the stationary nature of the random process, the concentration field and the probability of reaching a given location are independent of t .

Having a relatively long record of time series of currents V_i , the impact probabilities and consequently the risk assessment of pollution at a given location can be evaluated. After travel time $T = (n + m) \Delta t$, Equation 4.57 takes the form

$$\vec{r}(t, t + T) = \sum_n^{n+m} \vec{V}_i \Delta t \quad i = n, \dots, n + m \quad (4.58)$$

Counting of the particles at each location is done by using a grid overlay as in the case of random walk simulations. Pollutant concentrations are proportional to the number of particles located within every square of the grid.

The simulation procedure is illustrated in Figure 4.2 for the particular case of wind-generated currents. The statistical characteristics of the current velocity components are given in Table 4.2. It may be recognised that standard deviations are larger than average values. This indicates the high temporal variability of currents.

Table 4.2 Statistical characteristics of velocity components u and v (Figure 4.2).

Variable	u (cm/s)	v (cm/s)
Sample size	2047	2047
Average	5.06	-1.39
Variance	53.19	14.38
Standard deviation	7.29	3.79
Minimum	-19.22	-18.39
Maximum	-25.25	11.92
Range	44.47	0.31

The autocorrelation functions of current velocity components u and v are shown in Figure 4.21. The form of these functions means that after an initial time lag greater than about 500 s, the autocorrelation function has very small values. This means that that wind-generated velocities become uncorrelated or random and that autocorrelation tends to zero as time tends to infinity.

The results of this simulation are shown in Figure 4.22 in the form of lines of equal probability. The pollution field varies with time T after the first release of particles which represent the discharged wastewater.

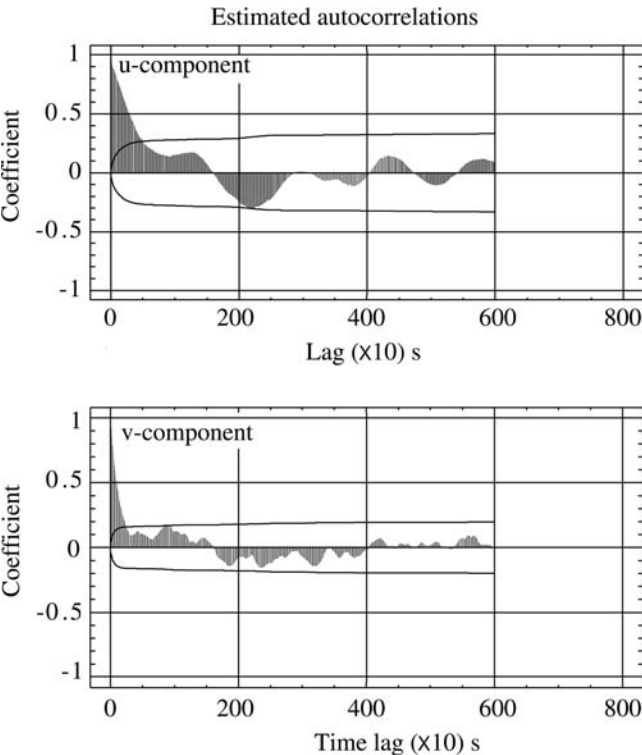


Figure 4.21 Autocorrelation functions of current velocity components u and v .

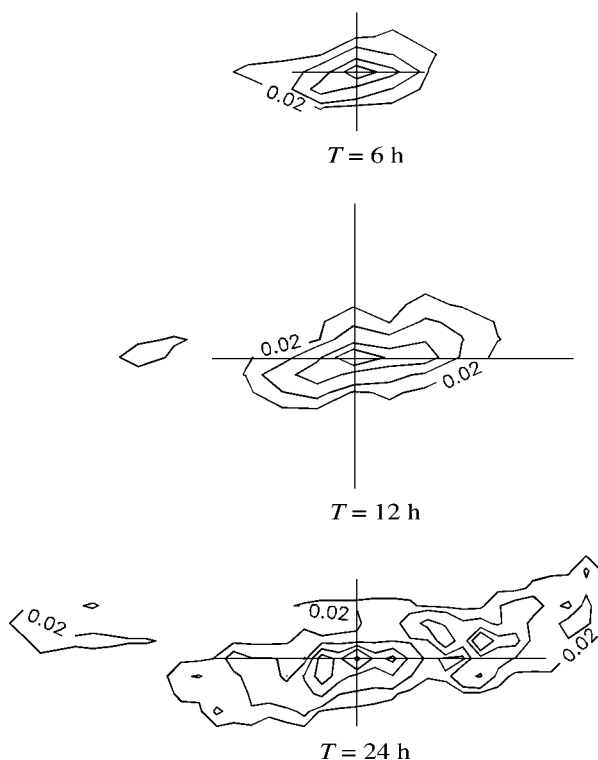


Figure 4.22 Contours of equal environmental impact probability after time $T = 6, 12$ and 24 h from initial release (wind generated currents and continuous constant discharge with $C_0 = 1$ from a point source).

4.2

Risk in River Water Quality

4.2.1

Introduction

Rivers and streams are natural drainage systems not only for rainfall water but also for different substances which may be dissolved in various concentrations. Overland flows discharge pollutants from non-point sources into rivers and streams distributed over the entire catchment area. Also wastewaters of industrial, domestic and agricultural origin are discharged into rivers. In situations where there are relatively low quantities of pollutant loads, turbulent mixing, re-aeration, sedimentation and re-suspension in rivers transport wastewaters away from the source into the sea (James, 1993).

If, however, wastewater loading from municipal sewage overcomes the receiving capacity of the river, negative effects may appear, as shown in Figure 4.23:

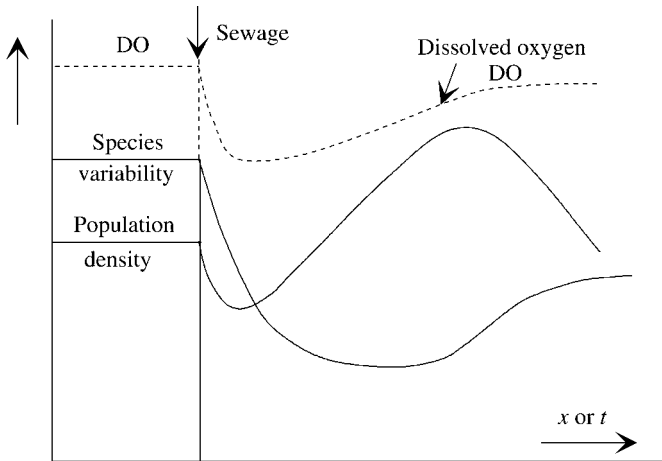


Figure 4.23 Effects on river water quality and species populations from sewage disposal.

- (1) decrease in the concentration of dissolved oxygen (DO),
- (2) increase in organic matter (BOD) and nutrients,
- (3) increase in the population density of certain microbes,
- (4) decrease in the variability of different species.

If toxic substances are discharged into a river, then biological species may disappear within a certain distance from the discharge point (Figure 4.24a). A rapid decrease followed by a progressive increase in populations may be observed (Figure 4.24b) in cases with large amounts of suspended solids that are discharged into the river.

To assess the risk of river pollution, different mathematical models have been developed. Most of them refer to the relationship between organic matter (BOD) and dissolved oxygen (DO). Apart from these, models describing the transport and fate of nitrates in rivers have also been developed. Numerical simulation, application of the Monte-Carlo technique and analysis of time series of water quality data may be used to quantify the risk of pollution. The above are briefly discussed in the following sections.

4.2.2

Mathematical Modelling and Simulation

4.2.2.1 Physically Based Mathematical Models

For river water quality, physically based mathematical models describe the mechanisms controlling the transport and fate of pollutants in one-dimensional space. These are:

- (1) advection, with mean velocity U ,
- (2) turbulent dispersion, with coefficient D_T ,
- (3) biochemical interactions.

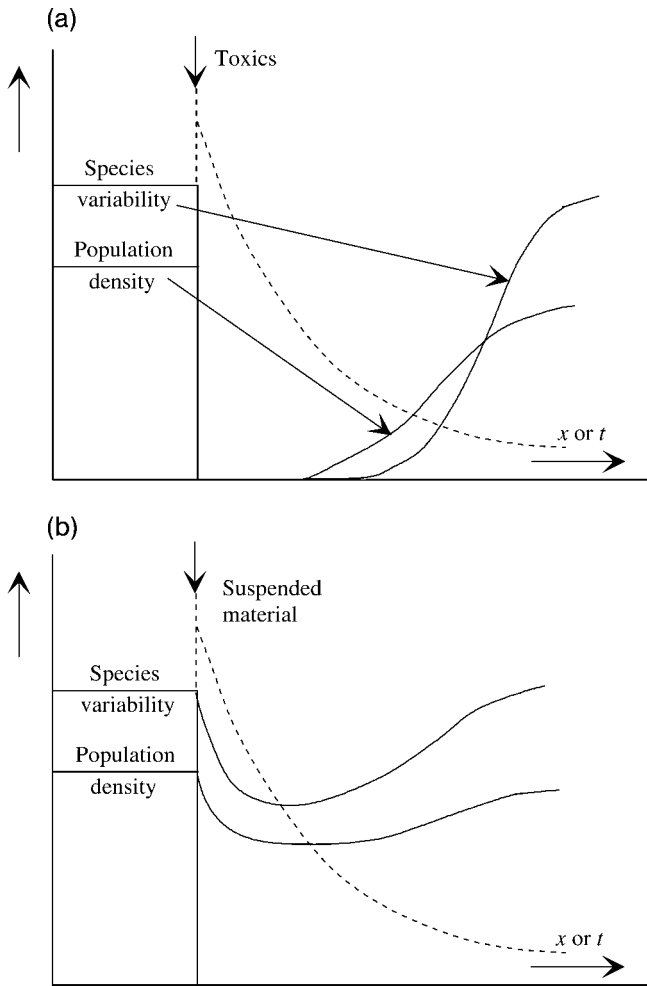


Figure 4.24 Impacts on the ecosystems in the river from the disposal of (a) toxic chemicals and (b) suspended solids.

The mass conservation of n related chemical species C_i , $i=1, 2, \dots, n$ may be expressed by a set of n coupled, nonlinear, partial differential equations of the form

$$\frac{\partial C_i}{\partial t} + U \frac{\partial C_i}{\partial x_i} = \left(\frac{1}{\Sigma} \right) \frac{\partial}{\partial x} \left\{ (D_T \Sigma) \frac{\partial C_i}{\partial x} \right\} + f_i(C_1, C_2, \dots, C_n) \quad (4.59)$$

where Σ is the cross-section of the river and $f_i(C_1, C_2, \dots, C_n)$ the temperature-dependent biochemical production or depletion rate of species i .

If only one pollutant is considered, for Σ and $D_T = \text{constant}$, Equation 4.59 is reduced to the one-dimensional advective dispersion equation in the form

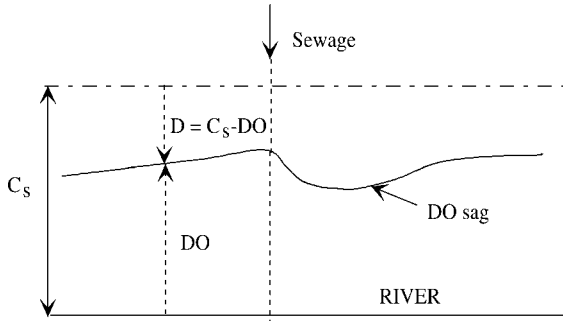


Figure 4.25 Schematic view of the oxygen deficiency sag curve.

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} = D_T \frac{\partial^2 C}{\partial x^2} \quad (4.60)$$

In the classic work of Streeter and Phelps, two species are considered, such as

$$C_1 : \text{organic matter BOD} \rightarrow C \quad (4.61)$$

and

$$C_2 : \text{oxygen deficit } D = C_s - \text{DO} \quad (4.62)$$

where C_s is the saturation dissolved oxygen (Figure 4.25). Using the symbols in Equations 4.61 and 4.62, the relationship between the rate of BOD discharge and the resulting concentration of DO take the form of the following two coupled, partial differential equations

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} = D_T \frac{\partial^2 C}{\partial x^2} - K_1 C - K_3 C \quad (4.63)$$

$$\frac{\partial D}{\partial t} + U \frac{\partial D}{\partial x} = D_T \frac{\partial^2 D}{\partial x^2} + K_1 C - K_2 D \quad (4.64)$$

where

K_1 is the *deoxygenation rate* (T^{-1}), a function of temperature and composition of the organic matter; K_1 is of the order of 10^{-6} s^{-1} ;

K_2 is the *reaeration rate* (T^{-1}), which depends on the turbulent flow near the free surface of the water, the wind speed, and so on. An empirical relationship gives

$$K_2 = \frac{4.5 \times 10^{-5} \times U^{1/2}}{H^{3/2}} [\text{s}^{-1}] \quad (4.65)$$

where

U is the mean velocity (m/s), and H the mean water depth (m).

Solution of Equations 4.63 and 4.64 gives the oxygen sag curve (Figure 4.25): near the site of wastewater disposal the BOD is high and the oxygen deficit will increase downstream. Then the deficit will gradually decrease because of reaeration.

More sophisticated physically based models may be developed by use of Equation 4.59. For example, first order chemical kinetics may be used to represent the nitrification process, that is, the decay of organic nitrogen and ammonia-nitrogen to nitrate-nitrogen through nitrite-nitrogen conversion (Thomann *et al.*, 1971).

4.2.2.2 Numerical Simulation

Various numerical algorithms have been introduced during the last decade for computer simulation of the governing differential equations. Algorithms based on finite differences and finite elements suffer from numerical diffusion and trailing effects. Lagrangian models seem to be accurate for describing the nitrogen kinetics in one-dimensional, unsteady, non-uniform river flow (Jobson, 1987). In this case chemical reactions are negligible and the analytical solution of Equations 4.59 or (4.60) is known for one-dimensional, constant velocity flow.

Numerical methods for integrating Equation 4.60 are classified into three types: (a) Eulerian, (b) Lagrangian and (c) Eulerian–Lagrangian.

Eulerian Methods These are based on a discretization of the various terms included in Equation 4.60 over a regular or irregular fixed grid. Finite differences are usually introduced using Taylor series expansions over a regular grid. Finite elements based on the Galerkin method are used over an irregular grid, adapted to the geometrical form of the boundaries.

Eulerian methods are not suitable for modelling the convective part of the dispersion equation (Equation 4.60) because various errors are introduced. These are due to the fact that only a limited number of terms in the Taylor series expansions are taken into account. Explicit algorithms suffer from so-called numerical diffusion. This is an artificial coefficient related to truncation errors. It is superimposed on physical diffusion leading to excessive attenuation of the input signal. Implicit finite difference algorithms introduce trailing effects because the initial signal is propagated at a different speed than the physical signal.

Lagrangian Methods These are based on the Monte Carlo or random walk principle, as explained in Section 4.1.3.

At time $t = 0$ a large number of particles N are considered. According to the random walk simulation the probability of finding a particle at a given position after time Δt follows a Gaussian law of mean value 0 and variance $\sigma^2 = 2\Delta t D$, where D is the diffusion coefficient. The accuracy of the method is illustrated in Figures 4.17 and 4.18.

Eulerian–Lagrangian Methods These are based on the method of characteristics. To illustrate the procedure consider the one-dimensional convective diffusion equation

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial x^2} \quad (4.66)$$

subject to the boundary condition

$$C(0, t) = 1, \quad t \geq 0 \quad (4.67)$$

The analytical solution has the form

$$C/C_o = \frac{1}{2} \left\{ 1 - \operatorname{erf} \left(\frac{x - Ut}{2\sqrt{Dt}} \right) \right\} \quad (4.68)$$

A set of moving points (Lagrangian) is used over a stationary grid (Eulerian) to solve numerically the convection equation (Equation 4.69)

$$\frac{dx}{dt} = U \quad (4.69)$$

Then, the diffusion part given by Equation 4.70

$$\frac{dC}{dt} = D \frac{d^2 C}{dx^2} \quad (4.70)$$

is superimposed by use of a finite difference algorithm.

Each moving point p , located at x_p , y_p is assigned a concentration C_p . From the physical point of view each point may represent a large number of fluid or pollutant particles.

At time $n + 1$ the new positions of particles are

$$x_p^{n+1} = x_p^n + U\Delta t$$

According to Equation 4.70 the change in concentration due to dispersion is

$$\Delta C_i^n = \Delta t D (\Delta_x^2 C_i^*)$$

where C_i^* is equal to the average of the particle concentrations C_p^n of all the particles which lie in the rectangle i . The new particle concentration is

$$C_p^{n+1} = C_p^n + \Delta C_i^n$$

and the new point concentrations are computed by using the following relationship

$$C_p^{n+1} = C_i^* + \Delta C_i^n$$

A mixed Lagrangian–Eulerian algorithm has been tested in one- and two-dimensional flows. The reliability of the numerical simulations is checked by comparing the numerical results with the analytical solution (Equation 4.68).

These comparisons are shown in Figures 4.26 and 4.27 for Peclet numbers 1, 10 and 100.

In Figure 4.27 the pure advection case of a Gaussian hill is considered. No deformations due to numerical diffusion or dispersion have been found.

4.2.3

Time Series of Water Quality Data

Data concerning water quality in a river may be collected in the form of a time series. Such data can be used for two purposes:

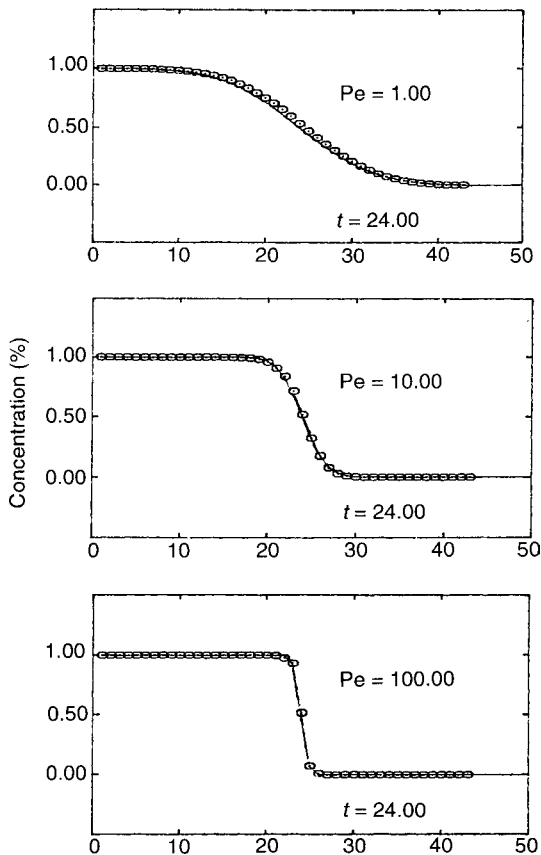


Figure 4.26 Comparison between the analytical solution (solid line) and numerical simulation (o), based on the mixed Lagrangian–Eulerian algorithm.

- (1) statistical analysis of time series to determine the trends, seasonality, autocorrelation and other statistical characteristics (Argyropoulos and Ganoulis, 1992);
- (2) estimation of coefficients for dispersion, reaeration, nitrification, denitrification, and so on.

Combination of time series water quality data with computer mathematical models may be used to assess the risk of water pollution in a river. A typical example of a time series for NH_4^+ concentration in a river is shown in Figure 4.28.

4.2.4

Risk Assessment

Variabilities in time and space of water quality characteristics in a river may be taken into consideration together with the environmental quality objectives to assess the risk of pollution.

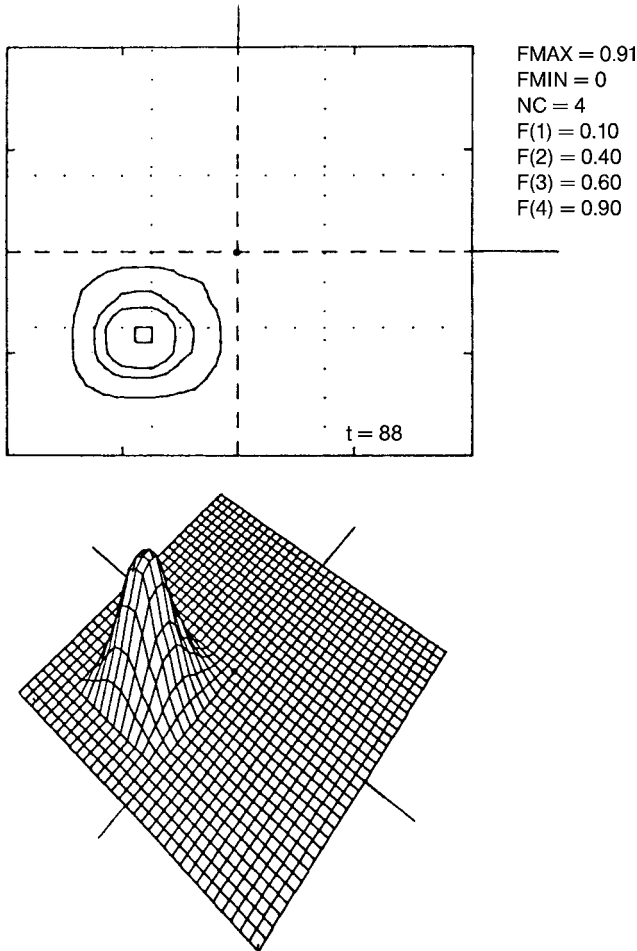


Figure 4.27 Numerical simulation of the two-dimensional rotation of a Gaussian hill (Lagrangian–Eulerian algorithm).

The mass balance equation for pollutant species i , may be used together with Monte-Carlo simulation to generate outputs in the form of frequency distributions of pollutant concentrations at given locations (Figure 4.29). The following statistical variations may be included

- (a) probability distribution of river flow rate;
- (b) frequency distribution or time series of pollutant loads (cross section 1, Figure 4.29);
- (c) concentration after initial dilution;
- (d) frequency distribution or time series downstream (location 2, Figure 4.29) using modelling.

Attention should be paid to the appropriate time scales for mathematical simulation. For example, processes like eutrophication and DO variation may indicate

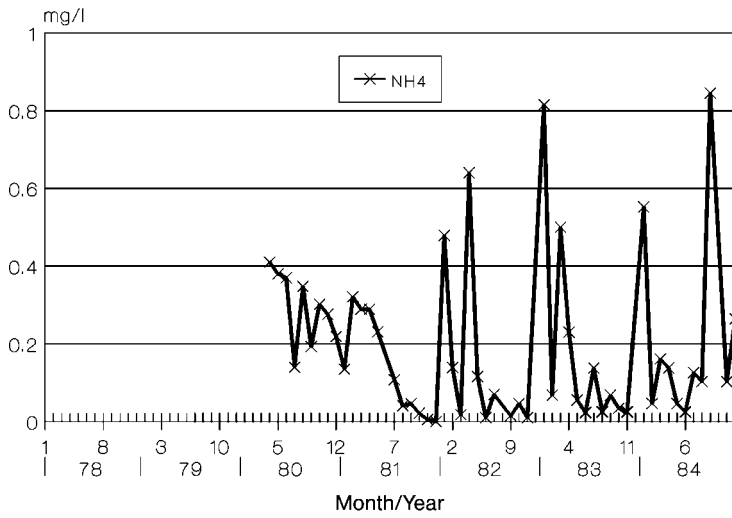


Figure 4.28 Time series of NH_4^+ concentration in a river.

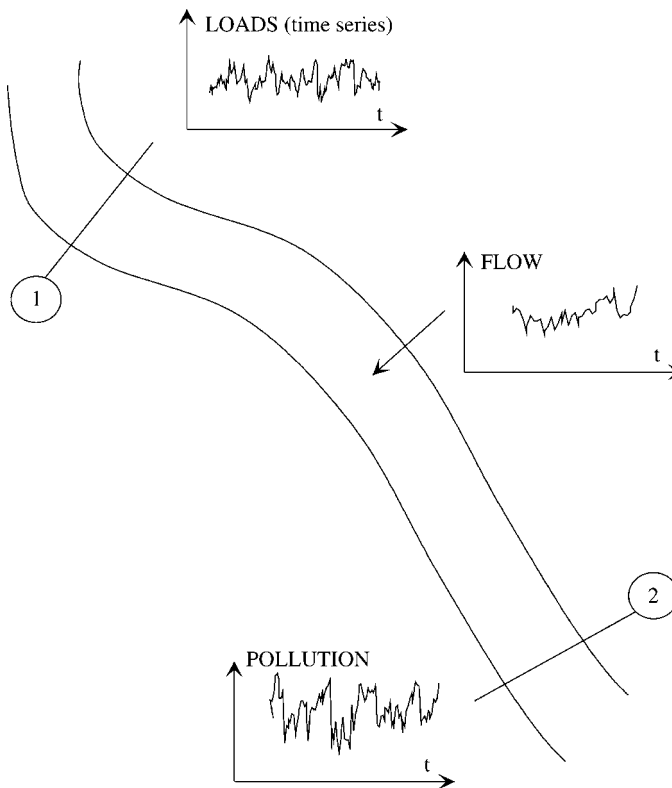


Figure 4.29 Risk assessment of river pollution.

significant diurnal variation. Time lags of ecological processes are not taken into consideration.

Statistical independence is assumed between random variables, such as flows and pollutant loads, although some correlation frequently occurs.

4.3

Risk in Groundwater Contamination

About 75% of the inhabitants of the EC member states depend on groundwater for their water supply. Public water supply requires a reliable source, which means that the quality, as well as the quantity, should be ensured beyond all doubt in relevant areas. Both groundwater quality and quantity are of essential importance for the diversity of ecosystems. Lower groundwater levels and changes in groundwater quality due to man-induced contamination cause loss of diversity of ecosystems and deterioration of natural reserves.

Groundwater is in danger of losing its potential functions due to its deteriorating quantity and quality. While aiming at sustainability of the use, the vital functions of groundwater reservoirs are threatened by pollution and overexploitation, as shown many times and for many places by Koshiek *et al.*, 1991.

One very important problem of groundwater quality deterioration is increasing salinisation near the soil surface and desertification of millions of hectares of irrigated land around the world. For example in Australia, it has been recognised (Tickell and Humphrys, 1984) that a rise in the groundwater table is one of the main causes of waterlogging and salinity increases near the top layer of the soil. As groundwater moves upward, salinity is increased by the dissolution of salts in the soil. The rising of the groundwater table originates from the effect of intensive actual irrigation combined with the disruption of the natural equilibrium between plants, soil and groundwaters. In fact, the intensive removal of deep-rooted vegetation in the past has reduced the natural drainage capacity of basins and destroyed the natural equilibrium between groundwater recharge and drainage. When the water table rises to a depth less than 2 m below soil surface, salt concentrations are further increased by evaporation and damage to vegetation and soils is then likely.

Protection of groundwater resources is based on different strategies involving either empirical or sophisticated methods. Various traditional strategies for groundwater protection range from the construction of groundwater vulnerability maps and the definition of protection perimeters around pumping wells, to the use of sophisticated optimisation multi-criterion decision-making techniques under risk conditions. A very characteristic example is the definition of adequate waste disposal sites in relation to the risk of groundwater contamination.

The main difficulty in designing groundwater development plans is that groundwater pollution is subject to several types of uncertainties. These are related to the high variability in space and time of the hydrogeological, chemical and biological processes involved. The principal task of engineering risk analysis is to assess the probability or risk so that groundwater quality standards are complied with in the areas to be

developed. For example, according to environmental quality standards, in groundwater used for irrigation the salinity concentration should not exceed 1000 ppm.

4.3.1

Importance of Groundwater Resources

4.3.1.1 Groundwater in the Hydrological Cycle

According to estimations by the US Geological Survey, 98% of the total fresh water available on the earth is stored in the ground. This water is located at depths up to 4000 m, and half of this quantity is technically available at depths of less than 800 m. It is interesting to note that rivers and lakes hold only 1.5% of the total amount of fresh water available on the planet, with the remainder being stored as soil moisture. Fresh water in polar glaciers and ice caps is excluded from these statistics, although this represents about 60% of the total fresh water on earth. The latter is estimated to total $40 \times 10^6 \text{ km}^3$ ($1 \text{ km}^3 = 10^9 \text{ m}^3 = 1 \text{ billion m}^3$), which is only 2.7% of the total (fresh and salt) water available.

From the above considerations it can be seen that, after glaciers, groundwater is the largest source of fresh water in the world. Although only a small fraction of the available groundwater resources is used, its development has recently become of increasing importance for many countries. This is mainly due to the fact that surface waters are over-used and increasingly polluted from domestic, agricultural and industrial discharges. Usually groundwater is still of very good quality, and its judicious use along with surface waters can give an optimum solution to specific problems of water demand. It is therefore important to realise the connection between groundwater and surface water in the world's hydrological cycle.

As shown in Figure 4.30 evaporation from the oceans exceeds precipitation. The remaining quantity of water ($47\,000 \text{ km}^3/\text{year}$) is transported over the continents, and together with continental evapotranspiration, gives $119\,000 \text{ km}^3/\text{year}$ of precipitation over the land. The major part (60%) of this water is re-evaporated, about 30% runs off

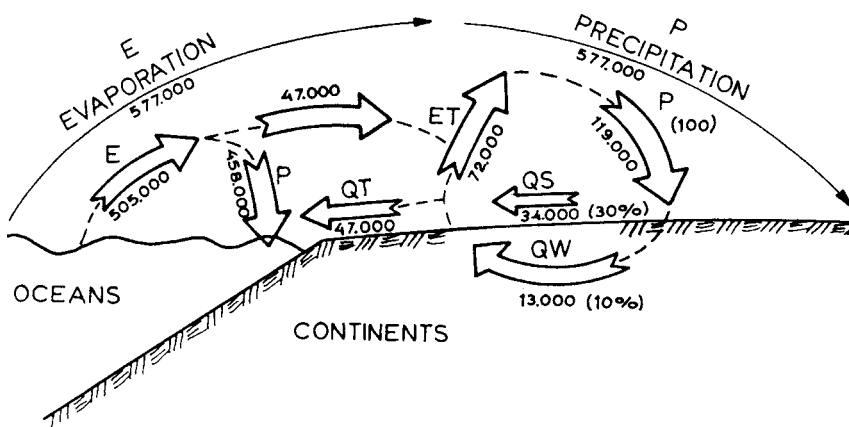


Figure 4.30 Global hydrological cycle. Units are in km^3/year .

Table 4.3 Mean annual precipitation and flows (surface run-off plus infiltration) in some EU countries (Bodelle and Margat, 1980).

Country	Precipitation		Efficient precipitation (surface run-off plus infiltration)	
	Mean annual volume $10^9 \text{ m}^3/\text{year}$	Mean annual height cm/year	$10^9 \text{ m}^3/\text{year}$	cm/year
France	440	80	170	30
Greece ^a	112	85	55	42
Italy	300	100	165	55
Spain	330	66	110	22
Germany	200	80	86	35
UK	250	100	160	65
Belgium	250	85	110	36

^a Estimate from incomplete data.

and the remainder (10%) percolates into the soil. This groundwater is termed meteoric and is of great importance for groundwater resources development. For shallow aquifers, and near the water table, water cycling takes a year or less, but in deep aquifers it is of the order of thousands of years.

For groundwater resources development it is not precipitation itself which is of importance but rather the so-called *efficient precipitation*. This is the amount of water resulting when evapotranspiration is subtracted from total precipitation. This amount represents the surface run-off plus the water infiltrating the soil.

For the development of water resources it is important to know the spatial distribution of the various components of the hydrological cycle. For groundwater resources development it is necessary to have geological, hydrogeological and hydrochemical information as well as field data allowing evaluation of the groundwater balance. Data for some European Union countries are summarised in Table 4.3.

4.3.1.2 Steps in Groundwater Development

From an engineering point of view, the ultimate goal is the optimum use of available groundwater resources. This quantitative question often relates to groundwater quality, when various pollutants are diluted into the soil and modify the chemical composition of the water in the aquifer. In several cases hot water and thermal springs offer the possibility of using geothermal energy for housing and various agricultural purposes (drying, greenhouses, etc.).

The quantity, quality and energy considerations in groundwater management form a multidisciplinary field requiring scientific cooperation between various disciplines, such as:

- *Hydrogeology*: geophysical and geological prospecting, drilling techniques, maps.
- *Groundwater hydrodynamics*: quantitative aspects of hydrogeology, mathematical modelling, calibration, prediction, and so on.

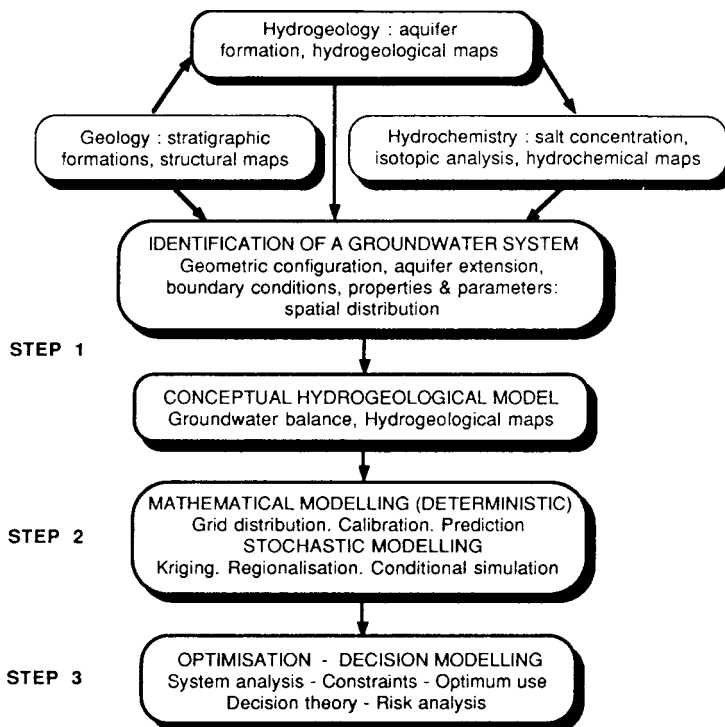


Figure 4.31 Steps in groundwater development.

- *Groundwater management*: systems analysis, optimisation techniques, risk and decision theory.
- *Hydrochemistry*: chemical composition of the soil and water.
- *Hydrobiology*: biological properties of groundwater systems.

Modern tools for groundwater development extensively use software, data bases and microcomputers, as shown in Figure 4.31

Identification of an aquifer and determination of its water balance is the first step, preceding mathematical modelling (deterministic or stochastic – step 2) and the management action (step 3). Step 1 forms the physical basis for all possible modelling simulations. As shown in Figure 4.32 it is very important to determine the extents of the aquifer system within the hydrogeological basin, which is connected to the surface hydrological basin. In terms of water balance we have:

$$(\text{Efficient precipitation}) = (\text{Total precipitation}) - (\text{Evapotranspiration})$$

$$PE = P - ETR$$

$$(\text{Total surface runoff}) = (\text{Efficient precipitation}) = (\text{Surface runoff}) + (\text{Infiltration})$$

$$QT = PE = QS + I$$

The efficient infiltration is found from the infiltration I if the evapotranspiration from the top soil layer is subtracted. The total groundwater flow rate Q_w is distributed in

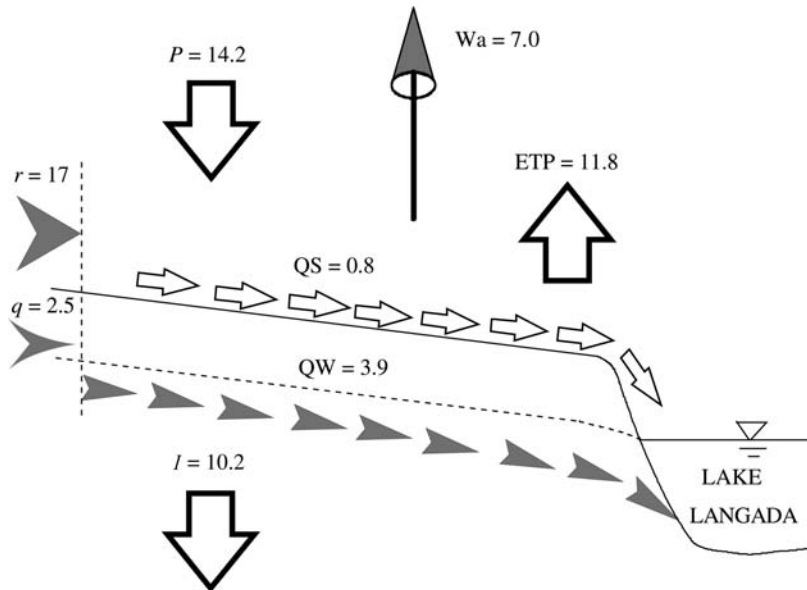


Figure 4.32 Water balance for the hydrological basin of Bogdana stream (millions of m^3/year).

the form of groundwater flow into the aquifer. In Figure 4.32 an example of water balance is given from a small basin of the Bogdana stream, located in Macedonia, Greece, close to the city of Thessaloniki.

4.3.2

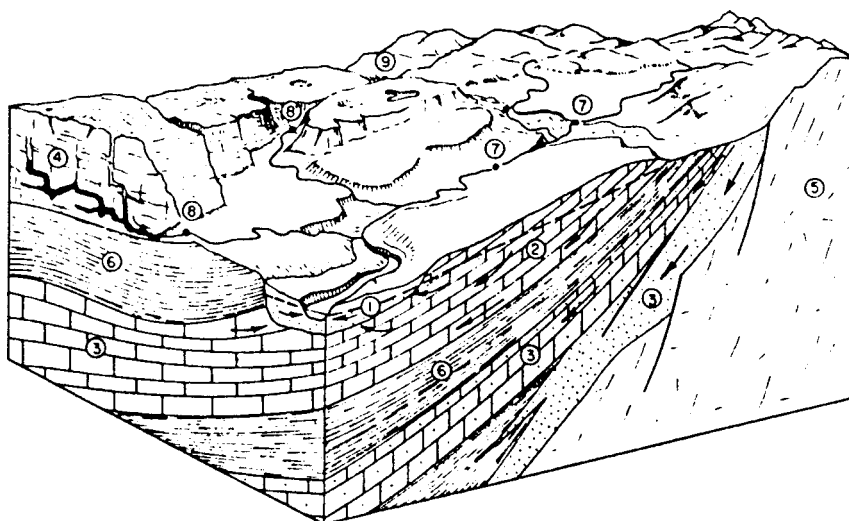
Properties and Field Investigation of Groundwater Systems

4.3.2.1 Water in Geological Formations

Water infiltrating the soil circulates through various geological formations. Depending on the boundary conditions (impermeable or semi-permeable layers of soil, atmospheric pressure, rivers and lakes) the groundwater forms various types of subsurface reservoirs, called aquifers. These are extensive permeable rock formations through which water partially accumulates and partially flows. Figure 4.33 gives an overview of different types of groundwater aquifers in various geological formations.

According to their geological formation characteristics, aquifers may be classified into three groups:

- (a) Alluvial and sedimentary aquifers;
 - (b) Limestone and karstic aquifers;
 - (c) Crystalline fractured aquifers.
- (a) The first category of sedimentary basins is characterised by successive layers of different hydrogeological properties: permeable, semi-permeable or impermeable. The water circulates in successive layers which consist mostly of gravels,



- | | |
|---|--------------------------------|
| 1 Alluvial aquifer in connection with a river | 6 Sedimentary impervious layer |
| 2 Regional aquifer with free surface in limestone | 7 Spring from phreatic aquifer |
| 3 Regional aquifer between impermeable layers | 8 Spring from karstic aquifer |
| 4 Karstic aquifer | 9 Leakage to a river |
| 5 Crystalline fractured rock | |

Figure 4.33 Groundwater in various geological formations (Bodelle and Margat, 1980).

sands, clays and silts. Phreatic, confined or semi-confined (leaky) aquifers are formed. Surface water recharging these aquifers comes mostly from the greater surface-drainage area, usually much wider than the hydrogeological basin. A typical example of a regional sedimentary basin is that of Paris (Figure 4.34), having a surface area of 140 000 km².

- (b) Limestone and karstic aquifers: solution processes by acidified rainwater increase the permeability of limestones and dolostones forming secondary aquifers. Karstic phenomena are extreme cases of such processes, creating subterranean

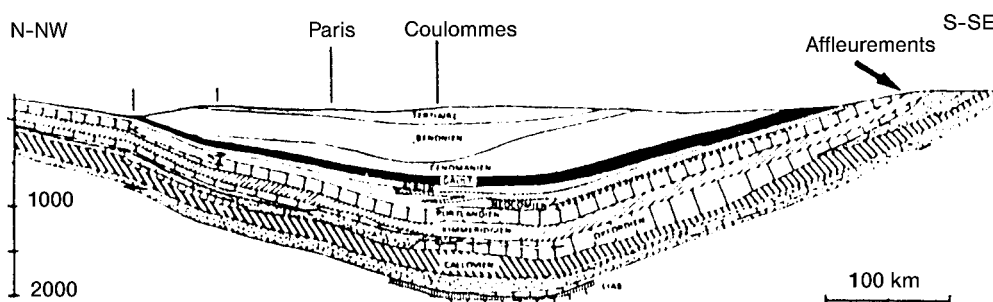


Figure 4.34 Geological section of the sedimentary basin of Paris (Castany, 1982).

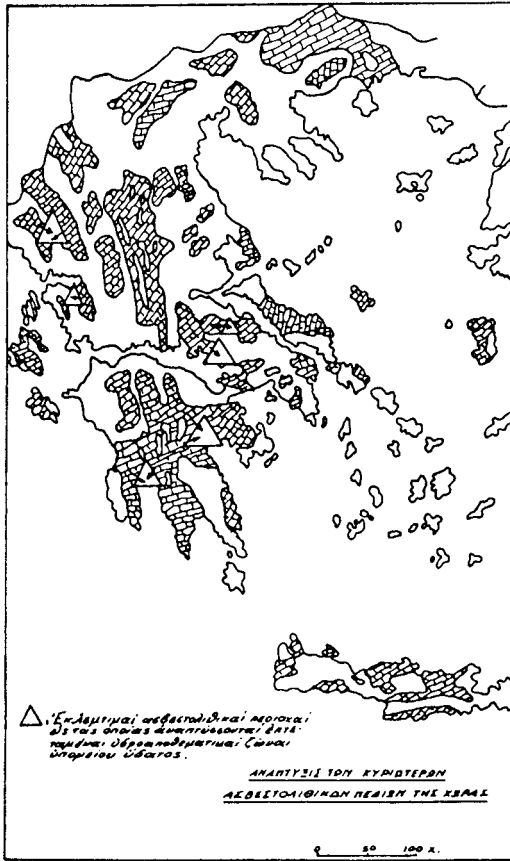


Figure 4.35 Karstic areas in Greece (cited by Therianos, 1974).

fractures and water conduits of high permeability. In karstic regions surface runoff is nil and large amounts of groundwater volumes can be found at various depths.

As shown in Figure 4.35 karstic formations cover about half of the total area of Greece. This represents a potentially valuable groundwater resource for the country. In northern Greece, known karstic aquifers are those of the Vermion mountain, the geothermal karstic reservoir of Katsikas-Petralona-Eleohoria, the thermal spring of Ag. Paraskevi, Chalkidiki and the spring of Aravissos, which actually provides part of the water supply for the city of Thessaloniki.

- (c) Crystalline rock aquifers: the importance of groundwater resources in these rocks depends on two factors, (1) the rate of fracturing and (2) the chemical weathering of the surface layer, through which precipitation water percolates into the rock. This geological formation is divided into several blocks by secondary and primary fractures.

4.3.2.2 Space and Time Scales

A space scale can be empirically defined as the extent of the flow domain. A time scale characterises the duration of a hydrological phenomenon. Using these definitions, multiphase flow in porous media is studied at various space and time scales. Three examples are characteristic in petroleum engineering:

- close to production or injection wells, multiphase flow is studied at the scale of a few metres;
- secondary oil production from the reservoir is simulated at the regional scale of some kilometres;
- formation conditions of the oil reservoir and historical case studies are conducted at the scale of the sedimentary basin, typically greater than 50 km.

A variety of time scales are used for time evolution. Some examples of unsaturated flow in hydrological applications include:

- the study of aquifer recharge following a heavy rainfall on a relatively short time scale (few hours);
- the estimation of renewable groundwater resources on seasonal or annual scales;
- the geological and hydrogeological study of groundwater formation on an historical time scale (for example, one century).

Space Scales A space scale is defined as the characteristic size of the spatial area in which the multiphase flow is studied. Five different space scales are distinguished.

(1) *Pore Scale*: $1\text{--}10^{-3}$ mm

This is the average length of a pore or solid particle. For soil materials (sand, clay or silt) typical lengths at this scale are of the order of 1 to 10^{-3} mm. The mathematical model of multiphase flow at this scale consists of the continuum fluid mechanics balance equations together with the boundary conditions over the fluid–fluid and fluid–solid interfaces. Physicochemical interactions due to molecular fluctuations close to these interfaces occur in the next lowest space scale, which is the molecular scale.

(2) *Sample Scale*: 10 cm

This is the scale of a small soil sample containing a large number of pores and solid particles. A 10-cm sized cube of rock with a volume of 1 l is a typical example of this scale, at which Darcy's law applies. Although microscopically the fluid–solid and fluid–fluid interfaces form a discontinuous medium, at this scale the continuum approach is possible by averaging.

(3) *Laboratory Scale*: 1 m

This is the scale of common experimental set-ups of the order of 1 m. At this scale the porous medium can be homogeneous or heterogeneous as, for example, in the case of microfractures in rocks.

(4) *Local Aquifer Scale: 100 m*

This is a characteristic length, in a horizontal direction, of the order of magnitude of the aquifer thickness. Multiphase flow is three-dimensional at this scale. Greater geological fractures cause heterogeneities of the aquifer.

(5) *Regional Scale: 10 km*

This scale is again in the horizontal direction but this time much larger than the aquifer thickness. Depth-averaged properties and flow variables are defined at this scale and two-dimensional models are used.

Depending on the particular space scale, the physical laws and mathematical models describing the multiphase flow take different forms. For example, the Navier–Stokes equations together with the kinematic and dynamic compatibility relations at the fluid interfaces and the boundary conditions at the solid walls form the mathematical model for multiphase flow at the pore scale. Darcy's law and the relative permeabilities of different fluid phases are used to compute the relative phase velocities at the macroscopic scale. From a theoretical point of view the mathematical model at the macroscopic scale is derived by averaging the corresponding mathematical model and the interface boundary conditions at the pore scale. This procedure is difficult because of the irregular form of the microscopic geometry and the presence of interfaces. Also, averaging gives rise to macroscopic coefficients, such as the relative permeabilities, which, although related to the local structure of the porous medium, display no explicit correlations with the local conditions.

Expressions for space averaging of local properties have been derived by many authors (e.g. Whitaker, 1967; Slattery, 1967 and Marle, 1967). The theory of regionalised variables (Matheron, 1965, 1967; Dagan, 1986; De Marsily, 1986) is based on the assumption that phase properties are stochastic variables. Macroscopic phase variables emerge as the ensemble averages of the microscopic variables. The quantitative description at the space heterogeneity scale is obtained by aggregation of the microscopic phase properties (Ganoulis, 1986). These length heterogeneity scales are the aggregation lengths which must be used to reduce the variance of the macroscopic phase properties to a certain limit, close to zero. By using this definition, the local macroscopic scale (Representative Elementary Volume, REV) is evaluated in terms of pore size or grain size distributions of the porous medium (Ganoulis, 1986). Figure 4.36 shows the different space which can be computed by aggregation. Note that at the pore level (scales l_a , l_b , l_s for the fluid phases a , b and solid phases) the porous medium is heterogeneous. At the local macroscopic scale l (REV) it is homogeneous but it becomes heterogeneous at the scale of heterogeneities L_1 , L_2 and L_3 (stratified porous medium). By aggregation at the larger scale, the aquifer may be considered homogeneous at the local aquifer scale L_a .

Time Scales In unsteady multiphase flow through porous media the time scale can be defined as the duration of the flow phenomena. From a specific event, such as a rainfall, and the resulting recharge of the aquifer in geological time, and taking 1 year as the time unit, the following time scales can be distinguished:

Time scale	Value (in years)
Event	10^{-3}
Seasonal	10^{-1}
Annual	1
Historical	10^2
Geological	10^4

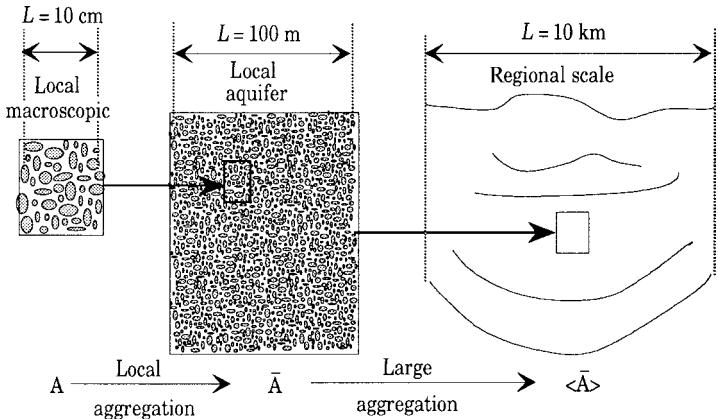


Figure 4.36 Change of scale in aquifers from local to regional level.

4.3.3

Aquifer Hydraulic Properties

4.3.3.1 Scale Effects

Aquifers are complex media consisting of networks of interconnected pores, fractures and cracks, through which water circulates. When all voids in the aquifer are completely filled by water, there is interaction between two phases: (a) the solid phase or solid skeleton or solid particles and (b) the liquid phase or groundwater. In the unsaturated soil zone above the phreatic water table, water and air are present in variable degrees. This is a special case of a two-phase flow of immiscible fluids (water–air). In several cases of practical interest oil, vapour and various pollutants may be present in the pores. The general problem involves multiphase phenomena of miscible or immiscible fluids, together with mass and heat transfer.

The physical and hydraulic properties of a porous formation (Bear, 1979) depend on the spatial scale to which they refer. At the pore scale the medium is heterogeneous because of the local variations in size of the solid particles and pores. The microscopic channels which are formed between the solid particles are irregular and tortuous in the 3-D space. The detailed description of these microscopic pathways is almost impossible, making an averaging procedure necessary.

By taking average values of the local properties (such as porosity and permeability), the irregular variations in space are smoothed out and some macroscopic properties of the medium are defined. The Representative Elementary Volume (REV) must have large enough dimensions to smooth out local variations, but be small enough for the theory of continuum mechanics to apply (De Marsily, 1986). The smoothing out procedure of REV can be extended into the regional scale of aquifers (Ganoulis, 1986).

As shown in Figure 4.36, by smoothing out the aquifer properties at a regional scale the medium can be considered as homogeneous (no spatial variation of its properties). The same aquifer is heterogeneous in the REV scale. The porous material within every REV is homogeneous, but not in the pore scale, where a pore-to-pore heterogeneity is present.

In order to describe the aquifer properties, one rational procedure can be applied: starting from the local properties and using a spatial averaging rule, the corresponding properties at the macroscopic level can be derived. For the practical application of this approach some idealised models of the porous media in the local space are necessary. Such models consist of interconnected capillary tubes or regular arrangements of spheres. Defining the porosity n as the ratio between the volume of pores to the total volume, in the cubic arrangement which represents the loosest state of packing, $n = 47.6\%$, while in rhombic packing the porosity may become as low as $n = 26\%$.

Specific Yield S_y A portion of the water present in the pore space is held by molecular and surface tension forces and cannot be drained by gravity. Expressed as a percentage of the total volume of the aquifer this quantity is called specific retention. Specific yield is the difference between porosity and the specific retention, S_r

$$S_y = n - S_r$$

This represents the water removed from a unit volume by pumping or drainage. It varies from 10 to 20% for alluvial aquifers to about 30% for uniform sands.

Grain Size Distribution This can be determined by conducting an analysis test in a nest of standard sieves with the coarsest on the top and the finest at the bottom. By counting the percentage of material passing through each sieve (finer material) a curve in a semi-logarithmic plot is obtained, as shown in Figure 4.37. This type of analysis permits the classification of the alluvial material from coarse gravel to fine clay (Table 4.4).

Table 4.4 Classification of aquifer material according to grain size.

Gravel	Sand			Silt			Clay
	Coarse	Medium	Fine	Coarse	Medium	Fine	
> 2 mm	0.2 – 0.6 mm	0.6 – 0.2 mm	0.2 – 0.006 mm	0.06 – 0.02 mm	0.02 – 0.006 mm	0.006 – 0.002 mm	<0.002 mm

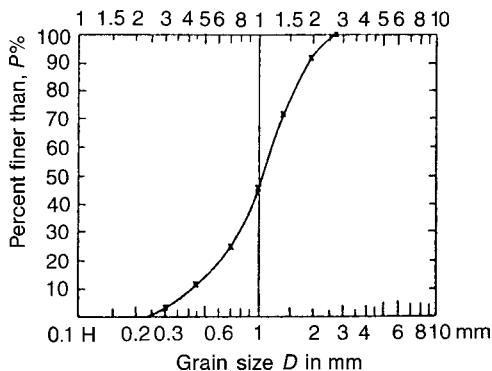


Figure 4.37 Grain size distribution of an aquifer sample.

The effective size d_{10} is the grain size corresponding to 10% of the material being finer and 90% coarser.

Coefficient of Permeability K If we denote the seepage or bulk velocity of groundwater flow as V and the hydraulic gradient as i , then empirically, Darcy's law takes the form

$$V = K i$$

where K is the coefficient of permeability (m/s), depending on fluid viscosity and the geometrical properties of the porous medium. The intrinsic or specific permeability k (m^2) is a function of the structural characteristics of the porous medium given by

$$k(m^2) = \frac{\nu}{g} \cdot K(m/s)$$

where ν is the kinematic viscosity of the fluid.

Various models have been proposed to explain Darcy's law and relate the permeability of the aquifer to the microscopic characteristics of the porous medium, such as the pore size and the grain size distributions. In fact, at the pore scale the local flow is laminar through the microscopic channels and

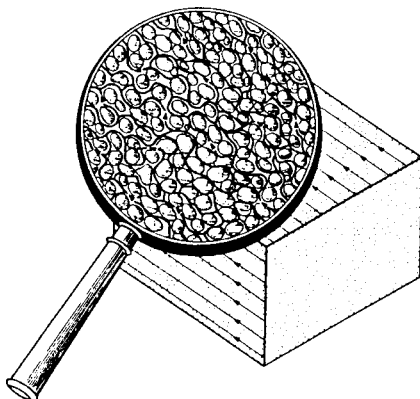


Figure 4.38 Microscopic and macroscopic streamlines in porous medium.

the Navier–Stokes equations apply, as shown in Figure 4.38. The local flow accelerates and decelerates as the pores are not cylindrical. The averaging of the Navier–Stokes equations to the level of Darcy’s law remains an intriguing problem.

An empirical relation between the grain size d_{10} and the intrinsic permeability k is

$$k(\text{m}^2) = 0.001d_{10}^2(\text{m})$$

Transmissivity Coefficient T (m^2/s) This is the product of field permeability $K(\text{m/s})$ and the saturated thickness of the aquifer. For a confined aquifer of width b (m) the transmissivity is $T = Kb$; for a phreatic aquifer of thickness h above the impervious layer the transmissivity is $T = Kh$.

Storage Coefficient S (Dimensionless) This represents the water volume discharged from a unit prism of aquifer (vertical column of aquifer standing on a unit area of 1 m^2) when the piezometric level falls by one unit depth (1 m). For phreatic aquifers the storage coefficient is the same as the specific yield and ranges from 0.05 to 0.30.

For confined aquifers, water is released because of the compression of the granular material of the aquifer and the reduction in water density. In this case the storage coefficient ranges from 5×10^{-5} to 5×10^{-3} and it can be computed using the formula

$$S = b(\rho g)n(\alpha + \beta)$$

where b is the aquifer thickness (m), ρ the fluid density (kg/m^3), n the porosity, α the aquifer compressibility (m^2/N), and β the water compressibility (m^2/N). The storage coefficient is usually estimated from field data by means of pumping tests.

4.3.3.2 Measurements and Field Investigations

The properties of regional aquifers vary from site to site. This is due to different reasons such as field variations in the geological conditions, inequalities in surface topography, non-uniform vegetation, and so on. The space variability of aquifers requires various measurements at several selected sites and at various depths to represent the hydrogeological system satisfactorily. Measurements can be made directly in the field or from soil samples taken to the laboratory.

One fundamental question concerns the number of measurements which are necessary for estimation of aquifer regional properties within a fixed and acceptable error. The answer to this question depends upon the particular type of aquifer, the parameter of interest and the variation of the properties in the region of concern. At present this problem can only be solved empirically.

If the spatial dependence of the aquifer properties is ignored, classical statistical analysis may be applied to estimate mean values, variances and statistical distributions. Modern theories use variograms and kriging analysis.

Typical laboratory measurements on field soil samples comprise the following:

(A) *Measurements of bulk density and porosity*

(1) *Bulk Density (ρ_b)*

This is the weight or mass of a bulk volume of soil. If the soil is dry, then the resulting bulk density is called dry bulk density ρ_{dry} .

(2) *Porosity (n)*

$$n = 1 - \frac{\rho_{dry}}{\rho_s}$$

where ρ_s is the density of the solid particles. For mineral soils ρ_s is taken as $2.650 \times 10^3 \text{ kg/m}^3$.

(B) *Measurement of saturated hydraulic conductivity*

- Constant head method.
- Falling head method.

(C) *Measurement of the soil–water content*

- Gravimetric method.
- Gamma-ray attenuation.

(D) *Measurements of soil–water capillary pressure and unsaturated hydraulic conductivity*

The most popular field measurements are pumping tests. Their principal use is in the determination of the hydraulic properties of an aquifer and the relevant methods are described in the second part of this book. Other field measurements are concerned with

- the soil–water content, using the neutron scatter method,
- the soil–water capillary pressure, using tensiometers,
- hydraulic conductivity: Lefranc and Lugeon type tests.

4.3.4

Conceptual and Mathematical Models

4.3.4.1 Conceptual Models and Flow Equations

Aquifer formations are complex hydrogeological systems with properties and hydrodynamic characteristics varying both in space and time. Any planning strategy for groundwater resources development and protection depends upon two main points: (a) the ability to predict the consequences of alternative operations imposed on the aquifer under study and (b) the ability to establish quantitative criteria to judge the quality of the results of each operation policy.

The first condition may be fulfilled through various modelling techniques. The second can be based on optimisation methods and risk analysis. Although important progress has been made in these theories, final judgements are actually based on various social and political considerations. However, modelling, optimisation and application of risk and reliability techniques may be useful tools for decision makers.

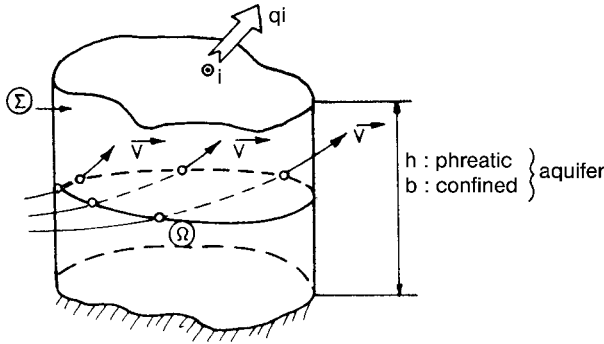


Figure 4.39 Control volume of an aquifer.

Conceptual models are idealisations of the natural aquifer systems (form, areal extension, physical properties of the aquifers) and their constituent processes (flow conditions, boundary conditions). Vertically integrated equations are usually used to represent flow in regional aquifers. These equations are obtained in the horizontal plane x - y by application of two basic laws

- the law of mass conservation and
- Darcy's law.

Consider the control volume Ω in an aquifer shown in Figure 4.39. This volume is defined by the surface Σ of height h (phreatic) or b (confined aquifer). The mass balance gives

Introducing the *piezometric head* or *hydraulic head* h as the sum of the *pressure head* $p/\rho g$ and the *elevation* z , that is $h = p/(\rho g) + z$ and using the definition of the storage coefficient S , the mass balance in the volume Ω takes the following form

$$\int_{\Omega} \rho S \left(\frac{\partial h}{\partial t} \right) d\Omega = - \int_{\Sigma} \rho (\vec{V} \cdot \vec{n}) d\Sigma$$

where \vec{V} is the vertical mean velocity of the flow.

Using the Gauss formula, we have

$$\int_{\Omega} \left[\rho S \left(\frac{\partial h}{\partial t} \right) + \text{div}(\rho \vec{V}) \right] d\Omega = 0$$

or

$$S \left(\frac{\partial h}{\partial t} \right) + \text{div} \vec{V} = 0 \quad (4.71)$$

This is the general mass balance differential equation for confined or unconfined aquifer flow. The vertical mean velocity \vec{V} is expressed by Darcy's law

$$\vec{V} = -(KC) \nabla h \quad (4.72)$$

where K is the coefficient of permeability and $C = b$ for confined aquifers or h for phreatic aquifers. Introducing Equation 4.72 into the mass balance equation (Equation 4.71) we obtain

$$S \left(\frac{\partial h}{\partial t} \right) = \nabla (KC \nabla h) \quad (4.73)$$

When pumping and recharging wells at points i is taking place, Equation 4.73 takes the following form

$$S \left(\frac{\partial h}{\partial t} \right) = \nabla (KC \nabla h) - \sum_i q_i \delta_i \quad (4.74)$$

where: $q_i > 0$ for pumping wells (in $\text{m}^3/\text{s}/\text{m}^2$); $q_i < 0$ for recharging wells (in $\text{m}^3/\text{s}/\text{m}^2$); and δ_i is the Dirac delta function for point i .

From Equation 4.74, two cases may be distinguished:

Confined Aquifer

$$S \left(\frac{\partial h}{\partial t} \right) = \nabla (T \nabla h) - \sum_i q_i \delta_i \quad (4.75)$$

($T = Kb = \text{transmissivity}$)

Phreatic Aquifer

$$S \left(\frac{\partial h}{\partial t} \right) = \nabla (Kh \nabla h) - \sum_i q_i \delta_i \quad (4.76)$$

4.3.4.2 Analytical Solutions

By using a number of simplifying assumptions, Darcy's law and the mass continuity principle can be used to obtain analytical solutions. Some steady flow idealised cases will be considered to illustrate this point.

Steady Flow to a Trench Through a Confined Aquifer As shown in Figure 4.40, a bed of porous material, having permeability K and thickness b , overlies an impermeable

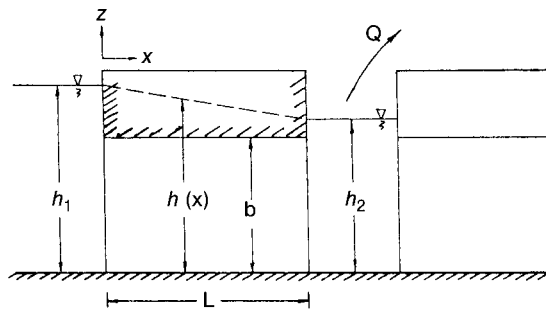


Figure 4.40 Steady flow to a trench through a confined aquifer.

base. The porous bed is itself overlain by an impermeable layer. A canal and trench of width m are separated from one another by a distance L . Let a head h_1 be maintained in the canal at the left while a head h_2 is maintained in the trench, both h_1 , and h_2 being greater than b . Because the cross-sectional area of the flow (mb) is constant, using Darcy's law

$$V = -K \frac{dh}{dx}$$

the flow rate Q becomes

$$Q = -K(mb) \frac{dh}{dx} \quad (4.77)$$

Integrating Equation 4.77 between h_1 , $x = 0$ and some arbitrary point at a distance x , the following linear relationship is obtained

$$h(x) = h_1 - \frac{Qx}{K(mb)} \quad (4.78)$$

Steady Flow to a Trench through a Fully Unconfined Aquifer In the case of unconfined flow illustrated in Figure 4.41 the situation is more complicated because the cross-sectional area of flow $m h(x)$ is unknown a priori. Due to the sloping water table the flow in the vertical plane x - z is two-dimensional. To resolve this problem the following two assumptions were first suggested by Dupuit in 1863: (a) for a small slope of the water table the flow is considered horizontal (one-dimensional) and, (b) the hydraulic slope dh/dx does not vary with depth and is equal to the slope of the water table.

From Darcy's law we have

$$Q = -K(mh) \frac{dh}{dx} \quad (4.79)$$

in which K , m and Q are constants. Integration of Equation 4.79 between $h = h_1$, $x = 0$, and $h = h_2$, $x = L$ gives

$$Q = \frac{Km}{2L} (h_1^2 - h_2^2) \quad (4.80)$$

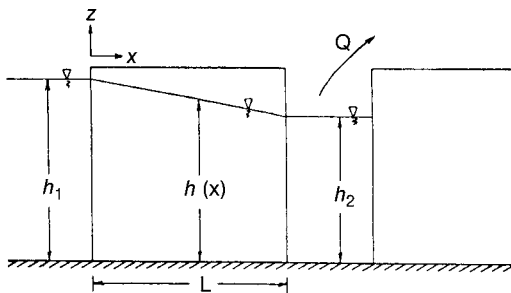


Figure 4.41 Steady flow to a trench through a fully unconfined aquifer.

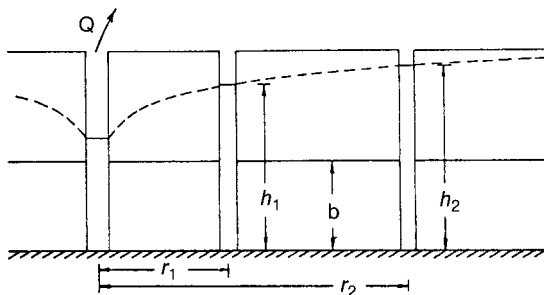


Figure 4.42 Confined flow to a pumping well.

Integrating Equation 4.79 between $h = h_1$, $x = 0$ and some arbitrary point at x gives a parabolic shape of the water table

$$h = \sqrt{h_1^2 - \frac{2Qx}{Km}} \quad 0 \leq x \leq L \quad (4.81)$$

Confined Flow to a Pumping Well As shown in Figure 4.42, a constant flow rate Q is discharged from a well in a confined aquifer of infinite extent. At 'steady' flow conditions let h_1 and h_2 be the piezometric heads observed at distances r_1 and r_2 from the well, respectively. The flow crosses the area $(2\pi r)b$ and Darcy's velocity toward the well is given by

$$V = K \frac{dh}{dr}$$

The constant flow rate Q is

$$Q = (2\pi r)bV = (2\pi Kb)r \frac{dh}{dr} \quad (4.82)$$

Integrating between r_1 , h_1 and r_2 , h_2 we obtain

$$\ln\left(\frac{r_2}{r_1}\right) = \frac{2\pi bK}{Q}(h_2 - h_1) \quad (4.83)$$

Thiem (1901) proposed this equation as a method for determining permeability K .

Unconfined Flow to a Pumping Well Using the Dupuit assumptions as in the case of unconfined flow to a trench, the following relationship is obtained

$$Q = 2\pi Kh \, dh/dr \quad (4.84)$$

By integrating between r_1 , h_1 and r_2 , h_2 (Figure 4.43) it follows that

$$\ln(r_2/r_1) = [(\pi K)/Q](h_2^2 - h_1^2) \quad (4.85)$$

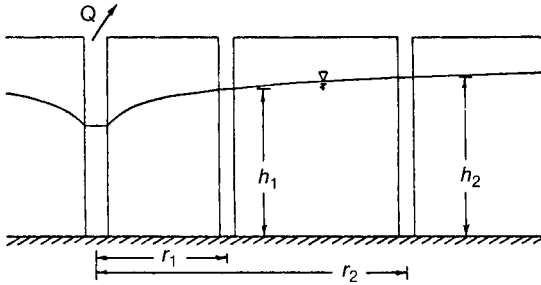


Figure 4.43 Unconfined radial flow to a pumping well.

4.3.5

Spatial Variability and Stochastic Modelling

4.3.5.1 Uncertainties in Aquifer Contamination Studies

Because of the natural variability in space and time, the main problem for evaluating the risk of groundwater contamination is the fact that physical parameters and variables of the system show random deviations. To this randomness, one must add various other uncertainties due to the scarcity of the information concerning the inputs, the value of the parameters (measurement and sampling uncertainties) and also the imperfection of the models (modelling uncertainties).

Figure 4.44 shows the random variation in space of porosity in a case of an alluvial aquifer. Because of such random variations of physical characteristics, it follows that the output variables are also not deterministic, and also show random variations. For dealing with randomness and uncertainties, risk analysis provides a general framework. The various steps to be undertaken for a comprehensive application of engineering risk analysis to groundwater contamination problems are the following: (1) identification of hazards, (2) risk quantification, (3) consequences of risk, (4) perception of the consequences and (5) risk management. Methods and tools used in groundwater contamination problems are: uncertainty analysis, stochastic simulation and the fuzzy set approach (Ganoulis, 1991d).

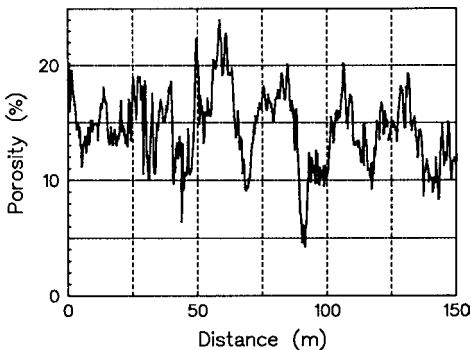


Figure 4.44 Random porosity variation in an alluvial aquifer.

For predicting groundwater flows in regional aquifers, several analytical solutions, approximate analytical techniques and, more recently, numerical models and algorithms of various levels of sophistication have been used. These methods for simulating aquifer hydrodynamics have been validated using physical models in the laboratory and *in-situ* measurements in real aquifers that are relatively homogeneous and of limited extent. However, the merit of these models for practical applications at the scale of hydrological basins is still unclear.

The primary reason for this unfortunate state lies in the very great variability in space and time of the hydrogeological parameters. *In-situ* measurements at the basin scale have demonstrated that the physical properties of the soils and the hydrological variables such as infiltration rate and piezometric levels are highly irregular. This natural variability can be understood as the combination of a deterministic and a stochastic component. The latter is analysed using probabilistic and statistical concepts. In fact, in recent years, the number of publications appearing on the application of stochastic methods to groundwater flow in aquifers has steadily increased. This indicates that more and more scientists are engaged in this area and that the stochastic modelling and management of groundwater resources is an active area of research.

Before exposing the principles of stochastic modelling in groundwater flows it may be useful to clarify briefly the following two points: (a) what is the relation between the deterministic and the stochastic approach for studying a natural groundwater system, and (b) what are the physical causes and mechanisms leading to the natural variability and stochastic behaviour of aquifers?

4.3.5.2 Stochastic Description

As shown in Figure 4.45, it is necessary to distinguish between: (a) the time variability of the aquifer boundary conditions and, (b) the spatial variability of the physical properties of the aquifer. The aquifer boundary conditions are irregularly distributed in time because they are related to the stochastic components of the hydrological cycle, such as precipitation, infiltration, time variation of river stage or they respond to randomly distributed water demands from the aquifer. Classical statistical methods and time series analysis can be applied to describe the structure of these boundary conditions.

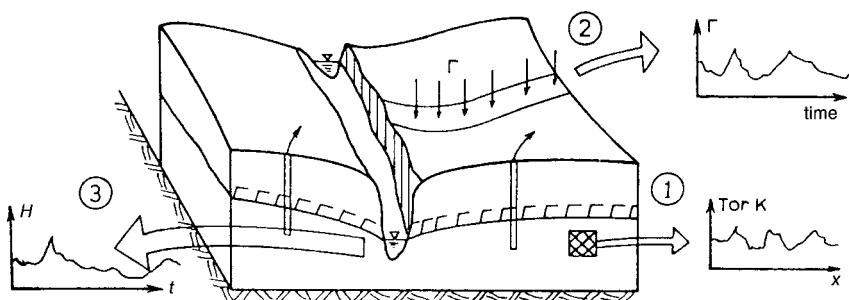


Figure 4.45 Variability of boundary conditions and physical properties in a river-aquifer system.

Because of various formation conditions such as deposition, successive glaciation, erosion, and so on, the physical properties of aquifers (porosity, hydraulic conductivity) display a great variability in space (De Marsily, 1986). If the spatial correlation of the stochastic variable is neglected, a probability law can be used to represent the statistical distribution of the variable. *In-situ* measurements indicate that hydraulic conductivity K follows a log-normal probability law. This means that if $Y = \ln K$, Y is a normal random variable with probability density:

$$f_Y(Y) = \left(\frac{1}{\sigma\sqrt{2\pi}} \right) \exp \left[-\frac{(Y - \langle Y \rangle)^2}{2\sigma^2} \right]$$

where the symbol $\langle \cdot \rangle$ is used for the mean values.

The non-dimensional variance of permeabilities or transmissivities is usually large:

$$\sigma_k^2 / \langle K \rangle^2 \text{ varies from } 0.25 \text{ to } 1 \text{ or } 2$$

The spatial stochastic structure of a variable Y can be described by using the covariance function $C(\xi)$, where ξ is the distance between two point measurements. This function is defined as

$$C(\xi) = \langle (Y(x) - \langle Y(x) \rangle) (Y(x + \xi) - \langle Y(x) \rangle) \rangle = \langle Y(x) Y(x + \xi) \rangle - \langle Y(x) \rangle^2 \quad (4.86)$$

Another way of doing this is through the variogram $\gamma(\xi)$, which is defined as

$$\gamma(\xi) = \frac{1}{2} \langle [Y(x + \xi) - Y(x)]^2 \rangle = \frac{1}{2N} \sum_1^N [Y(x_i + \xi) - Y(x_i)]^2 \quad (4.87)$$

The two functions $C(\xi)$ and $\gamma(\xi)$ are related. In fact it can be shown that

$$\gamma(\xi) = C(0) - C(\xi) = \sigma_Y^2 - C(\xi) \quad (4.88)$$

where σ_Y^2 is the variance of Y .

The stochastic approach for the description of aquifer systems is actually based on two main assumptions:

(a) *Stationarity*: This means that all the probability distribution functions of the random functions $Y_i(\xi)$ are invariable by translation. A practical consequence of this assumption is shown in Figure 4.46. The mean value of $Y(x)$ given by

$$\langle Y(x) \rangle = \mu$$

is constant and not a function of x . The same is valid for the covariance function $C(x)$ defined by the relationship:

$$C(x) = \langle (Y(x) - \mu) (Y(x + \xi) - \mu) \rangle = \langle Y(x) Y(x + \xi) \rangle - \mu^2$$

For a stationary random process the covariance is only a function of ξ , that is, $C = C(\xi)$.

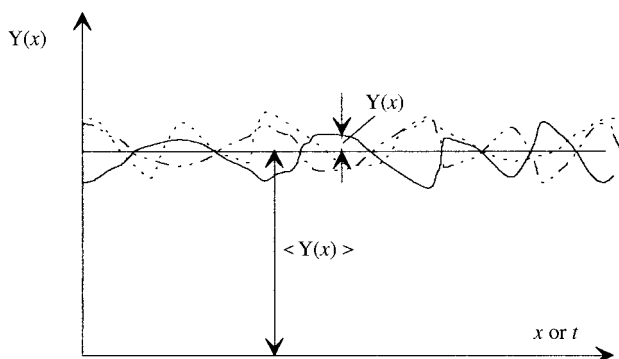


Figure 4.46 Realisations of a stationary random function.

(b) *Ergodicity*: According to this assumption, the statistical properties of the random variable $Y(x)$ can be found by analysing one single realisation of the function in space. This means that one unique realisation of the property has the same probability density function as the ensemble of all possible realisations. This assumption allows the analysis of the statistical characteristics of a random variable at one point, by knowing the distribution of this property in the aquifer space. For example, having the results of pumping tests at several points in the aquifer, the statistical analysis shows that the aquifer transmissivity T is a log-normal random variable in space. Using the ergodicity hypothesis it can be concluded that the probability density function of $T(x)$ at a point x is also log-normal.

The two mathematical assumptions above are actually necessary for analysing stochastically the hydrological behaviour of aquifer systems. However they are rather restrictive for real world problems at the basin scale. At the regional scale, the aquifer system is highly heterogeneous. One can distinguish between two components of heterogeneity: (a) the deterministic heterogeneity and (b) the stochastic heterogeneity.

Deterministic heterogeneity describes the space and possibly the time variation of the physical properties of the aquifer in a relatively regular manner. This kind of variation in physical parameters can generate unstable flow situations or hysteretic effects. In principle it is possible to take into account this heterogeneity in numerical modelling and simulation but in fact it is significantly limited by the large amount of computer time required and the difficulties in calibrating the physical parameters.

The stochastic heterogeneity of regional aquifers is actually not well known. The observed irregularities can present systematic trends, so that the stationarity hypothesis is not valid.

4.3.6

Risk Assessment of Groundwater Pollution

4.3.6.1 Immiscible Fluids

The intrusion of one fluid in a porous medium which is completely filled by another immiscible fluid (Figure 4.47) may be formulated as a stochastic process. The risk

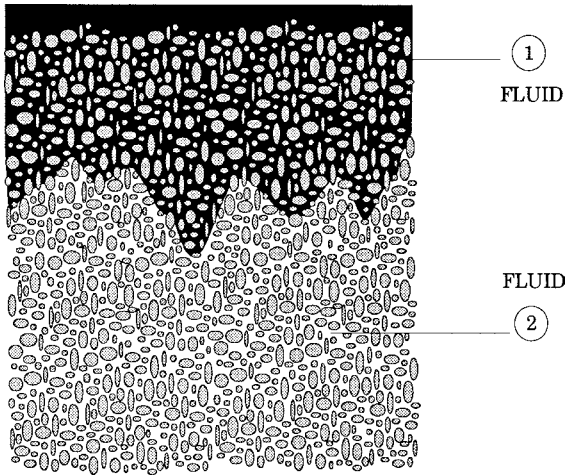


Figure 4.47 Intrusion of one fluid into a porous medium saturated by another immiscible fluid.

and hazard rate of intrusion of a non-wetting fluid, which may be considered as a pollutant, have been evaluated in Example (3.11), Section 3.3.3. This is the case of soil pollution where two immiscible fluids are present.

Multiphase flow and pollution in porous media are complex phenomena, because of the movement of the microscopic interfaces (menisci). The interpretation and modelling of the multiphase retention properties of porous media including the risk of pollution, must take into account the complexity of the internal geometry of the microscopic channels. Despite the fact that a 'pore' is a term which is poorly defined, functions describing the 'pore size distributions' were introduced very early in the literature (Ritter and Drake, 1945). These distribution functions in volume or in number of pores are based on a given conceptual model of the porous medium.

The risk of filling a certain pore depends on the continuous pathway which is needed to connect the invaded pore to the injected liquid phase (Figure 4.48). Because several large pores may be surrounded by sub-critical pores of small size (see Example (3.11), Section 3.3.3), 'entrapment' of the residual wetting liquid phase may occur. This phenomenon of closing-up of the wetting phase has been experimentally observed (Figure 4.48) in a two-dimensional transparent network of interconnected pores (Ganoulis, 1973, 1974; Ganoulis and Thirriot, 1977) and in transparent models composed of cylindrical capillaries of rectangular cross-section (Lenormand *et al.*, 1988). An example of the results of such experiments is given in Figure 4.48.

By increasing the capillary pressure p_c in discrete steps, the non-wetting fluid 1 pushes the wetting phase 2 further along (Figure 4.48): the degree of saturation θ_2 decreases as well as the critical pore radius r_c . This displacement is called *drainage*. It is characterised by a drainage curve relating the capillary pressure to the non-wetting phase content θ_1 . By the end of the experiment, even if the capillary pressure is tremendously increased, the non-wetting phase never completely saturates the porous medium. In this case the wetting phase reaches its *irreducible* or *residual* phase content value.

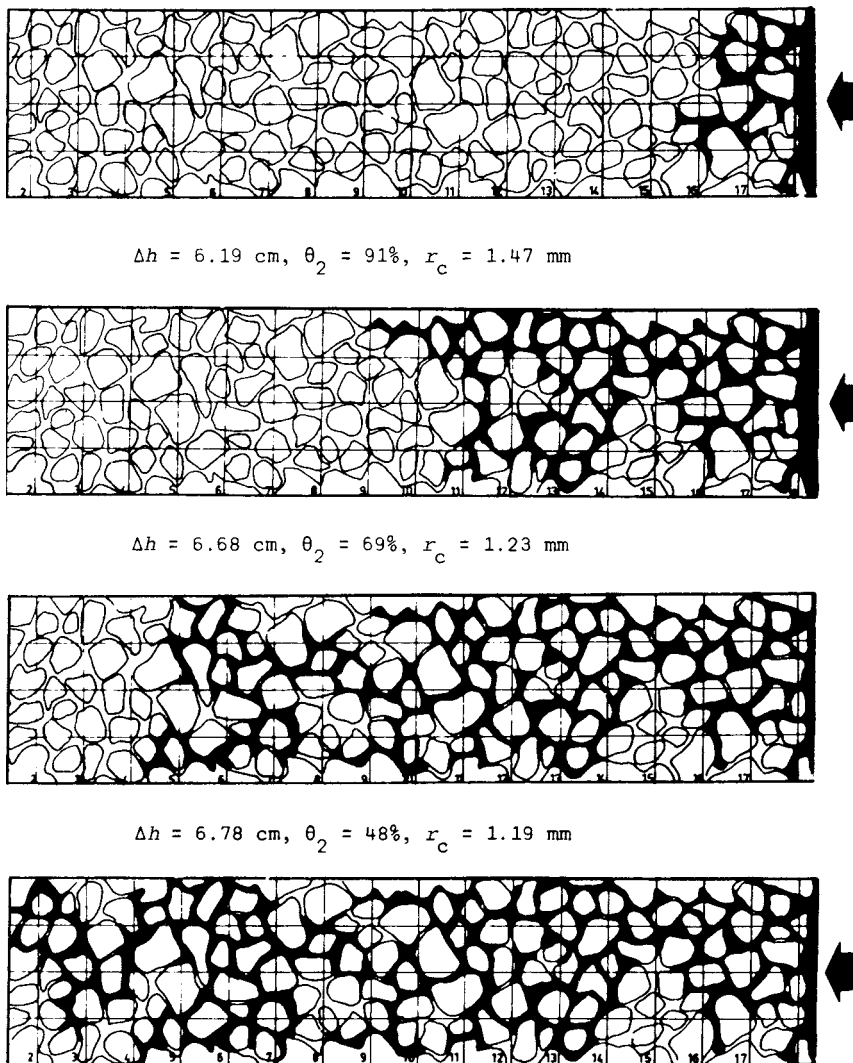


Figure 4.48 Experimental observation of a dark fluid 1 penetrating a porous medium saturated by another immiscible fluid 2.

Capillary networks of several forms of lattices having random pore size distributions adequately represent real capillary pressure curves and pollution phenomena of liquids, which are immiscible with water. Capillary networks have been intensively used in *percolation theory* (Mason, 1988). They are characterised by two main components: *bonds* and *sites*. Depending on the capillary pressure, bonds are 'active' or 'conductive' or they can be 'inactive'. A pore space is represented by a site where bonds meet. A site is filled by the injected fluid if at least one continuous pathway of active bonds exists between the site and the injection point. At a particular value of the

capillary pressure *infinite clusters* of conductive bonds appear. This is the condition of the *percolation threshold*. The capillary network models display this property and can adequately simulate hysteresis.

More about groundwater contamination by pollutants immiscible with water or 'Non-Aqueous Phase Liquids' (NAPL) may be found in the literature (Schiegg and Schwille, 1989).

4.3.6.2 Solute Transport and Random Walks

In the case of miscible fluids or solute transport, depending on the composition of the pollutant (organic, radioactive, microbiological), its interaction with groundwater and soil may be expressed as a function of the pollutant concentration.

For non-conservative pollutants in groundwater flows, as for example nitrates, the physicochemical interactions between liquid phases and with the soil should be taken into account. Nitrification and denitrification of chemical species c_i in the soil are assumed to follow irreversible first-order kinetics (Starr *et al.*, 1974). Together with advection and dispersion, adsorption of NH_4^+ by the soil is generally significant. It can be simulated by using the retardation factor R_i in the following mass conservation equation

$$R_i \frac{\partial C_i}{\partial t} + V_j \frac{\partial C_i}{\partial x_j} = D \frac{\partial^2 C_i}{\partial x_j \partial x_j} + f_i(C_1, C_2, \dots) \quad (4.89)$$

where C_i is the solution phase concentration of species i and D the dispersion tensor in the aquifer.

For conservative pollutants, such as saline waters, this interaction is negligible and for regional 2-D groundwater flows, the following dispersive convection equation may be used

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} \quad (4.90)$$

where: $C(x, y, t)$: is the pollutant concentration (M/L^3); $u(x, y, t)$ and $v(x, y, t)$ are the groundwater velocity components (L/T); D_x and D_y are the dispersion coefficients (L^2/T).

For two-dimensional flows the velocity field can be computed by using the technique of boundary elements (Latinopoulos *et al.*, 1982). As an example of its application, the computed water table elevation is shown in Figure 4.49.

In fact, Equation 4.90 is a random partial differential equation. The causes of randomness and variability are (1) the random variation of the velocity components (u, v) due to the spatial variability of the aquifer parameters (porosity, permeability), and (2) the variation of the dispersion coefficient D as a result of the random fluctuations of the velocity components. Generally speaking, stochastic simulation and risk analysis techniques can be used to quantify the effect of various uncertainties in the dispersion process (Ganoulis, 1986, 1991a, 1991b; Ganoulis *et al.*, 1991).

Several particle-oriented models in hydrological applications have been developed in the past (Bear and Verruijt, 1992). According to the method of characteristics (Konikow and Bredehoeft, 1978) a large collection of computer-generated particles,

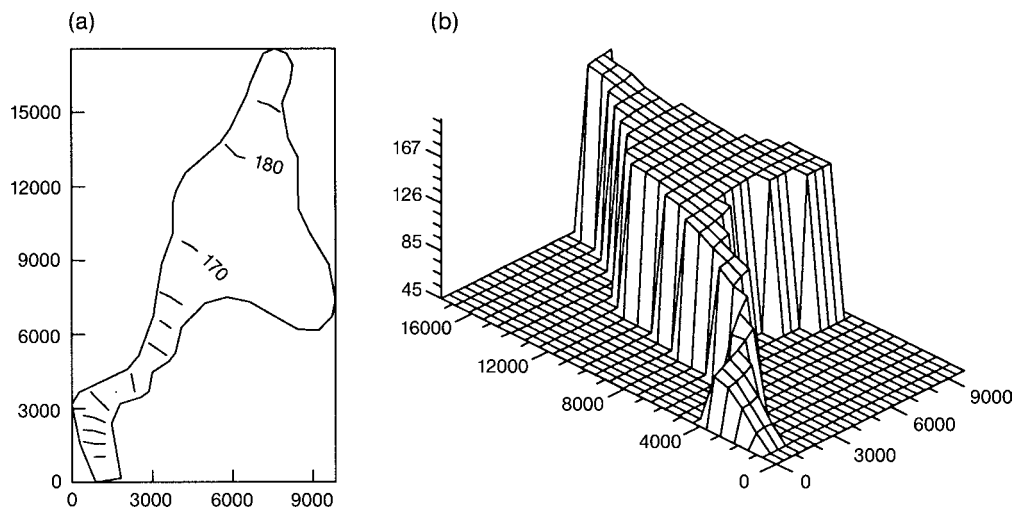


Figure 4.49 2-D (a) and 3-D (b) views of the water table elevation (in [m]) for the Gallikos aquifer.

each of which is assigned a value of concentration, move along streamlines. Then, these concentration values are corrected to take into account the influence of diffusion by a standard Eulerian finite difference or finite element technique. This method is rather complex and difficult to use and sensitive to the boundary conditions.

It seems that particle methods based on random walks are more flexible and easy to use and lead to relatively accurate results (Ganoulis, 1977; Kinzelbach, 1986).

Consider at time $t = n \Delta t$ a large number of particles N located at

$$\vec{r}_{n,p} = (x_{n,p}, y_{n,p}) \quad p = 1, 2, \dots, N \quad (4.91)$$

According to the random walk principle the probability of finding a particle at a given position after time Δt follows a Gaussian law of mean value 0 and variance $s^2 = 2\Delta t D$.

D is the dispersion coefficient (cf. Equation 4.47). The particles move from time $t = n\Delta t$ to time $t + \Delta t = (n + 1)\Delta t$ according to the relationships

$$x_{n+1,p} = x_{n,p} + u\Delta t + \xi \quad (4.92)$$

$$y_{n+1,p} = y_{n,p} + v\Delta t + \eta \quad (4.93)$$

where u, v are the velocity components of groundwater and ξ, η are random variables following a normal distribution of mean value 0 and variance $s^2 = 2\Delta t D$.

This is illustrated in the case of the Gallikos aquifer, located near the river Axios in Macedonia, Greece. It is an unconfined alluvial aquifer, from which the city of Thessaloniki receives part of its water supply. Vulnerability of the groundwater to pollutant sources has been investigated using random walks and Lagrangian methods. Introducing the corresponding velocity field, the groundwater pollution from a point source is studied, using the random walk simulation algorithm (Figure 4.50).

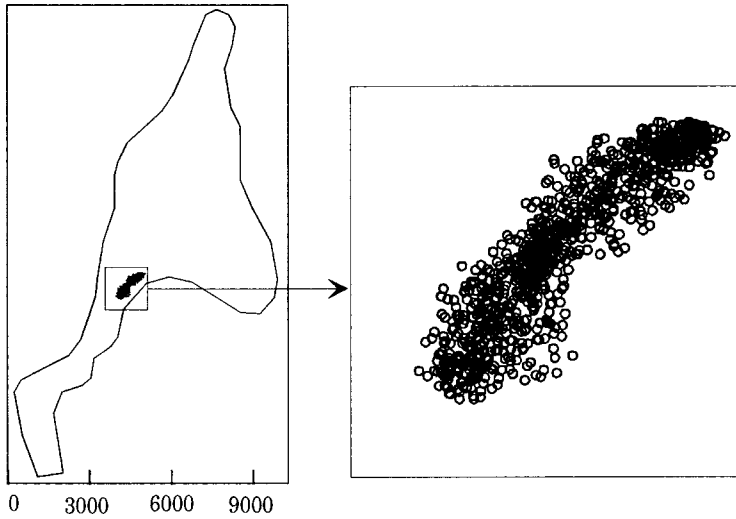


Figure 4.50 Random walk simulation in the Gallikos aquifer (dimensions are in metres).

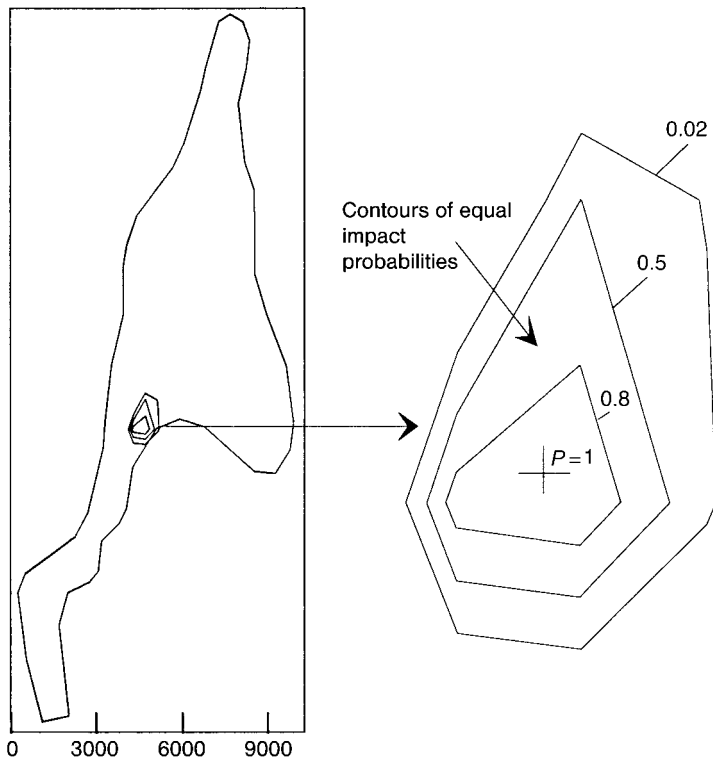


Figure 4.51 Impact probabilities in the Gallikos aquifer (dimensions are in metres).

To evaluate impact probabilities (Figure 4.51) and concentrations of the particles, the area is covered by a regular grid. Knowledge of the velocity components u , v at the grid points permits particle velocities to be computed by linear interpolation. The probability of reaching a given grid cell and consequently the particle concentrations are evaluated by counting the number of particles which fall within the grid square.

4.4

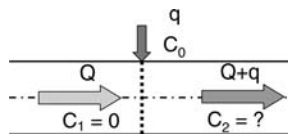
Questions and Problems – Chapter 4

Risk in Coastal Water Pollution

- What are the three main mechanisms of the fate of pollution in coastal waters?
- What is the physical meaning of Fick's law?
- What are the differences and similarities between molecular diffusion, turbulent diffusion and dispersion?
- How can the growth rate of bacteria in wastewaters be described mathematically?
- What are the main causes of water currents in coastal areas?
- How can coastal currents be effectively simulated numerically?

Risk in River Water Quality

- A pollutant is released at a constant rate q and a constant concentration C_0 at a river's cross-section as shown in the figure below. If Q is the river flow rate and $C_1 = 0$ the pollutant concentration upstream, apply the mass balance equation in order to define the pollutant concentration C_2 downstream.



- How can the oxygen deficiency curve or sag curve in a river flow be defined and what are the effective numerical simulation techniques for modelling it?

Risk in Groundwater Pollution

- Describe different types of aquifer using as a criterion the boundary conditions.
- Describe different types of aquifer using as a criterion the aquifer's geological characteristics.
- How can the variogram of an aquifer's property be defined?
- Consider the groundwater flow from a channel to a trench through a fully unconfined aquifer of length $L = 30$ m, width $m = 50$ m. If $h_1 = 20$ m is the water head in the channel and $h_2 = 1$ m the water head in the trench, calculate the groundwater flow rate if the groundwater conductivity or permeability is $K = 10^{-3}$ cm/s.

5

Risk Management

The simple definition of engineering risk as the probability of failure does not reflect completely the characteristics of the physical system operating under risk. It gives only an indication of the state of safety of the system or how it would behave under various uncertainty conditions. In a more general way, we can say that engineering risk is just an index characterising the level of performance of the system. If the system is performing safely, then engineering risk tends to zero. Inversely, when risk approaches 1, the system is likely to fail.

In order to describe in more detail the behaviour of the water system under risk, some other *performance indices* and *figures of merit* will be defined in this chapter. Some of these factors are better known as resilience, grade of service, vulnerability and availability.

Among the figures of merit, perhaps the most important is the one incorporating the *consequences of failure*. The function L of consequences may be expressed in economic units (costs, benefits) or in more general terms, for example environmental consequences or lives. For every particular numerical value of engineering risk the consequences may be evaluated. As shown in this chapter, this should be an essential element for the management of risks and decision analysis.

In this chapter, the basic decision theory under risk is briefly presented. In simple cases an objective function can be formulated. The minimisation of losses or maximisation of profits may be used as design criteria. For multiple objectives the utility theory and probabilistic or fuzzy trade-offs should be considered by introducing multi-objective decision analysis under risk.

5.1

Performance Indices and Figures of Merit

A water resources system operating under risk should be designed in such a way that safety prevails during its life-time. On the other hand, the notion of safety does not imply that the system is risk-free. Because of uncertainties a value of risk always exists; for engineering purposes this should be maintained at as low a level as possible. However, the engineering risk as an index characterising the state of safety of the

system is not sufficient to indicate all the properties of a system under risk conditions. This is why performance indices and figures of merit have been introduced.

Performance indices (PIs) are measures indicating how the system performs when external conditions create adverse effects such as extreme loading. Sometimes incidents may occur that would render the system unable to accomplish its function. These incidents are not catastrophic events, and the system may possibly recover.

Characteristic examples of the behaviour of a hydrosystem under risk is a pipe distribution system which at some point in time can only deliver part of the demand, or a sewer system that overflows at certain time periods. PIs should provide quantitative information about the incident-related properties of the system.

Duckstein and Parent (1994) report nine different incident-related PIs which may be calculated at each time t

- PI₁: grade of service
- PI₂: quality of service
- PI₃: speed of response
- PI₄: reliability
- PI₅: incident period
- PI₆: mission reliability
- PI₇: availability
- PI₈: resilience
- PI₉: vulnerability

The latter two PIs are of special interest in describing the characteristics of the system in cases of incident or failure.

Resilience: This is a measure of the reaction time of the system to return to safe operating conditions. A system of *high resilience* responds quickly to a given incident and returns quickly to normal state. A *low resilience* system needs a long time to recover.

Vulnerability: This is an index measuring the degree of damage which an incident causes to a system. It is known that highly sophisticated systems are the most vulnerable: an incident could cause complete destruction of its components. Examples of highly vulnerable systems are complex electronic devices, sophisticated computer systems and structures such as arch dams with a very small safety margin.

Figures of merit (FMs) are defined as functions of the performance indices. In a sense they are considered to be ‘super criteria’. If PI₁, PI₂, ..., PI_k are different PIs, then a FM_i may be expressed as

$$FM_i = FM_i(PI_1, PI_2, \dots, PI_k)$$

Two FMs are of particular interest (Duckstein and Plate, 1987):

- (a) sustainability (SU) and
- (b) engineering risk (RI).

Sustainability is a combination of high resilience and low vulnerability.

Engineering risk may be generally expressed by a joint probability distribution over all possible FMs. For example risk may be defined as the probability of having a given reliability and resilience. A special case is to define the engineering risk as the complement of the reliability or as the probability of failure. Economic consequences, such as costs and benefits may be expressed as functions of risk.

5.2

Objective Functions and Optimisation

In the past, traditional engineering approaches for water resources management emphasised the effective use of economic resources in planning and operation. Whilst still providing a reliable framework, investment and maintenance costs were to be minimised. As shown schematically in Figure 5.1, the main objective was to minimise total costs while maintaining a given degree of technical reliability. If only one objective is taken into account, an optimisation problem can be formulated.

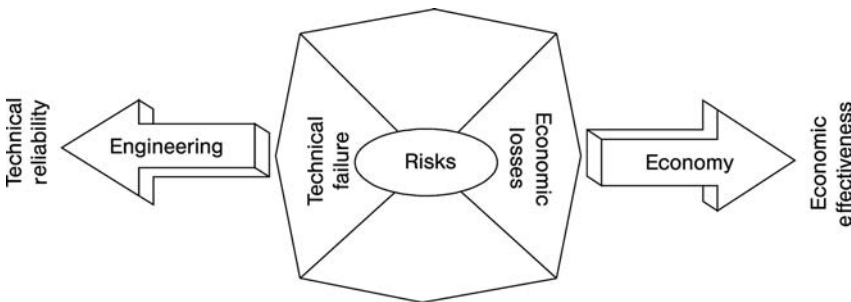


Figure 5.1 Economic effectiveness versus technical reliability.

5.2.1

Economic Optimisation under Certainty and under Risk

From engineering modelling and the design of the project a number of options or alternative solutions usually emerge. The selection process, which may be based on technical or other criteria, is part of the decision problem. We will first analyse some simple situations, where functional relationships may be found between the decision variables in order to formulate the objectives of the problem. In these cases analytical or numerical optimisation techniques can be applied (Ang and Tang, 1984; Mays and Tung, 1992). Using such techniques, maximisation or minimisation of the objective functions may be achieved under either certainty or risk conditions. This facilitates the choice of an 'optimum' solution.

Let us consider a very simple, one-dimensional decision problem. A flood levee is to be constructed having a crest height h above the mean water level h_o (Figure 5.2).

To select a value for the variable h , the uncertainty conditions and the objectives of the project should first be defined. If we consider the consequences of a flood, we will

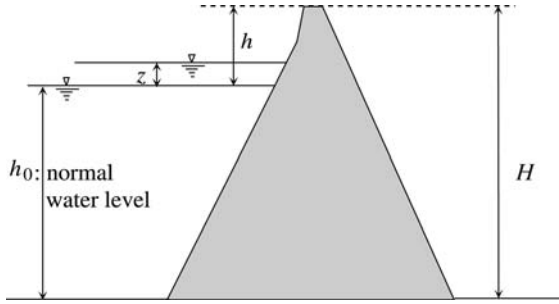


Figure 5.2 The flood levee problem.

find different kinds of damages: damage to properties, loss of lives, environmental consequences, decrease in aesthetic values, and so on. One reasonable objective should be to minimise the sum of both investment and damage costs.

If sufficient experience from other cases is available, then we can proceed under deterministic or certainty conditions. For example, let us assume that investment costs C_I increase proportionally to height h (Figure 5.3). The function $C_I(h)$ has the form

$$C_I = C_0 + Ah \quad (5.1)$$

Damage costs C_D may decrease exponentially with h (Figure 5.2), that is

$$C_D = Be^{-\lambda h} \quad (5.2)$$

The *objective function* $f(h)$ is written as

$$f(h) = C_I(h) + C_D(h) = C_0 + Ah + Be^{-\lambda h} \quad (5.3)$$

and the optimal solution (Figure 5.2) is at the minimum $f(h)$,

$$f_{opt} = \min f(h) \quad (5.4)$$

The decision problem usually involves uncertainties. These may be quantified in terms of risk.

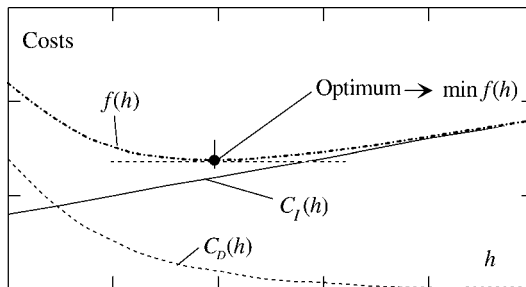


Figure 5.3 Optimisation of total costs under certainty.

Risk may be taken as a decision variable in optimisation. As a very simple example let us consider the hydrological risk for the case of the flood levee. The engineering risk p_F is the probability of overtopping. This may be expressed as

$$p_F = P(z + h_o \geq H) = P(z \geq H - h_o) = F(h) \quad (5.5)$$

where P is the probability; z is the elevation of the flood above the normal water level h_o , and $h = H - h_o$ is the height of the levee above h_o (Figure 5.2).

From Equation 5.5 a relationship between p_F and h becomes apparent. The objective function given by Equation 5.3 may be written as a function of p_F and the optimum solution may be found in terms of p_F or $(-\ln p_F)$. At every level of risk there are consequences implying potential damage. These may be expressed in terms of *damage costs* having monetary or non-monetary units. Protection against damage should imply some other costs, called *protection costs*.

For low risk, the damage costs are low and they increase as risk increases. The opposite is true for protection costs: high investment is necessary to keep the risk as low as possible. As the risk increases the protection costs decrease. Generally speaking, we can state that

- (a) *damage costs* increase as the risk increases and decrease as safety increases,
- (b) *protection costs* decrease as the risk increases and increase with safety.

To illustrate this situation let us consider a simple example in which the probability of overtopping is known.

Example 5.1

It is assumed that the probability density distribution of flood elevation above the normal water height is exponential (Ang and Tang, 1984), with a mean value of 2 m above h_o .

Find the risk corresponding to the economically optimum design and the corresponding height h of the water level above h_o . It will be assumed that only one overtopping is expected with damage costs ($C_D/\text{overtopping}$) = 70 000 US\$. The construction costs have the functional form given by Equation 5.1, with $C_o = 20$ 000 and $A = 7500$ US\$.

It is given that the probability density function of the flood elevation z above the normal water level is known. It can be expressed as an exponential distribution with a mean value of 2 m above h_o . We have

$$f(z) = \lambda e^{-\lambda z} \quad (5.6)$$

$$E(z) = \langle z \rangle = \frac{1}{\lambda} = 2 \quad (5.7)$$

The probability of overtopping, that is the probability of having $z > h$ (Figure 5.1) may be calculated as

$$\begin{aligned} P(h_o + z \geq H) &= \int_{z=H-h_o}^{\infty} f(z) dz = \int_{z=H-h_o}^{\infty} \lambda e^{-\lambda x} dx \\ &= -e^{-\lambda z} \Big|_{H-h_o}^{\infty} = e^{-\lambda(H-h_o)} = e^{-\lambda(H-h_o)/2} \end{aligned} \quad (5.8)$$

The probability of overtopping is by definition the *engineering risk* or *probability of failure* p_F . From Equation 5.8 it follows that

$$p_F = e^{-h/2} \quad \text{or} \quad h = -2 \ln p_F \quad (5.9)$$

Protection Costs: C_p

These are proportional to h . The general expression is

$$C_p = C_0 + Ah = C_0 - 2A \ln p_F \quad (5.10)$$

From Equation 5.10, C_p decreases as p_F increases.

Damage Costs: C_D

Suppose that B is the cost for every overtopping. Then the expected total damage costs are

$$C_D = \text{Expected cost} = \left(\frac{\text{cost}}{\text{overtopping}} \right) P(\text{overtopping}) = B p_F$$

The total costs are

$$C_T = C_p + C_D = C_0 - 2A \ln p_F + B p_F$$

The risk corresponding to the optimum (minimum) cost is (Figure 5.3)

$$\frac{\partial C_T}{\partial p_F} = -\frac{2A}{p_F} + B = 0 \Rightarrow p_F = \frac{2A}{B} = \frac{2(7500)}{70000} = 0.25 \Rightarrow \ln p_F = 1.39$$

It can be seen from Figure 5.4 that if safety ($-\ln p_F$) is chosen as variable, investment costs are an increasing function of safety, whereas damage costs decrease with increasing safety.

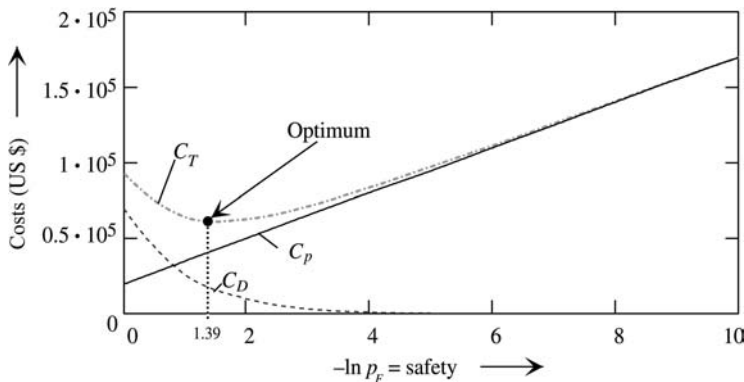


Figure 5.4 Optimisation of total costs under hydrologic uncertainty.

5.2.2

Optimisation Methods

To formulate the general optimisation problem, we should first consider the systems approach shown in Figure 5.4 (Biswas, 1976; Haimes, 1977; Hall and Dracup, 1970; Loucks *et al.*, 1981; Mays and Tung, 1992).

A system (natural or man-made) is a combination of elements or rules which are organised in such a way that, given a set of *inputs*, a number of *outputs* are produced. Inputs and outputs may be physical or economic variables which may be classified as controlled or uncontrolled (inputs), desirable or undesirable (outputs). Outputs sometimes have an influence on inputs (feedback). The system usually operates under a number of *constraints* (physical, legal, economic, etc.) (Figure 5.5).

Take, as an example, a natural system, such as an aquifer. Inputs may be *controlled*, such as

- pumping wells
- pumping rates
- artificial recharge
- pump tax
- interest rate

or *uncontrolled*, such as

- leakage from or to the aquifer
- natural recharge
- subsidence
- water demand

Outputs may be *desirable*, like the outflow from the aquifer which benefits the community, or *undesirable* such as the drawdown of the groundwater table resulting in costs to the community and subsidence. *Constraints* in the use of groundwater may be *physical* or *economic*: for example, the rule of keeping the groundwater table above a certain level, or the requirement not to pump more than the renewable groundwater resources or not to exceed a certain budget.

In general, let x_1, x_2, \dots, x_n be the n controllable inputs, project outputs and constraints. They are called *decision variables*. A function is defined in terms of x_1, x_2, \dots, x_n representing the *total cost* or the *total benefit*. This is the *objective function* which

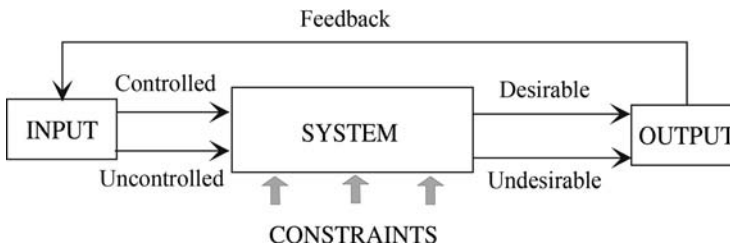


Figure 5.5 Definition of a system.

may be used to rank different designs. The optimum solution is obtained for the combination of the decision variables which maximise the benefit or minimise the cost function.

5.2.2.1 Mathematical Programming

If $f(x_1, x_2, \dots, x_n)$ is the objective function and $g_j(x_1, x_2, \dots, x_n) \leq = \geq 0$ $j = 1, 2, \dots, m$ are the constraints, the general optimisation problem is

$$\min (\text{or max}) f(x_1, x_2, \dots, x_n) \quad (5.11)$$

under

$$g_j(x_1, x_2, \dots, x_n) \geq = \leq 0, \quad j = 1, 2, \dots, m \quad (5.12)$$

The optimal design will correspond to values of the decision variables which satisfy the following equations

$$\frac{\partial f(x_1, x_2, \dots, x_n)}{\partial x_i} = 0, \quad i = 1, 2, \dots, n$$

We may notice that the constraints expressed by Equation 5.12 define in the n -dimensional space a set of points. Every solution may be represented by one of these points and the problem is one of selecting the point for which f becomes minimum (or maximum). The region defined by the constraints is called the *region of feasible solutions*.

Two classes of problems are usually defined in mathematical programming:

(a) Linear Programming

In this case the objective function and the constraints are linear functions of the decision variables, that is

$$f(x_1, x_2, \dots, x_n) = a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n = \max(\text{or min}) \quad (5.13)$$

subject to

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &\leq = \geq b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &\leq = \geq b_2 \\ &\dots \end{aligned} \quad (5.14)$$

$$\begin{aligned} a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &\leq = \geq b_m \\ x_i &\geq 0 \quad (i = 1, 2, \dots, n) \end{aligned} \quad (5.15)$$

(b) Dynamic Programming

We have:

Objective function

$$f(x_1, x_2, \dots, x_n) = f_1(x_1) + f_2(x_2) + \dots + f_n(x_n) = \max (\text{or min})$$

Constraints

$$\begin{aligned}
 a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &\leq \geq b_1 \\
 a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &\leq \geq b_2 \\
 &\dots \\
 a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &\leq \geq b_m \\
 x_i &\geq 0 \quad (i = 1, 2, \dots, n)
 \end{aligned}$$

where $f_j(x_j)$ and $a_{ij}(x_j)$ are non-linear functions of the decision variables.

Example 5.2

Two municipalities A (10 000 inhabitants) and B (25 000 inhabitants), located in the same valley, are to treat their sewage biologically before disposing of it into the river (Figure 5.6). Find the relative quantities of BOD that should be treated in order to minimise costs and respect environmental quality criteria.

Assuming a waste load of 60 g BOD per inhabitant per day, municipalities A and B would have a BOD production rate per day

$$A: (60 \times 10^{-3}) \times 10000 = 600 \text{ kg/d}$$

$$B: (60 \times 10^{-3}) \times 25000 = 1500 \text{ kg/d}$$

Let the treatment efficiency for BOD be 80%, with X the proportion of BOD treated in municipality A and Y the corresponding proportion in municipality B. The quantities of BOD that should be treated are

$$A: 600 (X) (0.8) = 480X \quad (5.16)$$

$$B: 1500 (Y) (0.8) = 1200Y \quad (5.17)$$

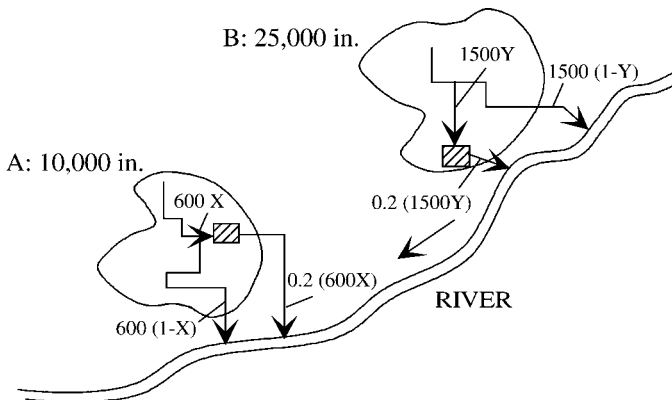


Figure 5.6 Sewage disposal in a river from two communities A and B.

The corresponding quantities which are to be discharged into the river are

$$A: 600(1-X) + (0.2)(600)(X) = 600(1-X) + 120X \quad (5.18)$$

$$B: 1500(1-Y) + (0.2)(1500)(Y) = 1500(1-Y) + 300Y \quad (5.19)$$

Let C be the cost per kg of BOD treated and P the penalty per kg of BOD rejected. The cost functions to be minimised are

$$A: C\{480X\} + P\{600(1-X) + 120X\} \quad (5.20)$$

$$B: C\{1200Y\} + P\{1500(1-Y) + 300Y\} \quad (5.21)$$

The constraint imposed on the municipalities aims to protect the river from excess BOD. Suppose that the total BOD per day allowed to be discharged by the two communities is 900 kg/day. From Equations 5.18 and 5.19 we have

$$\{600(1-X) + 120X\} + \{1500(1-Y) + 300Y\} \leq 900 \quad (5.22)$$

The linear programming problem can be now formulated. By adding Equations 5.20 and 5.21 the total cost function to be minimised is

Objective function

$$\frac{C}{P} \{480X + 1200Y\} + \{2100 - 480X - 1200Y\} \rightarrow \min$$

Constraints

The environmental constraint given by Equation 5.22 may be written as

$$1200 - 480X - 1200Y \leq 0$$

or

$$4.8X + 12Y \geq 12 \quad (5.24)$$

X and Y must be positive and less than 1, that is

$$X > 0, Y > 0 \quad (5.25)$$

$$X < 1, Y < 1 \quad (5.26)$$

For a unit treatment cost to penalty ratio C/P equal to 10, the objective function (Equation 5.23) becomes

$$4.32X + 10.80Y \rightarrow \min \quad (5.27)$$

The solution of the above problem is

$$X = 1 \quad \text{and} \quad Y = 0.6$$

Interpretation of the result (Equation 5.27) may be achieved by considering the graphical representation in Figure 5.7 of the constraints (Equations 5.24 to 5.26).

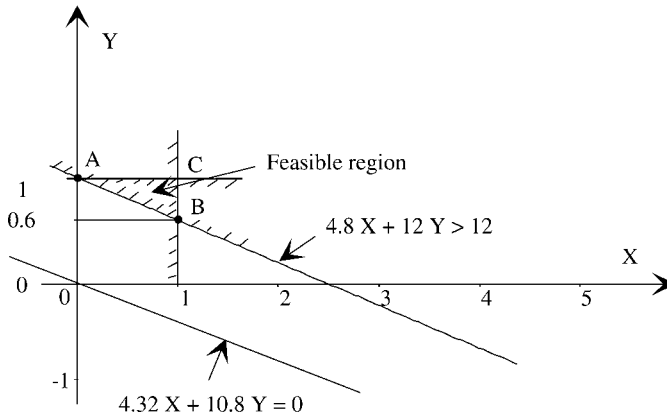


Figure 5.7 Graphical solution of the optimum sewage disposal problem.

The feasible domain is the triangle ABC, where AB is the line defined by the constraint shown by Equation 5.24. All points located in the upper half of the plane divided by the line AB satisfy the constraint (Equation 5.24). The solution which minimises the total cost should fall in the feasible region. Since the point representing this solution is not known, we may proceed by successive approximation.

Consider the objective line $4.32X + 10.80Y = 0$ (Figure 5.6). This line passes through the origin with a slope given by the objective (Equation 5.27). The line can be moved forward parallel to itself, until the first point within the feasible region is reached. This is point B (0.6, 0), which represents the minimum cost. This is because as the objective line is moved backwards and parallel to the original line, the costs decrease.

As a general rule we can state that in linear programming problems the optimum solution corresponds to one of the corners of the feasible region.

The cases we have examined before are very simple: they only have one objective function, which is analytically expressed in terms of the decision variables. In practice this type of one-dimensional optimisation problem is rather exceptional. Most of the time we have more than one objective, which cannot even be expressed as analytical functions.

Consider for example the risk management strategies for nitrate reduction (Dahab and Lee, 1991). Contamination of groundwater by nitrates is of major concern in several countries in Europe and also in the USA. An excess of nitrates may cause health problems, such as infant methemoglobinemia and gastric cancer. Among the several technical alternatives for nitrate reduction, such as bio-denitrification, ion-exchange, blending of two different types of water, and so on, there are several conflicting objectives such as

- (1) minimising costs,
- (2) maximising the safety level of human health, and
- (3) maximising the technical feasibility of nitrate removal.

5.2.3

Discontinuous Decision Problems

A more general presentation of the decision theory and multi-objective analysis is given in the following sections of this chapter.

Another simple case occurring in decision problems is the dichotomy between two alternatives: (a_1) taking an action and (a_2) none at all. Two characteristic *states of nature*, θ_1 and θ_2 , may correspond to these two possible actions, a_1 and a_2 (Rubinstein, 1986).

For example, a decision is to be taken on whether or not it is better to build a flood levee for the protection of an inhabited area (Duckstein and Bogardi, 1991). For a given protection level from floods Q_o , we have two possible states of nature: (θ_1) $Q \geq Q_o$: failure, and (θ_2) $Q < Q_o$: safety. For any combination of the two alternative actions and the two states of nature there are annualised protection costs and losses due to floods. In this case we have a discontinuous *economic loss function*, which can be displayed in matrix form (Table 5.1), where

C is the annualised protection cost, and
 K the annualised flood losses.

If the information on the states of nature is not perfect, we introduce the probabilities of occurrence of these states. In Table 5.1 we assume p to be the probability of failure and $(1 - p)$ that of safety. The economic losses are

$$\text{Action } a_1 : L(a_1) = Cp + C(1-p) = C$$

$$\text{Action } a_2 : L(a_2) = Kp + 0(1-p) = Kp$$

Action a_1 (protect) will be preferred if

$$L(a_1) < L(a_2)$$

and action a_2 (do not protect) will be better if

$$L(a_2) < L(a_1)$$

In more general cases we may have i possible actions and k states of nature. Uncertainty, which includes *aleatory* or *natural* randomness and *epistemic* or *controllable* uncertainties may be expressed by probabilities p_k .

Table 5.1 Discontinuous economic loss function.

Actions	States of nature	
	$\theta_1: Q \geq Q_o$ failure	$\theta_2: Q < Q_o$ safety
a_1 : protect	C	C
a_2 : not to protect	K	0
Probability	p	$1 - p$

Alternatively, the fuzzy set analysis or a mixed fuzzy set/probabilistic approach may be used. These are illustrated in the following two examples.

Example 5.3

In the summer of 1993 water supply reserves for the metropolitan area of Athens, Greece were estimated to be sufficient to meet demand only for the next few months. The main cause of this water deficiency was a multi-year drought. Having calculated both the cost of taking action (e.g. transport of fresh water by tankers or drilling wells for groundwater supply) and the economic losses from possible failure to meet the demand, find the optimal decision.

The key issue to the problem was whether or not precipitation in autumn would be sufficient to replenish the water reservoirs and satisfy the demand for the next hydrological year.

We could assume three possible states of nature ranging from the most pessimistic to most optimistic scenario:

θ_1 : weak precipitation will occur

θ_2 : normal precipitation

θ_3 : strong precipitation

To simplify this even further, we could assume two cases or states of nature (Table 5.2)

θ_1 : weak rainfalls (W)

θ_2 : strong rainfalls (S)

Assuming p to be the probability of weak precipitation and $(1 - p)$ that of strong precipitation we can write the economic loss matrix as Table 5.2 where

C is the cost for taking action and

K the economic losses in case of failure.

Let suppose that $K \gg C$ for example $K = 10 C$.

The opportunity losses for the case a_1 are

$$(1) L(a_1) = pC + (1-p)C = C$$

For the case a_2 , we have

$$(2) L(a_2) = pK + (1-p)0 = pK$$

Table 5.2 Economic loss matrix for the Example 5.3.

Actions	States of nature	
	θ_1 : (W)	θ_2 : (S)
a_1 : Take action	C	C
a_2 : Do not take further action	K	0
Probabilities	p	$1 - p$

Action a_1 should be preferable if

$$L(a_1) < L(a_2) \text{ or } C < pK \text{ or } p > C/K = 1/10$$

From hydrological data it could be estimated that the probability of having weak precipitation would be less than 1 : 10.

This means that $L(a_2) < L(a_1)$ and the action a_2 is preferable.

Example 5.4

Examination of samples of groundwater has indicated that contamination of groundwater has occurred from an industrial disposal site. The extent of groundwater pollution is such that there is risk of pollution in the river (Figure 5.8).

A decision has to be taken whether or not to undertake the drilling of a series of pumping wells in order to reduce the risk of water pollution in the river. Results from mathematical modelling of groundwater flow, including conditional simulation, turning bands and random walks, has indicated the probability of river pollution.

The risk of pollution p_1 if pumping wells are used has been found to range between 4×10^{-8} and 6×10^{-8} , whereas there is a higher risk of contamination p_2 , ranging between 5×10^{-3} and 8×10^{-3} if no action is taken. The risk here is the probability that the concentration of chemicals in the river C_{river} exceeds the maximum allowable by the standards, C_m , that is

$$\text{RISK} = P(C_{\text{river}} \geq C_m)$$

Because of the uncertainties, both risks p_1 and p_2 may be taken as triangular fuzzy numbers. We denote this as

$$\tilde{p}_1 = (4, 5, 6) \times 10^{-8} \quad \tilde{p}_2 = (5, 6, 8) \times 10^{-3}$$

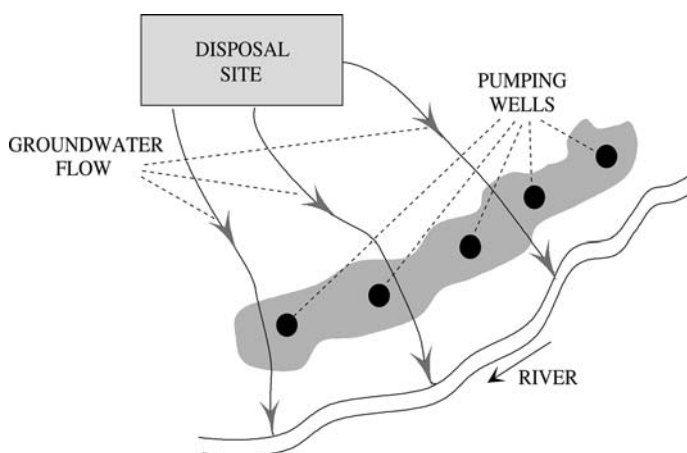


Figure 5.8 Groundwater and river pollution from an industrial disposal site.

Also the protection C and the damage costs K , due to possible pollution of the river, should be considered as fuzzy numbers. We have

$$\tilde{C} = (3, 4, 5) \times 10^4 \text{ US \$}$$

and

$$\tilde{K} = (5, 6, 10) \times 10^6 \text{ US \$}$$

both in the form of annualised costs.

In this situation we have two possible actions:

a_1 : undertake the river protection works

a_2 : do not act

Two states of nature exist, namely:

θ_1 : pollution of water in the river

θ_2 : no pollution

The decision problem may be represented as a decision tree (Figure 5.9). By using fuzzy arithmetic (Appendix B) we can estimate the fuzzy opportunity losses for actions a_1 and a_2 as

$$\tilde{L}(a_1) = \tilde{C}\tilde{p}_1 + \tilde{C}(1-\tilde{p}_1) = \tilde{C} = (3, 4, 5) \times 10^4 \quad (5.28)$$

$$\begin{aligned} \tilde{L}(a_2) &= \tilde{K}\tilde{p}_2 + 0(1-\tilde{p}_2) = \tilde{K}\tilde{p}_2 = \{(5, 6, 10) \times 10^6\} \cdot \{(5, 6, 8) \times 10^{-3}\} \\ &= (2.5, 3, 6, 8.0) \times 10^4 \end{aligned} \quad (5.29)$$

To compare two fuzzy numbers, we use the definition of a fuzzy mean, given in Appendix B. If \tilde{X} is the triangular fuzzy number (x_1, x_2, x_3) then its fuzzy mean is

$$FM(\tilde{X}) = \frac{x_1 + x_2 + x_3}{3}$$

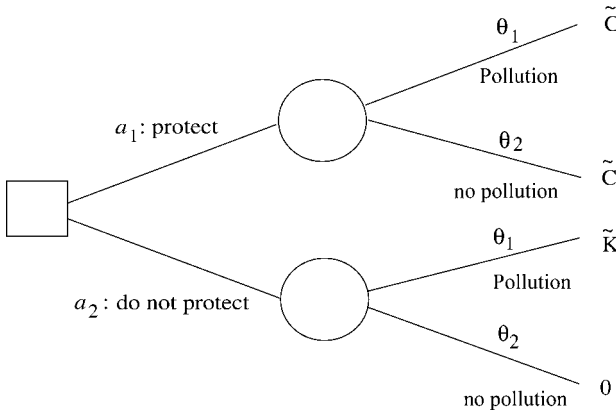


Figure 5.9 Decision tree representation of the river pollution problem.

Taking the $FM(\tilde{L}(a_1))$ and $FL(\tilde{L}(a_2))$ we have

$$FM(\tilde{L}(a_1)) = \frac{3+4+5}{3} = 4 < FM(\tilde{L}(a_2)) = \frac{2.5+3.6+8}{3} = 4.7$$

or

$$FM(\tilde{L}(a_1)) < FM(\tilde{L}(a_2)) \quad (5.30)$$

To conclude, *action a_1 (protect) is preferable* on a purely economic basis.

Considering the most confident values

$$C = 4 \times 10^4, K = 6 \times 10^6, p_2 = 6 \times 10^{-3}$$

we have, as for the case of Equations 5.28 and 5.29

$$L(a_1) = C = 4 \times 10^4 \quad L(a_2) = K p_2 = 3.6 \times 10^4$$

or

$$L(a_2) < L(a_1) \quad (5.31)$$

Thus, *action a_2 (do not protect) is preferable*.

It is interesting to note that if uncertainties were not taken into account as fuzzy numbers, the decision may have been the opposite.

5.3

Basic Decision Theory

5.3.1

Main Elements of Decision Making

Decision theory is concerned with alternative actions that an engineer or a decision-maker should undertake under different environmental conditions. By ‘environment’ we mean not only the physical environment but also economic, social, political and/or legal conditions.

There are three basic elements in a decision-making situation (Berger, 1985):

- (1) *Candidate alternatives* or *alternative actions*, designated as a_i . These are alternative design solutions, which engineers control and can select as candidate solutions to the problem. Because of various limitations, such as technological or modelling constraints, it is not possible to find all possible alternatives. This means that the set $\{a_i\}$ is *non exhaustive*. However, the members of the set $\{a_i\}$ are *mutually exclusive*. Any combination with a_i may be considered as a different alternative.
- (2) *States of nature* noted as $\{\theta_i\}$. These are environmental conditions in which any action a_i should operate. Such conditions are called ‘nature’. In a decision-making situation ‘nature’ can include technical, physical, political, economic and social considerations. These different conditions are not significantly influenced by the actions a_i , but significantly affect the consequences. The members of the set $\{\theta_i\}$ are *mutually exclusive and exhaustive*.

- (3) *Outcomes*, which are the consequences associated with an action and a state of nature. For every alternative action a_i , given a state of nature θ_j , an outcome Y_{ij} may be obtained. For every combination of action and state of nature, several measures of an outcome are possible. These may be expressed as costs, environmental impacts, social values, and so on.

Take, for example, the extension of an existing wastewater treatment plant in a big city, in order to remove nitrates.

In view of different technological processes which are available for nitrate removal, the following alternative actions could be taken:

- a_1 : bio-denitrification
- a_2 : ion exchange
- a_3 : blending of water
- a_4 : combination of a_1 and a_2
- a_5 : cancel the programme
- a_6 : delay the decision concerning a_1 , a_2 and a_4 to obtain more information

It may be observed that the list of a_1 – a_6 is not exhaustive and that a_1 – a_6 are mutually exclusive. Every candidate action a_i may be described by a set of variables called *control variables*. For example, a_1 depends on the amount of oxygen used and the type of bacteria. Action a_3 is defined by the proportion of two different types of water, such as groundwater and surface water.

Now, because of the particular size of the installation, there is uncertainty about the efficiency of each of the processes. We should distinguish three different states of nature, such as

- θ_1 : low efficiency
 - θ_2 : medium efficiency
 - θ_3 : high efficiency
- θ_1 , θ_2 , θ_3 are mutually exclusive and exhaustive

For every combination of an alternative a_i and a state of nature θ_j , we obtain an outcome Y_{ij} . This relates to wastewater containing a certain concentration of nitrates. Different measures may be used to describe each outcome, such as the cost and environmental impact.

Actions $\{a_i\}$, states of nature $\{\theta_i\}$ and outcomes $\{Y_{ij}\}$ may be represented in form of a *decision tree* (Figure 5.10) or a *decision table* (Table 5.3). A decision tree has two types of nodes

- (1) the decision nodes, from which alternative actions begin and
- (2) the chance nodes, which are the origin of the states of nature (Figure 5.10).

If the number of actions and states of nature is large, the decision tree becomes complicated.

A matrix representation may be preferable in such a case (Table 5.3).

From the above elements, two considerations are important in a decision problem

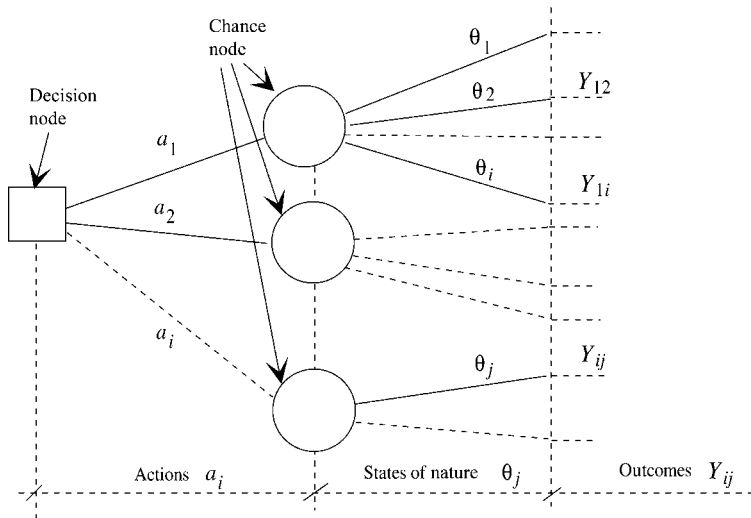


Figure 5.10 Elements of a decision tree.

- (1) Uncertainties
- (2) Preferences or criteria

Combination of (1) and (2) gives the *decision rule* which is the tool for taking the final decision.

Four main categories of decision situations should be distinguished in relation to different types of uncertainties.

5.3.1.1 Decision under Certainty

There are different alternative actions and for each alternative only one outcome can occur with certainty. This is the case when only one state of nature is possible and one outcome is obtained for every alternative action. In effect, the decision matrix of Table 5.3 is reduced to a single column. The selection of the best alternative may be based on ranking the outcomes in order of preference and choosing the preferable

Table 5.3 Elements of a decision matrix.

Actions	States of nature					
	θ_1	θ_2	θ_3	...	θ_k	...
a_1	Y_{11}	Y_{12}	Y_{13}	...	Y_{1k}	...
a_2	Y_{21}	Y_{22}	Y_{23}	...	Y_{2k}	...
a_3	Y_{31}	Y_{32}	Y_{33}	...	Y_{3k}	...
...
a_i	Y_{i1}	Y_{i2}	Y_{i3}	...	Y_{ik}	...
...

outcome. The problem in this case consists of ranking the outcomes according to a preference.

5.3.1.2 Decision under Risk

Different states of nature may exist with known objective probabilities. For every action a_i and state of nature θ_j the engineer can deterministically identify the outcome Y_{ij} . This case may be represented by the decision tree shown in Figure 5.9 or by the decision matrix of Figure 5.10 except that objective probabilities P_k associated with the states of nature θ_k should be added.

5.3.1.3 Decision under Uncertainty or Imprecision

The decision-maker can evaluate the outcome, given an alternative action and a state of nature, but he is not able to express objectively and quantitatively the probabilities of the states of nature. The problem is to select the optimal alternative a_i under such imprecise conditions.

5.3.1.4 Decision under Conflict

The states of nature represent situations where an opponent tries to maximise his own objectives. This is the topic of game theory (Fraser and Hipel, 1984).

5.3.2

Decision Criteria

The different *criteria* or *preferences* we can use in order to define a decision rule depend on personal attitudes but also on the type of decision problem in relation to different conditions of uncertainty.

The most complicated case for decision making is when no objective information is available about the occurrence of the states of nature. This is the case of decision making under uncertainty.

5.3.2.1 Decision Making under Uncertainty

To clarify the discussion let us consider again the example of the water supply in Athens, this time formulated slightly differently.

Example 5.5

To ensure an adequate water supply for Athens, Greece for the next year, the following alternative actions are considered:

- a_1 : transport of water by tankers
- a_2 : transport of water by trucks
- a_3 : drilling of new wells

Three different states of nature are taken for autumn 1993:

- θ_1 : Wet period (W)
- θ_2 : Medium precipitation (M)
- θ_3 : Dry period (D)

Table 5.4 Economic loss matrix (in billions of Dra) for the Example 5.5.

Actions	States of nature		
	$\theta_1 : W$	$\theta_2 : M$	$\theta_3 : D$
a_1 : transport by tankers	2.5	3	5
a_2 : transport by trucks	0.5	4	7
a_3 : new drillings	0.2	2	10

Due to a severe drought over the past 7 years it was difficult to quantify the probabilities of the states of nature.

The various costs, in billions of Drachmas (Dra), associated with various combinations of actions and states of nature are given in Table 5.4.

The costs do not only include expenses for infrastructure (e.g. pumping the water from tankers and transportation to the water treatment plant) but also operation costs and economic losses in the case where the water supply is still not sufficient (e.g. the combination of new wells and a dry season).

We can distinguish two different decision attitudes ranging from a more pessimistic (MINIMAX or MAXIMIN) to a more optimistic (MAXIMAX or MINIMIN) point of view.

The Pessimist: (MINIMAX or MAXIMIN) The decision-maker is pessimistic or conservative, believing that the worst can happen. If we consider losses, then he first looks at the maximum loss (worst case) and then tries to minimise it (Table 5.5). Using this rule, action a_1 (transport by tankers) will be chosen. If we have gains or utilities instead of losses, then the pessimist tries to maximise the minimum utility (MAXIMIN rule).

The Optimist: (MINIMIN or MAXIMAX) An optimistic person thinks that nature works with him. In case of losses, for every action, he thinks the minimum loss will occur (best case). Then he will take the decision which minimises the minimum particular loss (Table 5.5). In this example, the action a_3 (new wells) will be chosen.

In the case of benefits or utilities, the optimistic decision-maker will try to maximise the maximum particular benefit. This is the *maximax rule*.

Table 5.5 Illustrative economic loss matrix (in billions of Dra) and decision rules under uncertainty.

Actions	States of nature			Pessimist or MINIMAX Row maxima	Optimist or MINIMIN Row minima
	W	M	D		
a_1	2.5	3	5	5	2.5
a_2	0.5	4	7	7	0.5
a_3	0.2	2	10	10	0.2

Table 5.6 Regret matrix (in billions of Dra) of Example 5.5.

Actions	States of nature			Row maxima
	W	M	D	
a_1 : tankers	2.3	1	0	2.3
a_2 : trucks	0.3	2	2	5
a_3 : wells	0	0	5	5

The Regretist/Loss of Opportunity: This is the person who has a tendency to compare the difference between the outcome he actually realises and the maximum he could have realised with the best possible action under the particular state of nature. This difference is called the *degree of regret* or *loss of opportunity*.

For example, if the state of nature *Wet* is considered in Table 5.4 and action a_3 is selected, the regret is 0. However, if a_2 or a_1 had been selected, the excess cost or the loss of opportunity would be respectively, 0.3 and 2.3 units.

The objective now is to minimise regret. To do this, first the original *economic loss* or *utility matrix* is rewritten with the outcomes representing the losses due to imperfect prediction. Such a matrix is called a *regret matrix*. For the example of Table 5.4 the regret matrix is written as follows (Table 5.6).

Suppose now that we would like to minimise maximum regret. This is the *MINIMAX regret criterion*, which guarantees a lower limit to the maximum regret.

Using this criterion, first the maximum regret for each row is recorded (Table 5.6). The action a_2 (transport by trucks) is selected, which corresponds to the minimum value of regret, in the column on the right-hand side of the regret matrix (Table 5.6). The outcomes in the regret matrix are also called *opportunity costs*.

5.3.2.2 Decision Making under Risk

Suppose now that a hydrologic analysis is made from available data over the past 100 years. Classification of the past 100 autumns has shown that 60 were wet (W), 30 were medium (M) and 10 were dry (D). If we suppose that there is no reasonable evidence that the future will be radically different from the past, we assign the probabilities of the different states of nature as 0.6, 0.3 and 0.1 respectively. The decision tree shown in Figure 5.11 incorporates the probabilities $P(\theta_1) = 0.6$, $P(\theta_2) = 0.3$ and $P(\theta_3) = 0.1$.

For every action, the mean expected cost may be evaluated as

$$\begin{aligned}
 E(L_1) &= 0.6(2.5) + 0.3(3) + 0.1(5) = 2.9 \\
 E(L_2) &= 0.6(0.5) + 0.3(4) + 0.1(7) = \underline{1.3} \\
 E(L_3) &= 0.6(0.2) + 0.3(2) + 0.1(10) = 1.6
 \end{aligned}$$

Since $E(L_2)$ is the *minimum expected cost*, action a_2 (transport by trucks) should be chosen.

In the case of benefits instead of costs, the decision rule becomes the *maximum expected benefit*.

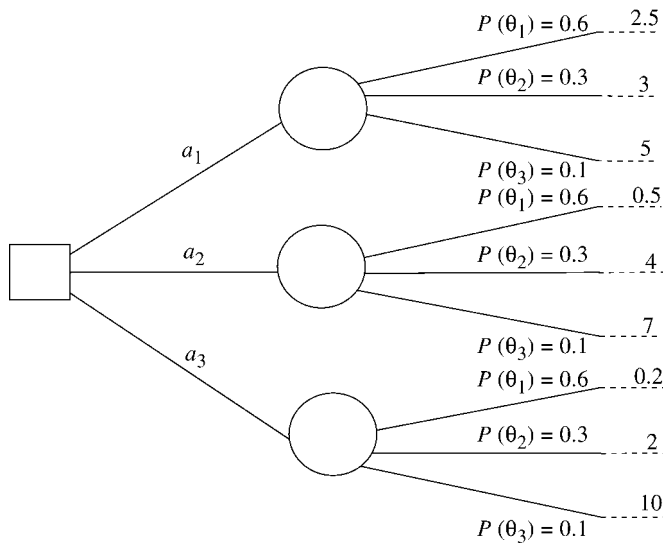


Figure 5.11 Decision tree under risk (Example 5.5).

5.3.3

Baye's Analysis and Value of Information

To simplify analysis consider a period of years, where, as in the example, we have two possible actions a_1 , a_2 and two states of nature $\theta_1 = W = \text{Wet}$, $\theta_2 = D = \text{Dry}$. The probabilities are $p(\theta_1) = p(W) = 0.6$ and $p(\theta_2) = p(D) = 0.4$. This may be considered as prior information. No additional information is given. The economic loss matrix is shown in Table 5.7.

The expected losses for actions a_1 and a_2 are

$$E[L(a_1)] = 0.6 \times 3 + 0.4 \times 5 = 1.8 + 2.0 = 3.8(\times 10^9) \text{ Dra} \quad (5.32)$$

$$E[L(a_2)] = 0.6 \times 1 + 0.4 \times 6 = 0.6 + 2.4 = 3.0(\times 10^9) \text{ Dra} \quad (5.33)$$

Because $E[L(a_2)] < E[L(a_1)]$ the action a_2 is chosen.

The corresponding decision tree is shown in Figure 5.12.

Table 5.7 Economic loss (in billions of Dra) matrix under risk with no additional information.

Actions	States of nature	
	$\theta_1 : (W)$	$\theta_2 : (D)$
Action a_1	3	5
Action a_2	1	6
Probabilities	0.6	0.4

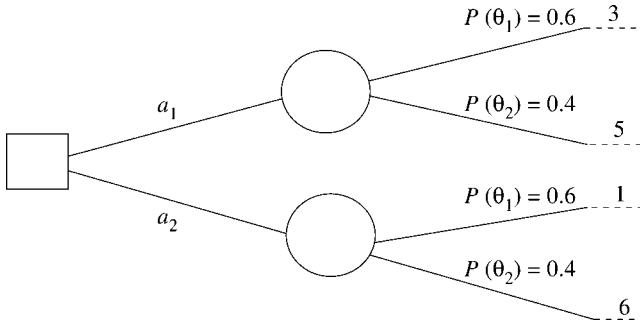


Figure 5.12 Decision tree under risk with no additional information.

5.3.3.1 Perfect Information

Suppose that a perfect source of information is available so that we know in advance the type of season (W or D). In order to minimise losses, when a wet season is predicted, action a_2 is decided upon and when the season is dry, action a_1 should be decided upon (Table 5.7). On average we will have 60% 'wet' and 40% 'dry' cases and the expected cost of perfect information is

$$E[PI] = 1 \times (0.6) + 5 \times (0.4) = 0.6 + 2.0 = 2.6 (\times 10^9 \text{ Dra}) \quad (5.34)$$

The difference between this value and the best we can expect without perfect information (Equation 5.33) is

$$3 - 2.6 = 0.4 (\times 10^9 \text{ Dra}) \quad (5.35)$$

This is known as the value of perfect information.

5.3.3.2 Imperfect Information

Suppose now that the probabilities 0.60 and 0.40 for the two states of nature are not perfectly known. An expert has predicted 50 wet seasons out of 60 and 32 dry seasons out of 40. The expert record is shown in Table 5.8 for 100 seasons. x_1 , x_2 are the predictions and θ_1 , θ_2 the actual or true states of nature.

If we use the expert information there will be 58 x_1 predictions and 42 x_2 predictions in a total of 100 seasons. For every prediction by the expert we can choose a_1 or a_2 . The new decision tree is shown in Figure 5.13.

Table 5.8 Actual and predicted states of nature (expert record over 100 seasons).

Actual state	Predicted state x_1	Predicted state x_2	Sum
θ_1	50	10	60
θ_2	8	32	40
Sum	58	42	

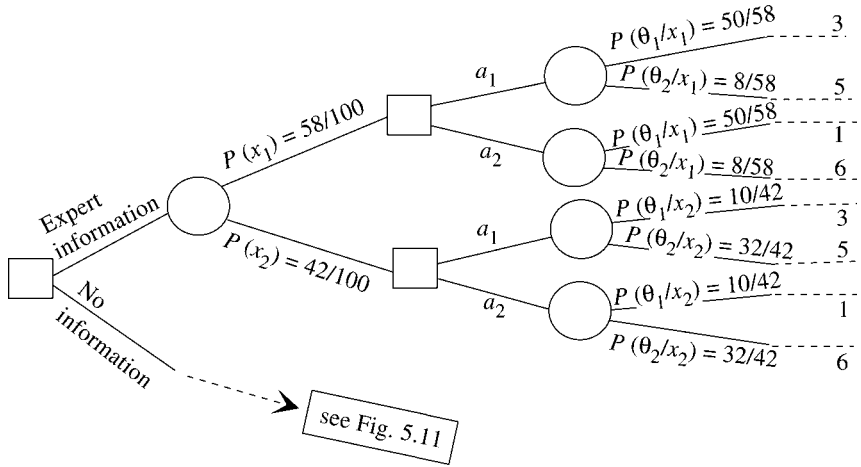


Figure 5.13 Decision tree with expert information.

When the expert predicts x_1 the expected costs from actions a_1 and a_2 are

$$E[L(a_1/x_1)] = \left(\frac{50}{58}\right) \times 3 + \left(\frac{8}{58}\right) \times 5 = \left(\frac{1}{58}\right) \times 190 \quad (5.36)$$

$$E[L(a_2/x_2)] = \left(\frac{50}{58}\right) \times 1 + \left(\frac{8}{58}\right) \times 6 = \left(\frac{1}{58}\right) \times 98 \quad (5.37)$$

Action a_2 should be chosen.

When the expert predicts x_2 the expected costs from actions a_1 and a_2 are

$$E[L(a_1/x_2)] = \left(\frac{10}{42}\right) \times 3 + \left(\frac{32}{42}\right) \times 5 = \left(\frac{1}{42}\right) \times 190 \quad (5.38)$$

$$E[L(a_2/x_1)] = \left(\frac{10}{42}\right) \times 1 + \left(\frac{32}{42}\right) \times 6 = \left(\frac{1}{42}\right) \times 202 \quad (5.39)$$

Action a_1 should be chosen.

The expected costs from using the expert information are

$$E[C] = \left(\frac{58}{100}\right) \times \left(\frac{98}{58}\right) + \left(\frac{42}{100}\right) \times \left(\frac{190}{42}\right) = 0.98 + 1.90 = 2.88 \quad (5.40)$$

Comparing this cost with results given by Equation 5.34 (cost with perfect information = 2.6) and Equation 5.33 (cost with no information = 3.0) we observe that

$$2.6 < 2.88 < 3.0$$

This means that the costs incurred with expert information are less than those incurred with no information but more than those resulting from perfect information.

5.4

Elements of the Utility Theory

Every outcome Y_{ij} corresponding to a given pair of action a_i and state of nature θ_j may have several measures or dimensions. For example, an outcome may be expressed as a cost or as a measure of environmental consequences or as a degree of efficiency. Such measures, which can influence decision making, are designated as *decision criteria* or *criteria*. The set of criteria is noted as $\{Y_j\}$.

The set of criteria may contain *quantified criteria* $\{y_j\}_N$ and *non-quantified criteria* $\{y_j\}_L$. For example, costs, technical characteristics and efficiencies, may be measured in dollars or other quantitative measures. Non-quantified criteria are related to aesthetics, human comfort or environmental values.

The difficulty with regard to evaluation and optimisation of several criteria is evident. Optimisation may be accomplished only with respect to a single criterion, which is a member of $\{Y_j\}$. Therefore, it is not possible to optimise all criteria simultaneously. One possibility is to apply a transformation reducing the set of criteria $\{y_j\}$ into a single characteristic figure of merit. Such a scalar measure of relative contribution to an optimum has been used with different names: *cost benefit*, *cost effectiveness* and *utility*.

Utility involves not only objective quantification but also the attitude of the decision-maker towards the risk. Summarising, we can state that utility is *a measure of the relative desirability of several alternatives*.

Let us consider two alternatives corresponding to two hypothetical lotteries A and B (Figure 5.14).

In case A there are two possible outcomes a_1 and a_2 which could occur with an equal chance of 50%. In case B the outcome will definitely be b (100%). Now let the monetary values of the outcomes be (case I)

$$L(a_1) = 1\$, L(a_2) = 0 \quad \text{and} \quad L(b) = 0.5 \$$$

In this case alternatives A and B may be indifferent.

Suppose now that in case II the monetary values are

$$L(a_1) = 100 \$, L(a_2) = 0 \quad \text{and} \quad L(b) = 50 \$$$

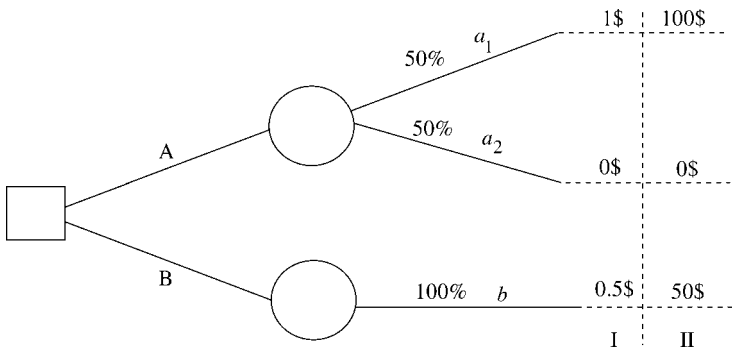


Figure 5.14 Illustration of the utility concept.

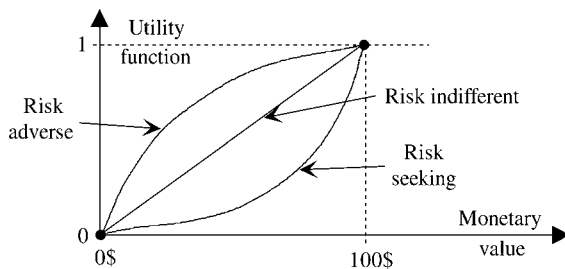


Figure 5.15 Different forms of the utility function.

Alternative B may be preferred, because a gain of 50 \$ is guaranteed. In alternative A there is a 50% chance of doubling this gain and a 50% chance of gaining nothing. Note that the expected monetary values are the same for the two alternatives.

This example illustrates the fact that preference may be influenced by the amount of money. Therefore the form of the relationship between the utility and the degree of uncertainty and the amount of monetary values (utility function) may reflect the attitude of the decision-maker to the risk. Three different attitudes can be distinguished (Figure 5.15):

- (a) risk adverse,
- (b) risk indifferent,
- (c) risk seeking.

The fundamental question is how to determine utilities given several measures of outcomes. Utility theory is a general methodology to determine numerical values of utilities given different possible outcomes in a decision-making situation.

Empirical relations have been proposed by several scientists in the past. D. Bernoulli (1730) proposed the following formula $u(A) = \log A$, where A is a certain amount of money and $u(A)$ the utility function. Buffon (French naturalist), stated that if a certain amount is added a to an existing sum of money A then $u(a)$ is given by

$$u(a) = \left(\frac{1}{a}\right) - \left(\frac{1}{(A+a)}\right)$$

Modern utility theory has been developed on an axiomatic basis by Von Neumann and Morgenstern (1947). More details may be found in the literature (Raiffa, 1968).

5.5

Multi-objective Decision Analysis

Preferences or decision criteria Y_j are not objectively defined; they rather reflect what the decision-maker wants. Four main criteria are usually considered necessary to achieve sustainability. As shown in Figure 5.16 these are:

- Technical reliability;
- Environmental safety;
- Economic effectiveness;
- Social equity.

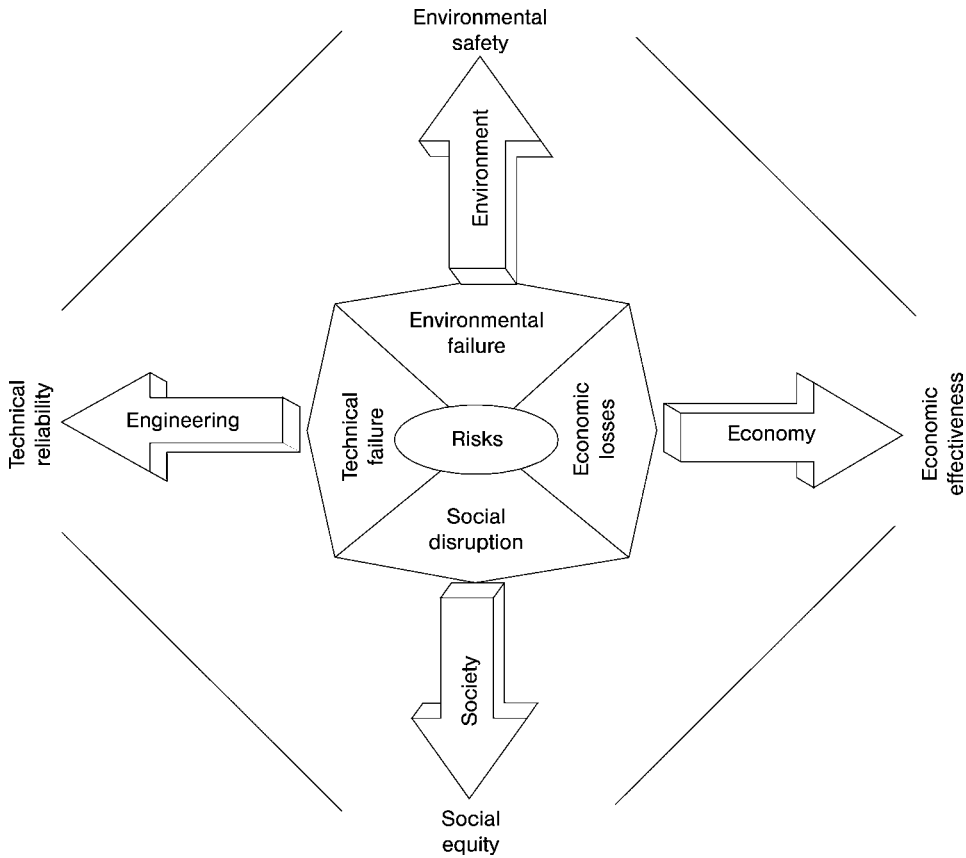


Figure 5.16 The four pillars for sustainable water resources management.

For every specific case of a given river basin, the above four objectives can be hierarchically structured into attributes and goals. This is the hierarchical Multi-Criteria Decision Analysis (MCDA) approach, shown in Figure 5.17 (Bogardi and Nachtnebel, 1994; Vincke, 1989).

MCDA techniques are gaining importance as potential tools for solving complex real world problems because of their inherent ability to consider different alternative scenarios, the best of which may then be analysed in depth before being finally implemented. (Goicoechea *et al.*, 1982; Szidarovszky *et al.*, 1986; Pomerol and Romero, 2000).

In order to apply MCDA techniques, it is important to specify the following:

- The attributes: which refer to the characteristics, factors and indices of the alternative management scenarios. An attribute should provide the means for evaluating the attainment level of an objective.
- The objectives: which indicate the directions of state change of the system under examination, and which need to be maximised, minimised or maintained in the same position.

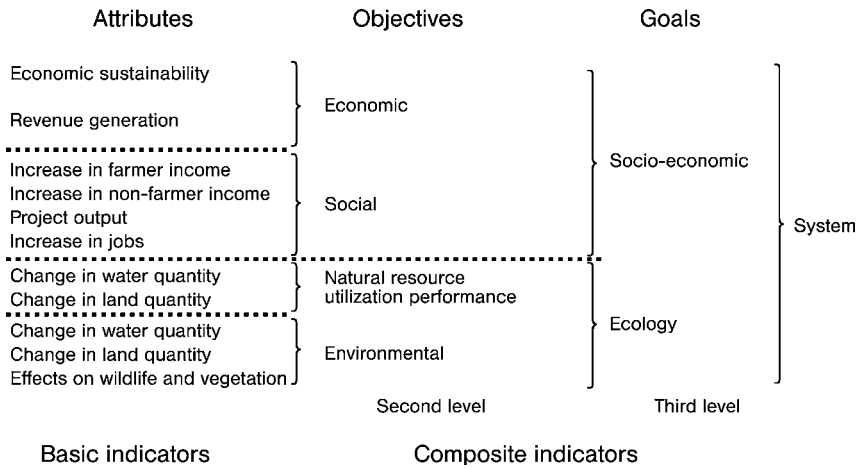


Figure 5.17 Attributes, objectives and goals for sustainable water resources management.

- The criteria: which can be expressed either as attributes or objectives.
- The constraints: which are restrictions on attributes and decision variables that can or cannot be expressed mathematically.

A multi-criterion programming problem can be formulated in a vector notation as:

$$\text{"Satisfy"} \quad f(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_I(\mathbf{x})) \quad (5.41)$$

$$\text{Subject to} \quad g_k(\mathbf{x}) < 0, \quad k = 1, 2, \dots, K \quad (5.42)$$

$$x_j \geq 0, \quad j = 1, 2, \dots, J \quad (5.43)$$

Here there are I objective functions (Equation 5.41) each of which is to be 'satisfied' subject to the constraint sets (Equations 5.42 and 5.43). The region defined by this constraint set is referred to as the *feasible region* in the J -dimensional decision space. In this expression, the set of all J -tuples of the decision variable \mathbf{x} , denoted by \mathbf{X} , forms a subset of a finite J -dimensional Euclidean space; in many other applications, \mathbf{X} is defined to be discrete. In the further special case where \mathbf{X} is finite, then the most satisfying alternative plan has to be selected from that finite set \mathbf{X} .

It is important to note at this point that the word 'optimum' which includes both the maximisation of desired outcomes and minimisation of adverse criteria is replaced by the word 'satisfactum' and 'optimise' is replaced by 'satisfy' in this discussion. The reason is that when dealing with two or more conflicting objectives, one cannot, in general, optimise all the objectives simultaneously (Simon, 1957) as an increase in one objective usually results in a deterioration of some other(s). In such circumstances, trade-offs between the objectives are made in order to reach solutions that are not simultaneously optimum but still acceptable to the decision-maker with respect to each objective (Goicoechea *et al.*, 1982; Roy, 1996).

In a mathematical programming problem such as that defined by Equations 5.41–5.42 and 5.43, the vector of decision variables \mathbf{X} and the vector of

the objective functions $\mathbf{f}(\mathbf{x})$ define two different Euclidean spaces. These are (1) the J -dimensional space of the decision variables in which each coordinate axis corresponds to a component of vector \mathbf{X} , and (2) the I -dimensional space \mathbf{F} of the objective functions in which each coordinate axis corresponds to a component of vector $\mathbf{f}(\mathbf{x})$. Every point in the first space represents a solution and gives a certain point in the second space that determines the quality of that solution in terms of the values of the objective functions. This is made possible through mapping the feasible region in the decision space \mathbf{X} into the feasible region in the objective space \mathbf{F} , using the I -dimensional objective function.

5.5.1

Feasible, Non-dominated and Efficient Solutions

In Multi-Criterion Decision Analysis (MCDA), the question is not to obtain an optimal solution as in the case of one objective. Instead of an optimum solution, we refer to a 'non-inferior' or 'non-dominated' solution. This is a solution for which no improvement in a single objective can be achieved without causing a degradation of at least another objective.

Let us consider, for example, the problem of 'maximising' two conflicting objectives Y_1 and Y_2 subject to a set of constraints:

$$g_j(x_1, x_2, \dots, x_n) \leq 0 \quad j = 1, 2, \dots, m \quad (5.44)$$

As shown in Figure 5.18, each couple of values Y_1 and Y_2 that satisfy the constraints lies within the feasible region or feasible space. This region is limited by a curve ABCD called a feasibility frontier. All points of this frontier form the set of 'non-inferior' or 'non-dominated' solutions. Every decision vector on this curve is defined by a maximum value of the objective Y_2 given a value of the objective Y_1 . This particular solution is 'optimal' in the sense that there can be no increase in one objective without a decrease in the value of the other.

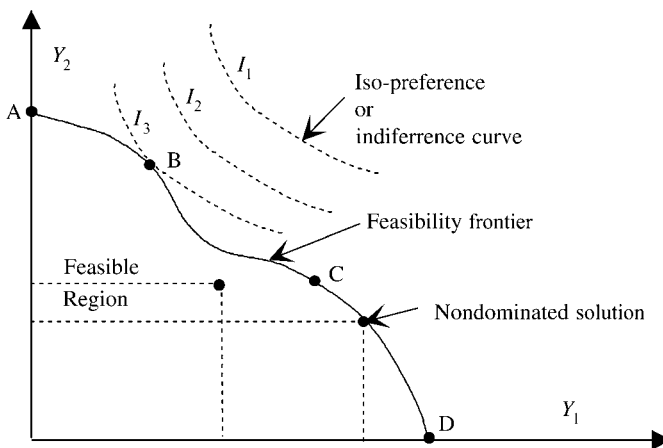


Figure 5.18 Non-dominated solutions for a two-objective problem.

A selection of one particular solution from a set of non-inferior solutions depends on the preferences of the decision-maker. This may be indicated by a family of iso-preference or indifference curves (Figure 5.18). In this figure, the efficient solution is defined by the point B on the feasibility frontier which has the maximum level of preference.

5.5.2

Solution Procedures and Typology of MCDA Techniques

Finding a set of efficient solutions to a mathematical programming problem is usually determined using a generating procedure, in which an objective function vector is used to identify the non-dominated subset of feasible decisions. This procedure deals mostly with the objective realities of the problem (e.g. the set of constraints) without necessarily taking into consideration the preference structure of the decision-maker.

In order to clarify the procedure for choosing a technique, the classification of MCDA models given in Tecle and Duckstein (1994) is now summarised. Five types are distinguished

- (1) Value or utility-type: which essentially coalesce the multiple objectives into a one-dimensional 'multi-attribute' function. It can be a value function that is deterministic or a utility function that includes a measure of risk.
- (2) Distance-based techniques: which seek to find a solution as 'close' as possible to an ideal point, such as compromise and composite programming or else, a solution as 'far' as possible from a 'bad' solution, such as the Nash cooperative game concept.
- (3) Outranking techniques: which compare alternatives pair-wise, and reflect the imperfection of most decision-makers' ranking process (Roy, 1996) namely, alternative $A(j)$ is preferred to alternative $A(k)$ if a majority of the criteria $C(i)$ is better for $A(j)$ than for $A(k)$ and the discomfort resulting from those criteria for which $A(k)$ is preferred to $A(j)$ is acceptable. As a result, non-comparability of certain pairs of alternatives is an acceptable outcome; this is in contrast to the previous two types of approaches where a complete ordering of alternatives is obtained. Techniques such as ELECTRE and PROMETHEE are recommended.
- (4) Direction-based: interactive or dynamic techniques where a so-called progressive articulation of preferences is undertaken.
- (5) Mixed techniques: which utilise aspects of two or more of the above four types. In planning problems, a general class of methodology has been developed to rank different alternatives with various conflicting objectives under risk (Goicoechea *et al.*, 1982).

One of the most promising methods is the Composite or Compromise Programming. First, trade-offs between objectives may be made in different levels to obtain some composite economic or ecological indicators. Then, ranking between different

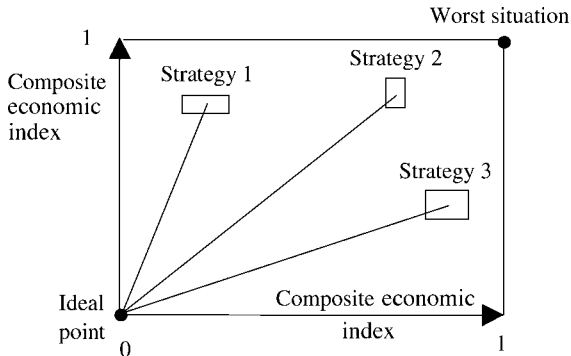


Figure 5.19 Ranking of different strategies expressed in terms of economic and ecological indices.

strategies or options may be done using different techniques, such as the one based on the minimum composite distance from the ideal solution (Duckstein and Szidarovszky, 1994) (Figure 5.19).

5.6

Questions and Problems – Chapter 5

Performance Indices and Figures of Merit

- How can performance indices and figures of merit of a water system be defined?
- If risk is considered as a figure of merit, explain the different definitions of risk.
- How are vulnerability, resilience and sustainability defined and how are they interrelated?

Objective Function and Optimisation

- How is an objective function developed?
- Under which conditions can a system be optimised?

Basic Decision Theory

- A new technique for wastewater treatment may have high (H) or low (L) efficiency. A decision should be made between two alternatives:
A: building a small scale experimental facility and
B: building a full size treatment plant.

Additional costs for the case AH (small scale-high efficiency) are 100 units for AL (small scale-low efficiency), 20 units for BH (full size-high efficiency) and 50 units for BL (full size-low efficiency).

- Draw the decision tree and the decision matrix.
- Decide between alternatives A and B using the MiniMax and MaxiMin rules (decision under uncertainty).
- If the probability of H (risk of high efficiency) is equal to 30%, decide between A and B (decision under risk).

- (b) Two alternatives are suggested for building a new dam:
 A: use traditional material having 99% reliability and 1.5×10^9 units cost,
 B: use new material with 90% reliability and 1.0×10^9 units cost.
- (b.1) Draw the decision tree and the decision matrix.
 (b.2) Decide between alternatives A and B using the minimum expected cost.

Utility Theory

- (a) What is the difference between benefit and utility?
 (b) The following two alternatives are suggested:
 A: 100% probability of winning €100 M, and
 B: 89% probability of winning €100 M
 1% probability of winning nothing, and
 10% probability of winning €500 M

Most people choose A.

Find the alternative that gives the maximum expected utility. Compare this with the popular choice A and explain why most people choose alternative A (this is the so-called 'Allais paradox').

- (c) A poll was conducted asking people if they prefer music composed by Beethoven (Be), Bach (Ba) or Mozart (Mo). According to the results

1/3 of population responded	Be > Ba > Mo
1/3	Mo > Be > Ba, and
1/3	Ba > Mo > Be

Then a 2/3 majority prefers:

Be > Ba
 Ba > Mo
 Mo > Be

Which means that 2/3 of the people prefer Beethoven to Mozart and 2/3 Mozart to Beethoven.

How can you explain this paradox in peoples' preferences?

Multi-objective Decision Analysis

- (a) Why can't all multiple conflicting objectives be maximised or minimised?
 (b) What is the definition of 'feasible' and 'dominant' or 'non-dominated' solutions?
 (c) List five different methodologies for multi-objective decision making. What is the principle of the 'Compromise' or 'Composite Programming' methodology?

6

Case Studies

Many semi-enclosed bays and coastal areas in the Mediterranean are heavily polluted, mainly by domestic sewage. Collection and analysis of water quality data is the basis for assessing the environmental situation and the risk of pollution. Mathematical models based on stochastic variables are useful tools to quantify the risk of pollution and explore the efficiency of different remedial measures for restoring coastal ecosystems. This is illustrated by the case of the Thermaikos Gulf (NE Mediterranean sea).

Of considerable concern in recent years is the increase in atmospheric CO₂ due to its influence on global climate. Studies so far have concentrated primarily on precipitation and air temperature changes from an assumed doubling in CO₂ (the $2 \times \text{CO}_2$ scenario). The risk of climatic change, which may influence the water quality in a coastal region, is analysed and a case study from the Thermaikos Gulf in Macedonia, Greece, is presented.

Water drained by rivers carries pollutant substances such as organics, nutrients, fertilisers and pesticides, with an ultimate destination of coastal waters in estuaries. Major rivers discharging into the Mediterranean, such as the Po, Rhone, Ebro and the Nile are heavily polluted. It has been recognised that coastal water pollution from rivers is nowadays one of the most crucial environmental problems. Apart from local sources discharging wastewaters into the river, pollution in river estuaries and deltas originates mainly from diffuse sources, scattered within the entire river basin. Agricultural activities may overload soils with fertilisers and pesticides. Washing-off of the soil by rainfall produces large concentrations of nitrates, phosphorus and toxic chemicals in rivers and coastal waters. Time series data of pollutant concentrations in rivers show high variability both in time and space. Uncertainties related to various kinds of variability in data should be analysed and then quantified by means of adequate mathematical tools. This is shown for the case of the Axios river (Macedonia, Greece).

Surface, coastal and ground waters should be considered in a unified framework. The role of groundwater is vital in the economy for public health and to protect ecological systems. About 75% of the inhabitants of the European Union member states depend on groundwater for their water supply. Public water supply requires a reliable source, which means that the quality, as well as the quantity, should be beyond all doubt in relevant areas.

Groundwater contamination is the most critical among the various types of pollution that can occur in the water cycle. This is a consequence of the large time scales of the phenomena and the irreversible character of the damage caused. Due to the very slow movement of groundwaters, pollutants can reside in the aquifer for a very long time. As a consequence, groundwater remains polluted for centuries even if the pollutant sources are no longer active. At the same time, because of the complex interaction between pollutants, soil and groundwater, remediation of contaminated subsurface is a very delicate operation. In most cases total removal and cleaning up of the contaminated soil is necessary, or biological techniques need to be applied for a long period of time to become efficient. In addition, both quality and quantity of the groundwater are of essential importance for the diversity of ecosystems. Lower groundwater levels and changes in groundwater quality due to man-induced contamination cause loss of diversity of ecosystems and deterioration of natural reserves. Groundwater is in danger of losing its potential functions due to the deterioration of quantity and quality. While aiming at sustainability of use, the vital functions of groundwater reservoirs are threatened by pollution and overexploitation. This is shown for a characteristic case study of groundwater salinisation in the Campaspe area, Victoria, Australia.

6.1

Coastal Pollution: the Thermaikos Gulf (Macedonia, Greece)

Using the results of monitoring at 12 stations from 1984 to 1990, the water quality in the Thermaikos bay area is presented. At all these stations temperature, salinity, pH, dissolved oxygen, nitrites, nitrates, ammonia, phosphates, silicates, heavy metals, total coliforms and *E-coli* were measured in the water column with seasonal frequency. There is a general trend for water pollution to increase from south to north and from the open sea to the river estuaries. This reflects the effect of pollutant loads from the human population in the northern region and from river flow. Mathematical modelling of the transport and fate of pollutants in the bay are used to assess the risk of pollution. The use of models in analysing various combinations between the choice of the disposal site and the degree of sewage treatment is discussed.

Meteorological and local climatic information is essential when analysing the long-term quality characteristics of coastal waters. More specifically, with respect to any global warming, it is useful to see the likely effect of the speculated climate change scenarios on coastal water quality. This can be studied by simulation, as presented below for a typical case in the Mediterranean, the Thermaikos Gulf, Macedonia, Greece. The question is what would the consequence on the water quality of Thermaikos Gulf be if the carbon dioxide content of the atmosphere ($2 \times \text{CO}_2$ scenario) were doubled? In the case study, only the direct influence of temperature changes on water quality will be considered. Indirect effects, caused by variations in the amount of run-off or rainfall precipitation entering the water body, have not been included.

6.1.1

Description of the Thermaikos Gulf

The Thermaikos Gulf is located in the north-west of the Aegean Sea and has a width of 15 km at its maximum opening between the Aherada Peninsula in the west and Epanomi in the east (Figure 6.1). The maximum 'height' of the gulf, from north to south, is 45 km and its total surface area 473 km²; Figure 6.2 illustrates its bathymetry. The Thermaikos is open only on the south side. It constitutes the discharge basin for one major river (the Axios) and three minor rivers in terms of flow rate (the Aliakmon, the Loudias and the Galikos) (Figure 6.1). All three carry water year-round, with seasonal flow rates varying between 10 m³/s and 400 m³/s from summer to winter.

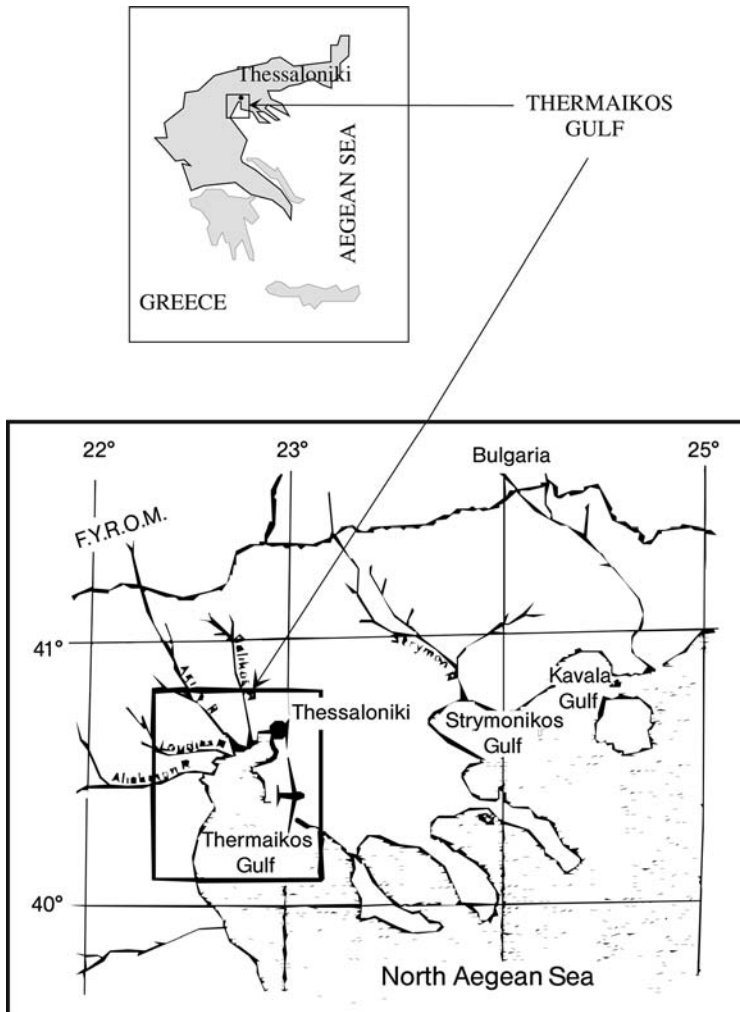


Figure 6.1 Geographic location of Thermaikos Gulf.

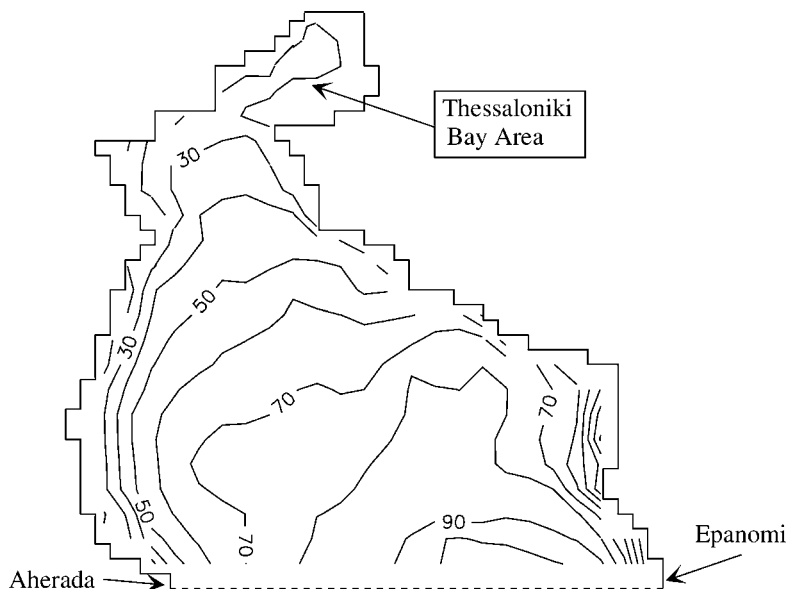


Figure 6.2 Bathymetry of Thermaikos Gulf (in m).

The great variation in flow rates is also due to irregular drainage from agricultural irrigation (Ganoulis, 1988a, 1990, 1991a). In addition, the sewage from the city of Thessaloniki (1 000 000 inhabitants) is also discharged into the gulf.

The main climatological data of the region are shown in Tables 6.1 and 6.2. The prevailing winds are S-SE during summer and N-NW during winter (Figure 6.3).

Table 6.1 Meteorological characteristics in the Thermaikos Gulf: temperature during the period 1930–1975 (from Ganoulis, 1988a).

Temp. (°C)	J	F	M	A	M	J	J	A	S	O	N	D	Year
Min.	3.0	2.9	6.3	12.1	17.5	22.6	25.4	25.4	20.2	14.2	9.5	5.2	15.3
Max.	10.5	11.3	13.7	17.4	22.3	25.4	28.3	28.4	25.4	21.5	14.5	11.5	17.5
Ave.	6.0	7.3	10.0	14.8	19.6	24.0	26.8	26.5	22.4	17.2	12.4	8.0	16.2

Table 6.2 Meteorological characteristics in the Thermaikos Gulf: precipitation during the period 1930–1975 (from Ganoulis, 1988a).

Month	J	F	M	A	M	J	J	A	S	O	N	D	Year
Precip. (mm)	41	35	40	41	49	37	27	20	31	51	56	55	483

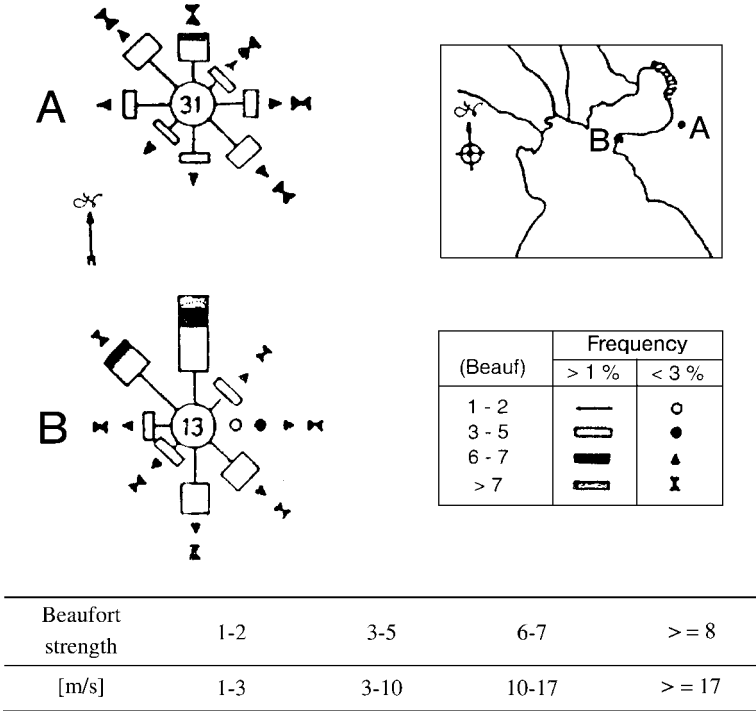


Figure 6.3 Meteorological characteristics in the region of the Thermaikos Gulf: average yearly wind roses at two locations (A = Thessaloniki Macedonia airport, B = Epanomi) during the period 1950–1968 (from Ganoulis, 1988a).

Strong winds (>15 m/s) are infrequent and of short duration, lasting one or two days and usually occurring in the winter.

The initial design of the sewerage system of the city of Thessaloniki is shown in Figure 6.4. The figure focuses on the upper part of the gulf, known as the Bay of Thessaloniki. The main sewer collector (SC) is a tunnel of 2 m in diameter, located at an average depth of 20 m. This pipe collects the entire city's sewage from the eastern to the western part of the greater Thessaloniki metropolitan area. The pipe ends at the sewage treatment plant (TP), located close to the Gallikos river (Figure 6.4). The plant uses advanced treatment procedures, including bio-oxidation of wastewater, after which the design initially provided for wastewater to be disposed into the Axios river using a twin-pipe system between the sewage treatment station and the river (Figure 6.4). However, because of environmental concerns about the water quality in the river and the estuary, the design was subsequently modified. This is because the flow rate of the Axios river has been constantly decreasing over the last few years, leading to lower wastewater dilution. At the same time, new water quality standards have to be applied for the protection of river and coastal waters according to directives issued by the European Union. The coastal area close to the mouth of the river is considered to be a protected area of very great importance from an ecological point of

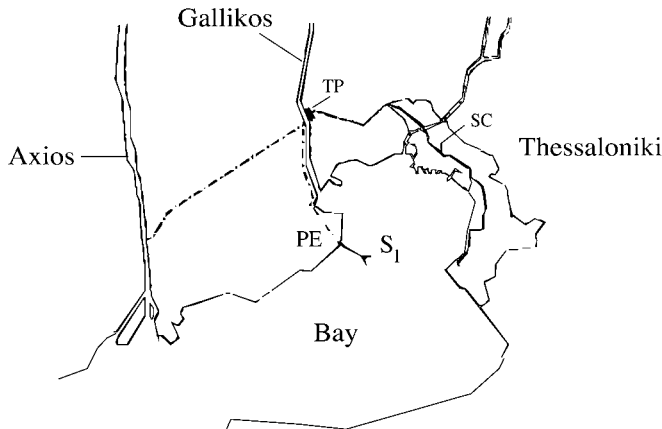


Figure 6.4 Sewage collection and treatment plant in the city of Thessaloniki.

view. According to the RAMSAR convention this area is a special protected estuary. An estimation of the pollutant loads discharging into the bay is given in Table 6.3.

Until such time as biological treatment of all wastewater is fully implemented, a preliminary operation system of the treatment plant was provided in 1992. During this transitional period, wastewater was disposed of in the upper part of the bay (point PE), by using a ditch parallel to the bed of the Gallikos river (Figure 6.4). The local environmental impacts in this area and especially the concentrations of coliforms were studied using risk assessment and mathematical modelling techniques (Ganoulis, 1991d; 1992).

Important questions raised for the design were:

- Is a submarine outfall (S_1 in Figure 6.4) needed?
- If so, what is its best location?
- What is the optimum degree of wastewater treatment in relation to possible eutrophication in the bay?

Table 6.3 Pollutant loads in the bay of Thessaloniki.

Pollutant sources	Flow rate (m^3/day)	BOD5 (kg/day)	N (kg/day)	P (kg/day)
Sewage	150 000	60 000	10 000	4000
Industrial waste-waters	60 000	10 000	5000	?
Axios	Winter $170 \text{ m}^3/\text{s}$ Summer $20 \text{ m}^3/\text{s}$	50 000	16 000	4000
Aliakmon	Winter $80 \text{ m}^3/\text{s}$ Summer $10 \text{ m}^3/\text{s}$	20 000	3000	900
Loudias	Winter $30 \text{ m}^3/\text{s}$ Summer $10 \text{ m}^3/\text{s}$	20 000	3000	900
Pumping stations	Winter $15 \text{ m}^3/\text{s}$ Summer $2 \text{ m}^3/\text{s}$	20 000	4000	400

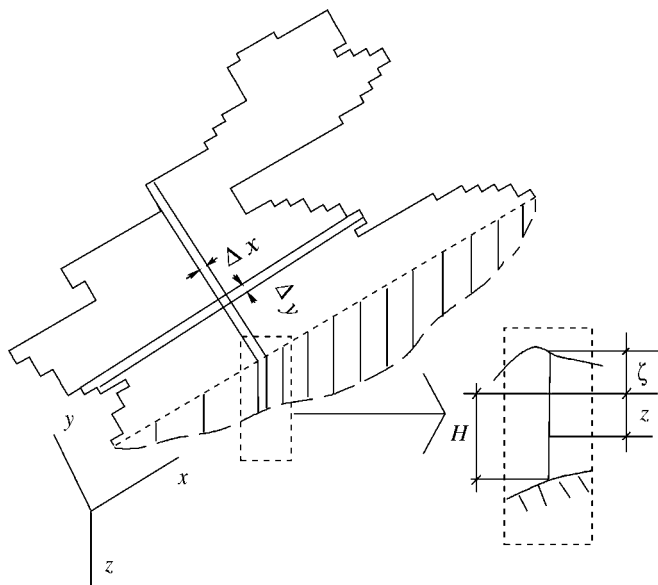


Figure 6.5 The 3-D grid used in hydrodynamic computations.

6.1.2

Water Circulation Patterns

A 3-D hydrodynamic model, which simulates the wind-induced circulation at various depths, was developed. The hydrodynamic model integrates the Navier–Stokes equations in the finite-difference grid in Figure 6.5; this is described on a regular Cartesian coordinate system, on the x - y plane, and the transformed coordinate $\sigma = (z - \zeta)/H$ along the vertical – z . H represents the water depth and the surface elevation.

On the basis of mathematical simulation work and *in situ* measurements the average water circulation patterns during winter and summer were determined and are shown in Figure 6.6 (Ganoulis, 1988a, 1990). Tidal currents are insignificant; total tidal elevation in the inner bay does not exceed 30 cm. The measurements show that during summer, strong stratification of the water occurs with the warmer surface layers remaining stable over the colder depth layers; this leads to relatively anoxic conditions at the bottom. In contrast, during winter, the colder and denser surface layers destroy the stratification and satisfactory vertical mixing in the water column results. Consequently, the worst conditions for pollution occur during summer.

The understanding of water circulation is of great importance. Previous measurements of currents using drogues, drift cards and current meters (Balopoulos and James, 1984; Ganoulis and Koutitas, 1981) and the application of hydrodynamic models (Ganoulis and Koutitas, 1981; Krestenitis and Ganoulis, 1987) led to the following conclusions: (a) tidal currents are very low (<5 cm/s); (b) external circulation from the N. Aegean sea creates a current entering the bay along the eastern coast and creates cyclonic circulation; (c) currents are mainly due to the winds.

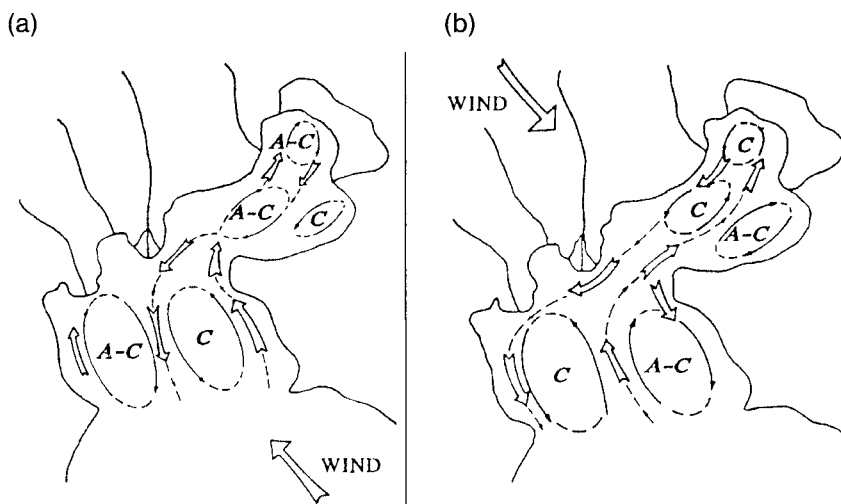


Figure 6.6 The prevailing water circulation patterns in the inner Thermaikos Gulf during (a) summer and (b) winter (A, Anticyclonic; C, Cyclonic circulation).

During the summer, sea breezes create residual water circulation, which is very characteristic of situations where pollutants are transported. In fact, this is the most critical circulation state for the pollutant advection because, as the currents are small, an increase in pollutant concentration is observed. In the subsequent development of mathematical modelling, steady state hydrodynamic conditions corresponding to the prevailing winds were used.

The convective dispersion model was used to assess the risk of pollution in the gulf. For the numerical integration of the equations involved, various numerical algorithms have been developed during the last decade. Algorithms based on finite differences or finite elements suffer from numerical diffusion and trailing effects. Lagrangian models based on random walk simulation (cf. Chapter 4) or using a mixed Eulerian–Lagrangian approach have been found to be reliable in simulating the fate of pollutants in the Thermaikos Gulf (Ganoulis, 1990, 1991a). These models have been tested in simple cases where analytical solutions are available and validated by using the data collected. They have been adopted as tools for studying the environmental impacts of several alternative remedial measures aimed at protecting water quality in the gulf.

6.1.3

Water Quality Assessment

The monitoring of water quality characteristics and data processing is the basis for formulating computerised mathematical models and deciding upon the appropriate remedial measures for environmental protection. The main objective of the Thermaikos Bay study was the assessment of the environmental situation in the bay

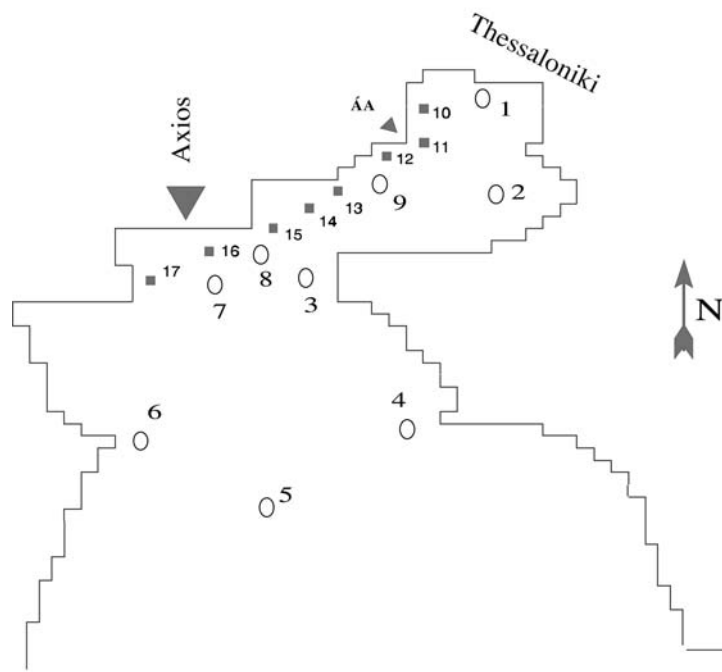


Figure 6.7 Location of sampling stations in the Thessaloniki Bay.

and the environmental impact analysis of the sewage works. As shown in Figure 6.7 appropriate sampling stations non-uniformly distributed in space were selected. Using the research vessel 'THETIS' (13 m long) from 1984 to 1990, more than 2500 water samples were collected and analysed. Apart from the currents and winds, the following parameters were seasonally monitored near the surface, at the mean depth and near the bottom of the water column:

- (a) Temperature, salinity, density, dissolved oxygen, pH
- (b) Nutrients such as NO_2^- , NO_3^- , NH_4^+ , PO_4^{3-} , SiO_4^{4-}
- (c) Total coliforms and *E-coli*
- (d) Heavy metals such as Cd, Pb and Cu

Heavy metals were also analysed in sediments. Variations in water quality parameters were very large both in time and space. As an example the time series of nitrates at station 1, located near the city of Thessaloniki, is shown in Figure 6.8. These variations are due to the irregular physical conditions which prevail in the Mediterranean. In fact, the tides are very weak and the wind-induced circulation is very unsteady and variable in space. In view of the large variations in the data a statistical analysis was carried out. The contour lines indicating equal concentrations of dissolved oxygen are shown in Figure 6.9. These are the mean values recorded over the time period 1984–1989 near the sea bed.

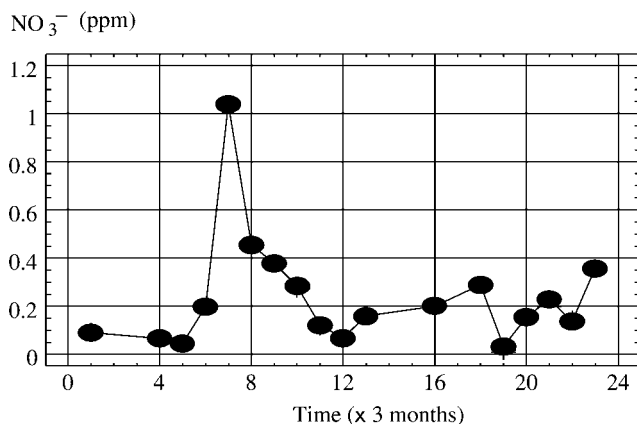


Figure 6.8 Time series of nitrates (NO_3^-) recorded at station 1 near the sea surface.

From these data a statistical trend was deduced for the increase in water pollution from south to north (high land-based population activities) and in the river estuaries (high pollutant loads). In fact four different zones were distinguished, ranging in water quality from very bad to excellent (Ganoulis, 1988a).

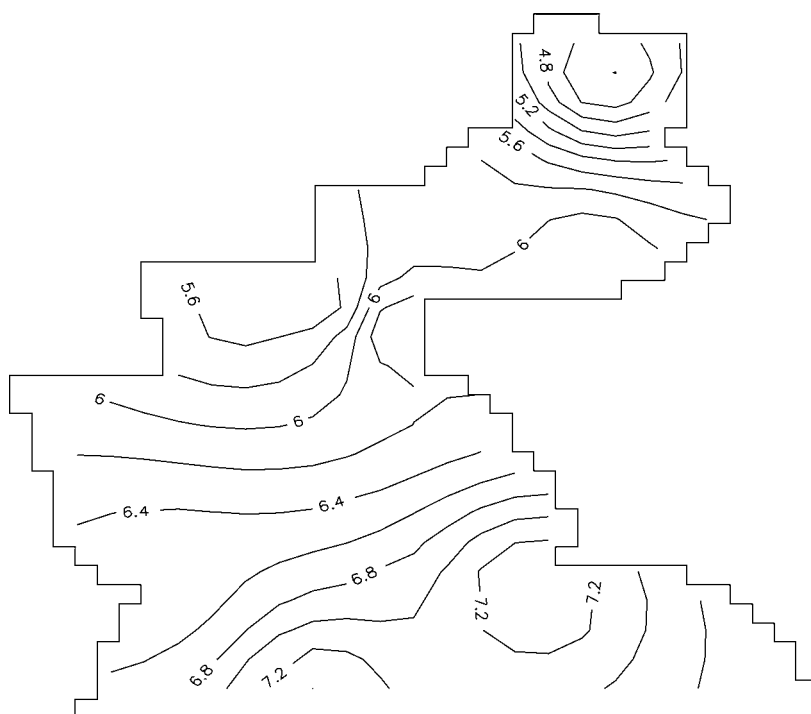


Figure 6.9 Distribution of dissolved oxygen (DO) in the Thermaikos Gulf (mean values during the period 1984–1989 measured near the seafloor).

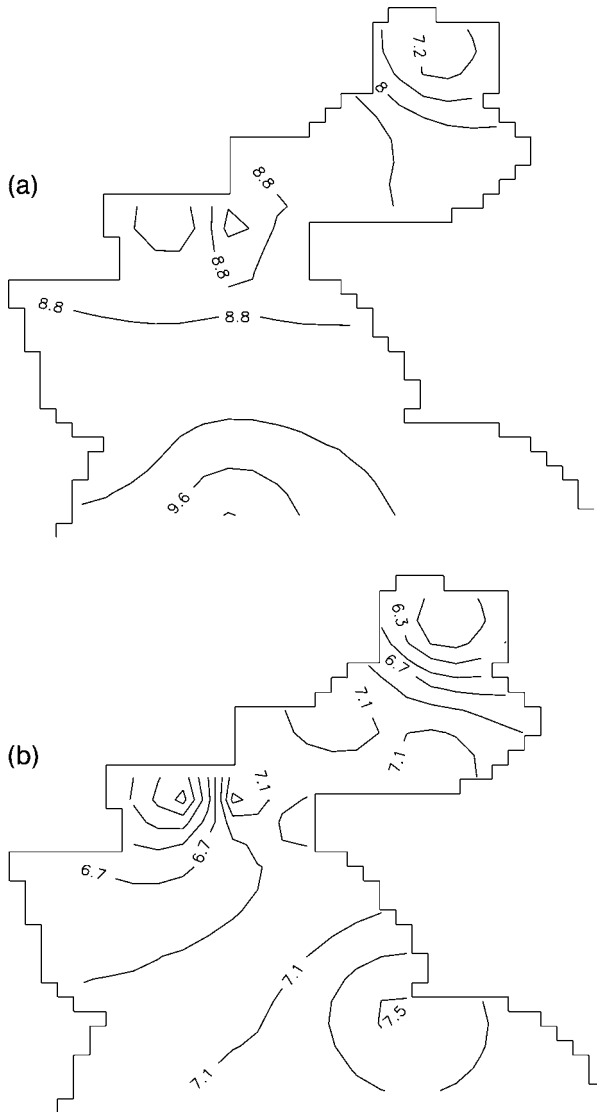


Figure 6.10 Experimental results for the distribution of DO near the bottom of the Gulf (averaged annual values; (a) 1992 and (b) 1991) (from Ganoulis, 1988a, 1990).

It should be noticed (Figures 6.9 and 6.10) that the mean annual values of dissolved oxygen are not constant, especially for 1991 and 1992. A general improvement can be observed in 1992, possibly due to the operation of the wastewater treatment plant (this started at the beginning of 1992).

Ecologically sensitive coastal zones in the bay area requiring special protection measures are shown in Figure 6.11. These include the major part of the western

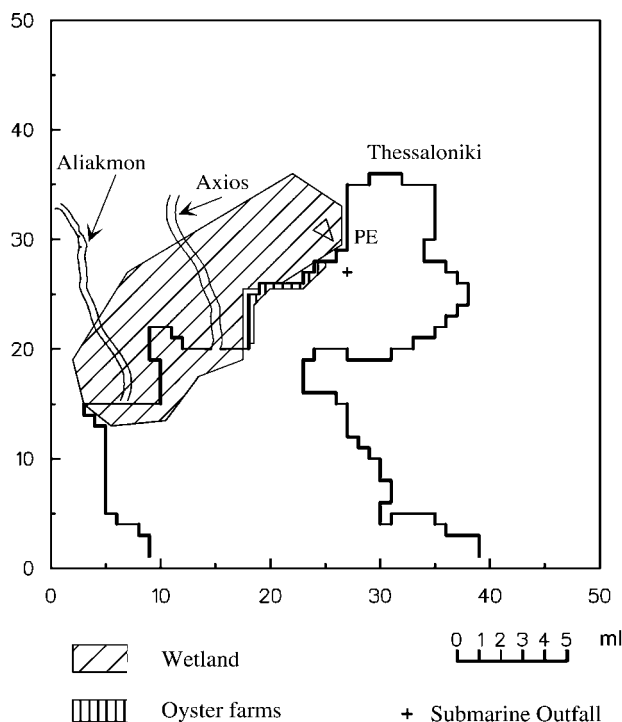


Figure 6.11 Sensitive zones in the Bay of Thessaloniki.

coast near the rivers, where the water is shallow and large quantities of nutrients are discharged from rivers.

Oyster farms have been established in this part of the bay, producing several millions of tonnes of oysters every year. With the operation of the new wastewater treatment plant, the risk of shellfish contamination by coliform bacteria should be evaluated. Chlorination for sewage disinfection has to be used very carefully (Ben Amor *et al.*, 1990) in order to avoid the formation of THM (Tri-Halo-Methanes) in coastal waters.

The assessment of water contamination risk was carried out using two methodologies, which are explained in Chapter 4:

- (1) the random walk simulation and
- (2) the use of data on wind-generated currents in the form of time series.

To validate the random walk simulation, the actual situation near the site where the wastewater is discharged (Paliomana, site PE in Figure 6.12) was studied.

The validation was based on a choice of the 'best' values of two parameters: the dispersion coefficient D and the time T_{90} of the bacterial decay. Using data from sampling, the best choice of these coefficients was made by calibration (Ganoulis, 1991d, 1992). This is the case shown in Figure 6.13a, where the values $D = 4 \text{ m}^2/\text{s}$ and $T_{90} = 5 \text{ h}$ were found.

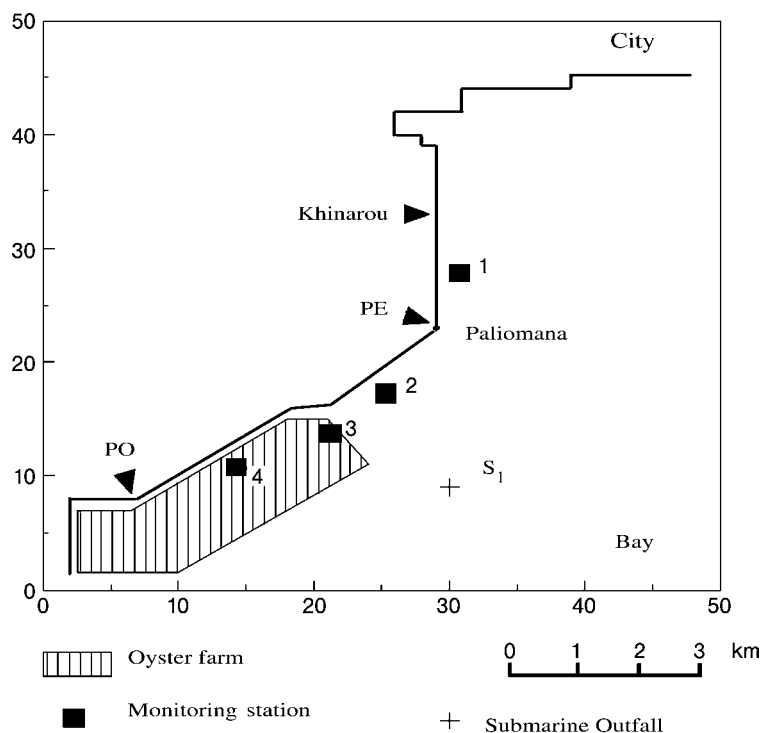


Figure 6.12 Sensitive environmental zones near Paliomana.

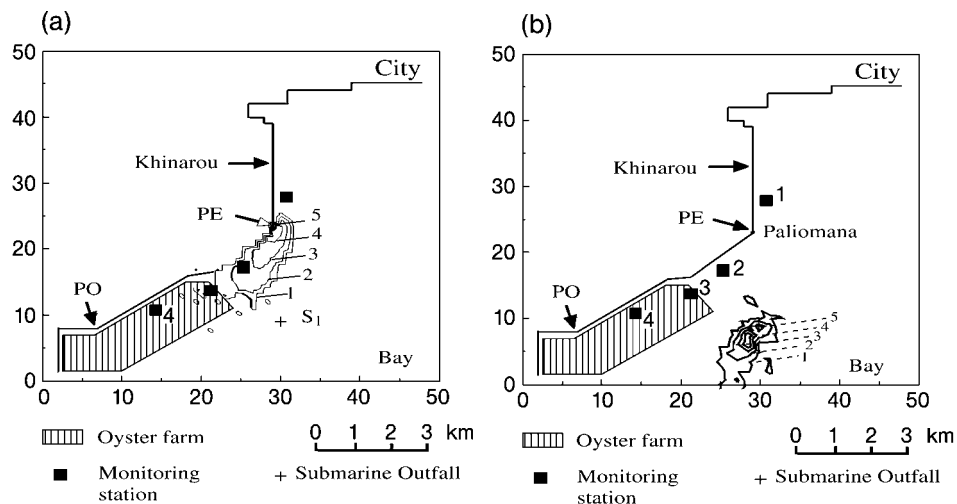


Figure 6.13 Contours of *E. coli* concentrations: simulation of the actual situation (a) and using a submarine outfall S_1 (b).

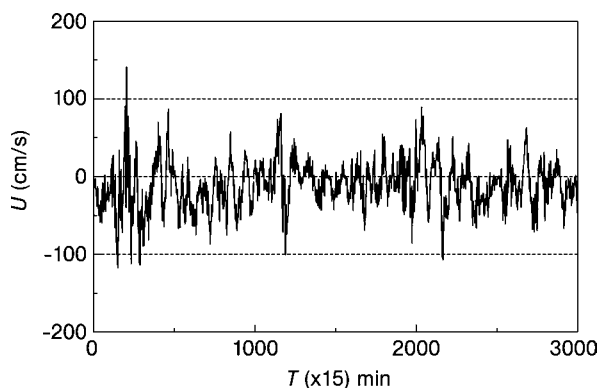


Figure 6.14 Time series of current velocity U .

A total of 10^4 particles were used over the entire time-simulation period. Small oscillations due to the statistical character of the method do not affect its application to any great degree. By using a fixed grid and counting the number of particles located in a given grid cell, lines of equal concentration are obtained (Figure 6.13a and b).

It should be noted that samples were also taken at night. The value $T_{90} = 5$ h represents a mean value between day and night time situations. The results of the simulation shown in Figure 6.13a are in good agreement with actual measurements (Ganoulis, 1992). Comparison was based on the C_{80} concentrations (80% of the samples having $C < C_{80}$). These concentrations must comply with EU standards within the oyster farming area. To obtain a further dilution of wastewater, the use of a short submarine outfall is a good solution (Figure 6.13b).

When a time series of currents is available (Figure 6.14), the direct simulation method, based on the displacement of particles with random current velocities, gives more realistic results. This method was used to evaluate the risk of pollution from two different discharge sites in the bay. The risk of coastal water pollution from discharge site A is shown in Figure 6.15 and from site B in Figure 6.16.

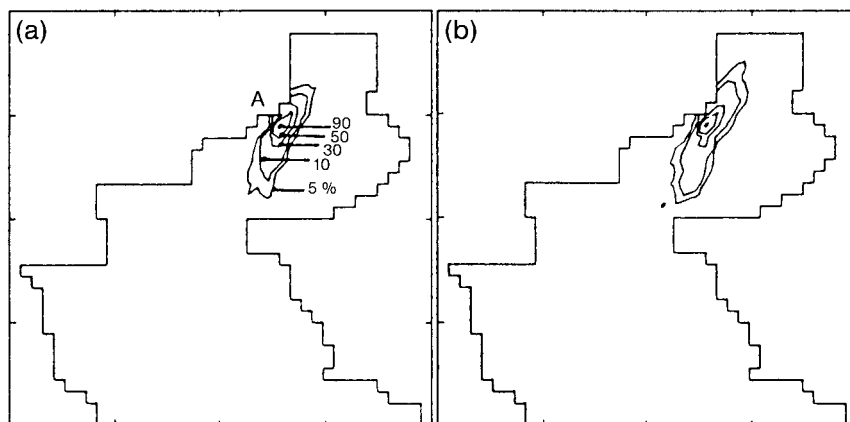


Figure 6.15 Contours of impact probabilities from wastewater within $T = 3$ h (a) and $T = 6$ h (b) after release. Discharge at site A.

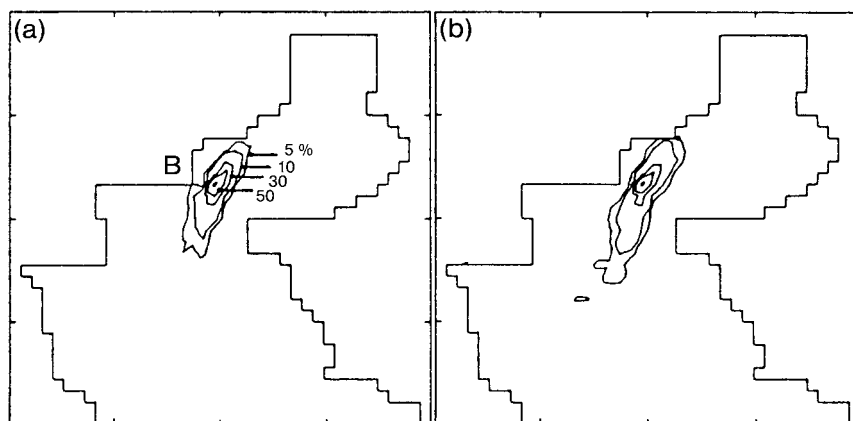


Figure 6.16 Contours of impact probabilities from wastewater within $T = 3$ h (a) and $T = 6$ h (b) after release. Discharge at site B.

6.1.4

Risk of Pollution under Climate Change

6.1.4.1 Temperature and Climate Change

The approach used in this work was based on the methodology initially developed by Bardossy and Caspary (1990) and further elaborated by Matyasovszky *et al.* (1993). The background of the method is that local meteorological characteristics, such as temperature and precipitation, may be related to global circulation patterns (CPs). Knowing the CPs over a grid covering the whole of the northern hemisphere, a downscaling to local meteorological parameters should be possible. For this operation a large sample of existing data was used.

The method took into account data at two different scales (a) a small scale for local meteorological data, and (b) a large scale for global circulation patterns. Linking the two types of data was achieved using a multivariate stochastic model together with historical data. This is a rather consistent and scientifically well-founded approach, as opposed to the approach which assumes different climate scenarios, such as an arbitrary increase or decrease in average temperatures. To generate local meteorological time series under global climate change conditions, the methodology (Bardossy and Plate, 1992) was applied in successive steps:

- (a) Characteristic types of daily circulation patterns (CP) were classified for the region under study and a probabilistic analysis of the frequency of appearance of the various CP types was undertaken.
- (b) Local meteorological variables such as temperature and precipitation were analysed probabilistically, with probability distribution functions conditioned to a given CP type.
- (c) A Markov space–time model was developed for linking local variables such as temperature, to various CP types.

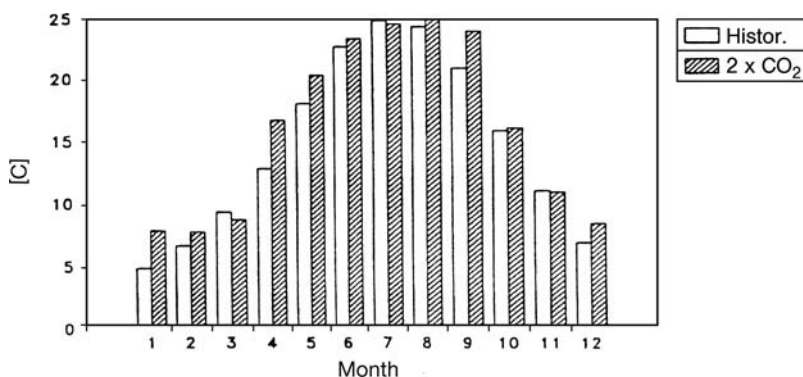


Figure 6.17 Monthly means of daily mean temperature at Macedonia airport.

- (d) CPs were reproduced by use of Global Circulation Models (GCM) and down-scaled local time series were generated and compared with historical data.
- (e) $2 \times \text{CO}_2$ CPs were simulated from GCMs and local time series of temperature were obtained, reflecting the effect of global climate change.

Results shown in Figures 6.17 and 6.18 refer to temperature data collected in the area of Mikra, Thessaloniki (N. Greece), where the city's Macedonia airport is located. As seen in Figure 6.17, in general the temperature has a tendency to increase under climate change due to a doubling of CO_2 . The increase is not equally distributed over the year: the highest increase is about 4°C in January, April, May and September, while there is no significant change in February, March, June, July, August, October and November. In order to more accurately predict the impact of climate change on temperature, coastal water quality and eutrophication, research is currently underway (Matyasovszky *et al.*, 1993) which takes into account data from other meteorological stations in N. Greece.

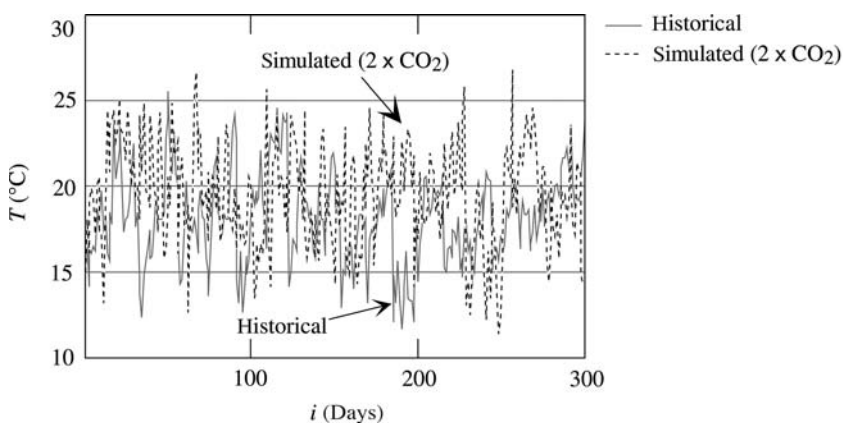


Figure 6.18 Historical and climate-induced daily temperature time series for September at Macedonia airport.

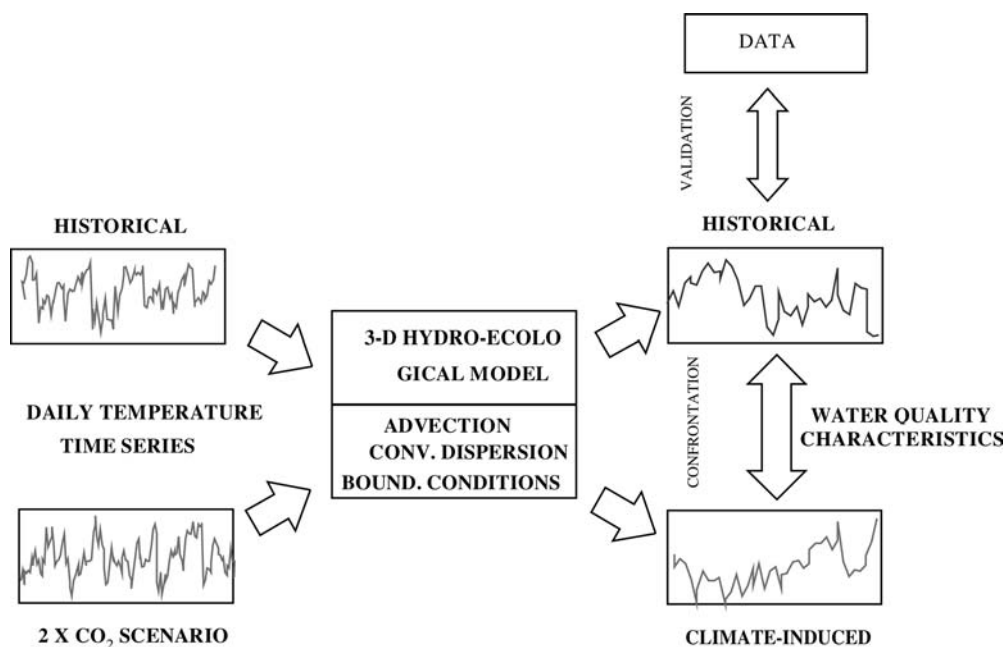


Figure 6.19 Operation modes of the hydro-ecological model.

6.1.4.2 Monte Carlo Simulation

As shown in Figure 6.19, temperature time series were introduced as input to a 3-D hydro-ecological model both as historical and as simulated under climate change data. Results of model simulation include circulation patterns and pollutant concentrations at various positions and depths, including phytoplankton or chlorophyll-a concentrations. They are used either to validate the model by comparing simulation results with available measured data (Ganoulis *et al.*, 1994; Ganoulis, 1988a; 1992), or to predict climate-induced temperature impacts on water quality.

The 3-D hydro-ecological model describes water quality and eutrophication in coastal areas by taking into account three main processes:

- convection by currents;
- dispersion due to turbulence;
- variation of phytoplankton biomass due to biochemical interactions with other physical and chemical systems.

Phytoplankton comprises many different forms of algae and it is customary to consider algal concentrations in terms of chlorophyll-a concentrations. As shown in Figure 6.20a and b the chlorophyll-a growth rate $S_A = dA/dt$ reflects the uptake of nutrients such as NH_3 , NO_3 and PO_4 .

Under the influence of solar insolation and temperature, chlorophyll is recycled by

- respiration,
- decay (non-predatory),
- settling.

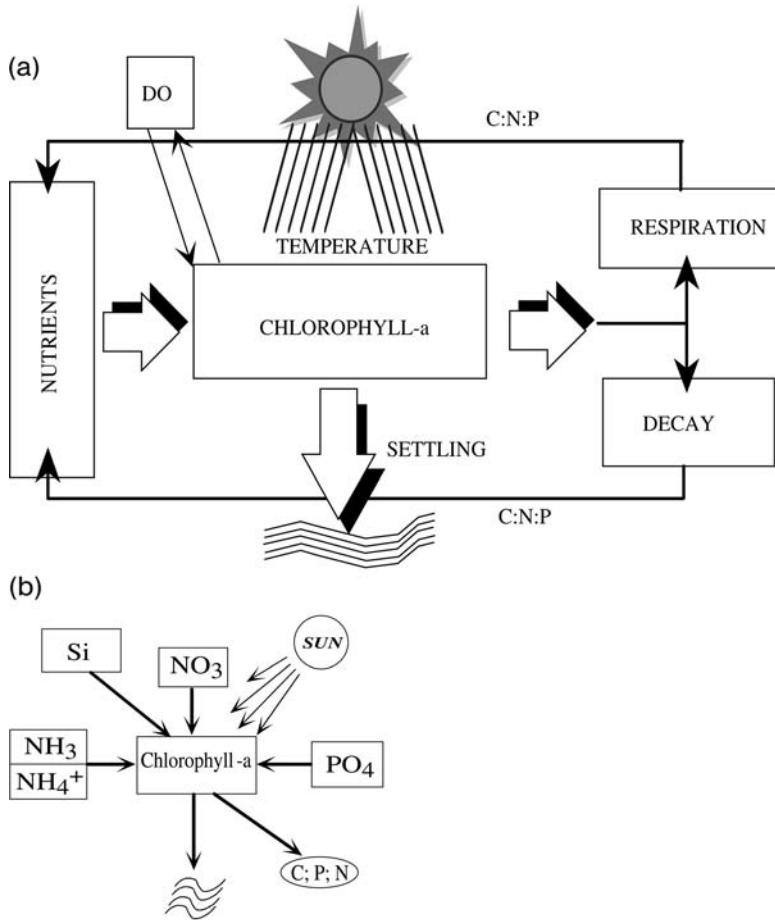


Figure 6.20 Phytoplankton kinetics: (a) physical mechanisms and (b) interaction between chemical systems.

Inorganic nutrients are reproduced by phytoplankton biomass and are recycled back into the system through respiration and non-predatory mortality. Organic matter is converted into dissolved inorganic substances at a temperature-dependent rate.

Concerning interactions with dissolved oxygen, although algae produce oxygen by photosynthesis in the euphotic zone, this is reversed at night due to respiration. Furthermore, algae settling at the bottom, contribute to oxygen uptake by biodegradation.

In terms of chlorophyll-a concentration A , phytoplankton kinetics may be described as

$$\frac{dA}{dt} = (\mu - r_A - e_x - s - m_A)A - G_A \quad (6.1)$$

where A is chlorophyll-a concentration (mass/volume); m , phytoplankton growth rate (T^{-1}); r_A , respiration rate (T^{-1}); e_x , excretion rate (T^{-1}); S , settling rate (T^{-1}); m_A , non-predatory mortality (T^{-1}); and G_A , loss rate due to grazing (mass/volume/time).

Temperature plays a key role in all biochemical transformations. The constants involved in usually, first-order kinetic relationships (i.e. oxidation, nitrification, denitrification) are related to temperature according to an Arrhenius-type relationship

$$m = m_{20^\circ\text{C}} \times T^{(t-20^\circ\text{C})} \quad (6.2)$$

The growth rate $S_A = dA/dt$ in chlorophyll-a expressed in Equation 6.1 should be incorporated in the advective–dispersion balance equation (Fischer *et al.*, 1979), which has the form

$$\frac{\partial A}{\partial t} + \frac{u\partial A}{\partial x} + \frac{v\partial A}{\partial y} + \frac{w\partial A}{\partial z} - \frac{\partial}{\partial x} \left(K_x \frac{\partial A}{\partial x} \right) - \frac{\partial}{\partial y} \left(K_y \frac{\partial A}{\partial y} \right) - \frac{\partial}{\partial z} \left(K_z \frac{\partial A}{\partial z} \right) = S_A \quad (6.3)$$

where A is phytoplankton concentration (mass/volume); u, v, w , velocity components (length/time); and $S_A = dA/dt$, rate of change of phytoplankton concentrations due to biochemical interactions as given by Equation 6.1 (mass/volume/time).

The ecological model expressed by Equations 6.1 and 6.3 is linked to a 3-D hydrodynamic model, which simulates the wind-induced circulation at various depths.

Water quality in the gulf is determined as follows. First, on the basis of prevailing summer winds, the water circulation in the gulf can be determined using the three-dimensional hydrodynamic model. The gulf is divided horizontally into a rectangular 2×2 km grid. The water circulation velocities to be fed into a suitable ecological model may be found anywhere in the gulf. For this task the Water Analysis Simulation Program (WASP) (di Toro *et al.*, 1981) was chosen. The model is based on a link-node representation of the flow field, divided into interconnected segments. Using the flow exchanges between neighbouring segments, it solves the transport and diffusion equations for each pollutant. Different program modules deal separately with toxic or non-toxic pollutants.

A less refined discretisation is needed for the ecological model than for the hydrodynamic model. The spatial relationship between the two grids on the horizontal plane is shown in Figure 6.21. The same segmentation is replicated along the vertical direction, following the transformed depth coordinate ζ . Software assuring the automatic linkage between hydrodynamic and ecological computations was developed especially for this purpose.

Though water temperature influences water circulation through density and salinity changes, at this stage this influence was not considered during the application of the hydrodynamic model. Instead, the water circulation was modelled using an average water temperature of 19°C , representative of mean summer conditions. The amount of waste load inflows is summarised in Table 6.3. The WASP software

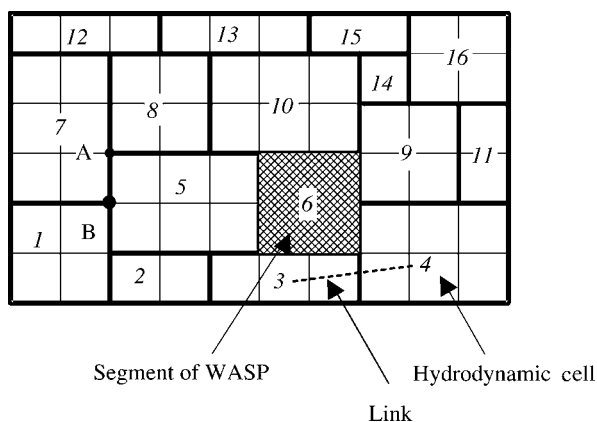


Figure 6.21 Relationship between the hydrodynamic (fine lines) and ecological grids (thick lines) on the horizontal plane.

was used for the case of full eutrophication, that is, using all eight available physical systems (Ammonia, Nitrates, Chlorophyll, CBOD, DO, Organic Nitrogen, Organic and Inorganic Phosphorus).

The modelling approach described above was used to predict the effect of climate change on the water quality of the gulf (Rafailidis *et al.*, 1994). Using the summer overall circulation pattern the ecological model was run with different daily temperature time series. Each consecutive run of the model resumed from the concentrations in each segment of the gulf at the end of the last time step. Thus, whilst safeguarding the long-term consistency of the data used by the model, a long run of many years duration was possible.

Two sets of ambient temperatures for Thessaloniki were used as inputs, both supplied from the methodology described in Section 6.1.4.1 (Bardossy and Plate, 1992; Ganoulis *et al.*, 1994; Matyasovszky *et al.*, 1993). One time series reflected the 'historic case', that is under the $1 \times \text{CO}_2$ scenario. The second time series was based on a speculative $2 \times \text{CO}_2$ scenario ('modelled case'). From each time series, covering a total of 30 years, the daily air temperatures in May were extracted to reflect typical summer conditions. A further reason for choosing this month was that in May, as shown in Figure 6.17, there is significant average global warming, so the effects on water quality, if any, would be easier to see.

Using the two temperature time series and with summer water circulation conditions, trends were obtained for CBOD and DO respectively. These trends reflect the minima and maxima of the respective water quality parameters throughout the body of the gulf, that is at all horizontal sites or depths. Thus, they represent the best- and worst-quality water environments due to pollution which marine life may encounter within the water body.

Daily time series of minimum dissolved oxygen (DO) under historical and climate-induced daily temperature time series are shown in Figure 6.22. Dissolved oxygen is a characteristic parameter, reflecting the overall influence of many pollutants and

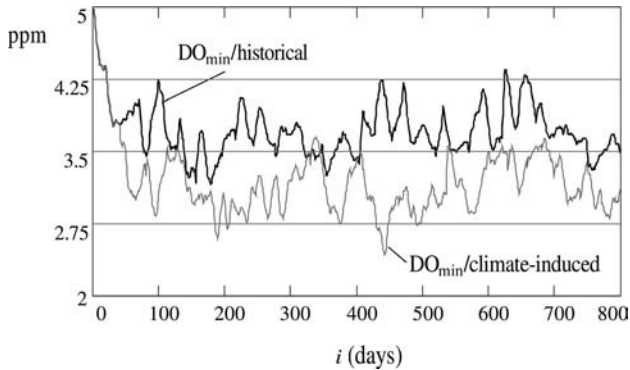


Figure 6.22 DO_{min} concentrations (ppm): historical and under climate change.

also eutrophication on the water quality. Simulated daily values of DO shown in Figure 6.22 are minimum values near the bottom. Comparing the DO time series presented in Figure 6.22 it can be seen that on a daily scale the decrease in DO due to a climate-induced increase in temperature may be much larger than the average decrease over a longer time scale. There is an average decrease of oxygen, which could threaten species living near the bottom.

The climate-induced change in phytoplankton is related to total nitrogen concentrations, whose minimum values over the total area of the bay are shown in Figure 6.23. An average increase in temperature of $4^{\circ}C$ also produces an increase in total nitrogen, which though rather small on average is more pronounced on a daily scale.

For the case study undertaken in the Bay of Thessaloniki (N. Greece), although no significant change was found in the average temperature over a year, there is a risk of oxygen depletion on a daily scale. Impacts on phytoplankton concentration and eutrophication change are currently under investigation.

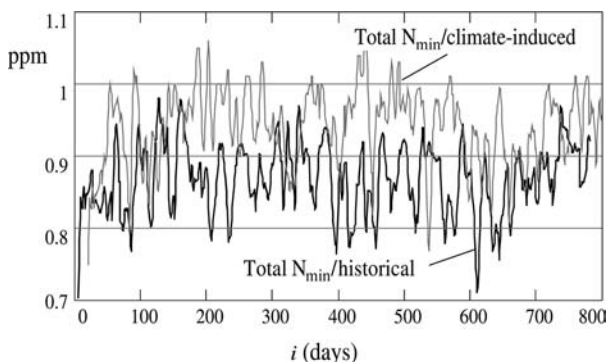


Figure 6.23 Total N_{min} concentrations (ppm): historical and under climate change.

6.2

River Water Quality: the Axios River (Macedonia, Greece)

The case of the Axios river has been studied in relation to the more general problem of systematic increases in nitrites in the surface waters around Greece and other European countries. This increase in concentrations constitutes a potential danger of pollution for river water resources. Although actually the nitrate concentration in surface waters in Greece does not generally exceed the critical values fixed by the EU standards, there are many cases of local water contamination by nitrates. This refers to rivers, lagoons and semi-enclosed coastal areas, into which large quantities of nitrates drain from surface waters (Giannakopoulou, 1990). In these areas water renewal is minimal and high nitrate concentrations enhance eutrophication phenomena.

The data obtained by monitoring Greek rivers with regard to nitrate concentration, show in many cases that there is a systematic trend for increased nitrate content. This trend should be taken into consideration together with the increasing use of fertilisers, which are among the principal sources of nitrate contamination. Other important sources of nitrate pollution are farming (not yet very intensive in Greece) and the use of septic tanks in urban areas where the sewer system has not yet been completed. In this section, an overview of the present situation regarding the nitrate contamination of waters in Greece is given first. Then, a case study on the Axios river (Macedonia, N. Greece) is summarised. In this study monthly sampling of nitrate concentration was carried out and related water quality parameters were analysed. After presentation of the data, various possible sources of nitrate contamination are considered. To prevent water pollution by nitrates many technical alternatives are possible. Depending on the degree of knowledge of the existing environmental situation as well as of the relationship between pollutant loads from external sources and the nitrate concentration in waters, these technical solutions can be very efficient. Mathematical modelling is a very useful tool for assessing the state of pollution in the water environment and predicting the efficiency of several alternatives in reducing pollution. Various numerical techniques for modelling the transport and fate of nitrates in the water environment, developed in the Hydraulics Laboratory, AUTH, are also briefly discussed in this section.

6.2.1

Present Situation

The morphology of the hydrologic basins in Greece, which are generally small and steep, does not favour the penetration of nitrates into the groundwater aquifers. In relation to the relatively non-intensive use of fertilisers in agriculture, the level of nitrate content in waters seems to be generally rather low. However, some of the available data show that in many cases there is a systematic trend towards an increase in nitrate concentrations.

Although the available data does not allow a complete picture of water contamination by nitrates to be presented, an attempt to summarise the present situation is given in Figure 6.24. In the Pinios (Thessalia), Axios (plain of Thessaloniki) and

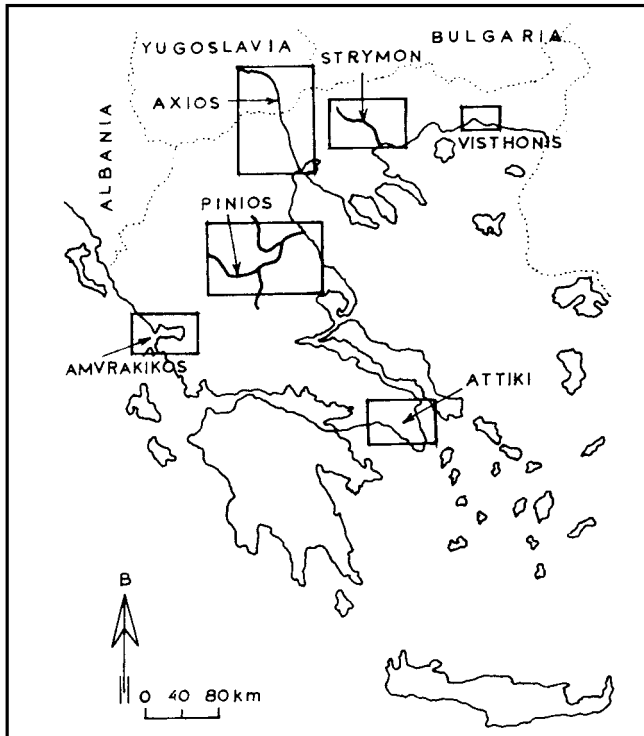


Figure 6.24 Areas at high risk for water contamination by nitrates in Greece.

Strymon (plain of Serres) rivers, the risk of the nitrate concentration increasing is high. This is also the case in the Bay of Amvrakikos (W. Greece) and Lake Visthonis (E. Greece). The groundwater aquifer in the Attiki area also contains high levels of nitrates (Figure 6.24).

In fact, following international trends the use of fertilisers in Greece has been steadily increasing over recent years. This has a bearing on the use of nitrogen (N) and commercial (NPK) fertilisers in Greece from 1970 to 1983 (OCDE, 1985).

6.2.1.1 Axios River

The Axios river was initially chosen for wastewater disposal from the greater Thessaloniki metropolitan area, after adequate treatment. This is the reason for undertaking an extensive monitoring programme to assess the water quality characteristics of the river. This programme was initiated in 1988 by the Hydraulics Laboratory, AUTH, to supplement the existing water quantity and quality data, collected by the Ministry of Agriculture.

As shown in Figure 6.25 the Axios river has a partial length of approximately 75 km between the Greek and the former Yugoslavian border and the sea. This corresponds to only 10% of the total basin area of 23 750 km², the remainder being located in the Former Yugoslav Republic of Macedonia (FYROM).

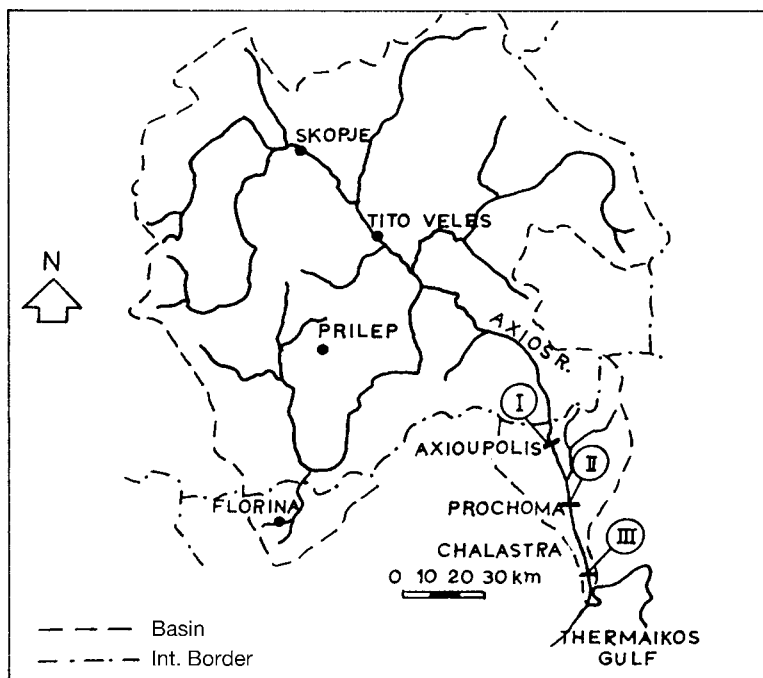


Figure 6.25 Monitoring stations on the Axios river.

The mean flow rate of the river is about $170 \text{ m}^3/\text{s}$ and the minimum, during the summer, $37 \text{ m}^3/\text{s}$. Samples were collected on a monthly basis at monitoring station III starting in April 1988 (Figure 6.25). The following water quality parameters were analysed: temperature, pH, dissolved oxygen, BOD, COD, suspended solids, salinity, conductivity, nitrites, nitrates, ammonia, total organic nitrogen (NK), phosphates, silicates and heavy metals.

The time series of the nitrogen-related parameters are shown in Figure 6.26. All these values (in ppm) are rather low and indeed far below the prescribed values for drinking water. By comparing the mean values of the nitrate-nitrogen concentrations with the corresponding values from 1981 to 1982 (Table 6.4), it can be seen that in 7 years the increase in nitrate-nitrogen was about 50%.

This trend is confirmed during the period 1988–1990 by the linear regression shown in Figure 6.27. A seasonal analysis of the existing data was undertaken in order

Table 6.4 Values of N-NO_3 concentrations (in ppm) in Axios river.

Period	Min.	Mean	Max.
1981–1982	0.50	1.05	1.76
1988–1990	0.52	1.56	2.75

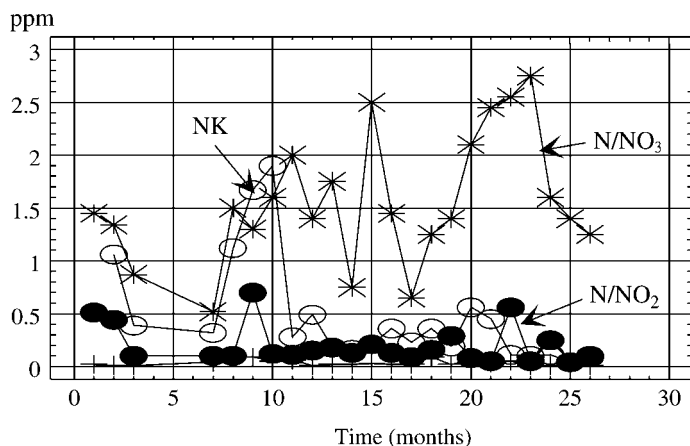


Figure 6.26 Time series of nitrogen-related parameters at station III (Axios river).

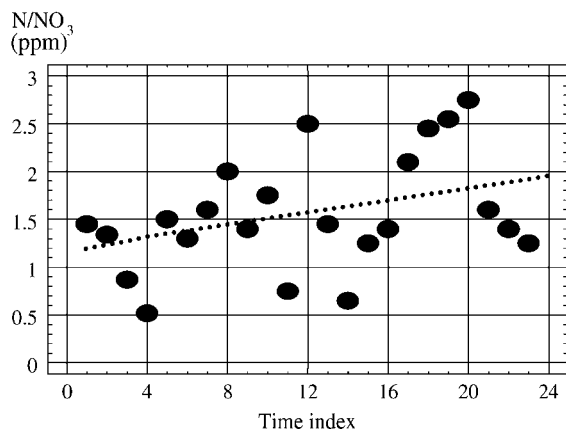


Figure 6.27 Linear trend for N-NO₃ concentrations at station III (1988–1990) (Axios river).

to explore possible sources of nitrate pollution in the Axios river. First, by computing the autocorrelation coefficient of the nitrate time series it was found that no significant correlation existed for a time period greater than 3 months. By plotting the seasonal sub series of N-NO₃ concentrations over 1 year, it can be seen from Figure 6.28 that the increase in nitrates is systematic between November and February each year. This corresponds to the rainy season and the subsequent washing-off of the soil by the drainage water.

6.2.2

Mathematical Modelling

To study the impact of sewage disposal on the Axios river, the mixed numerical technique based on an Eulerian–Lagrangian algorithm was applied (cf. Section 4.2.2.2).

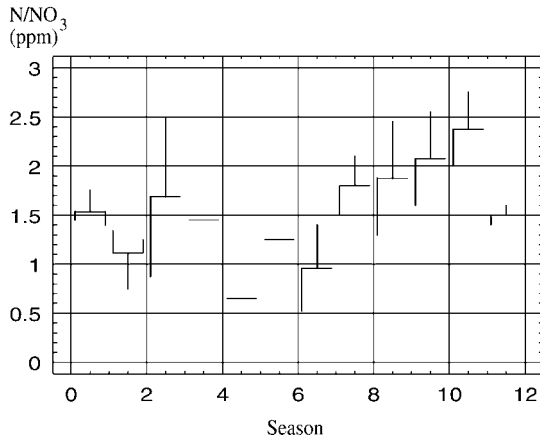
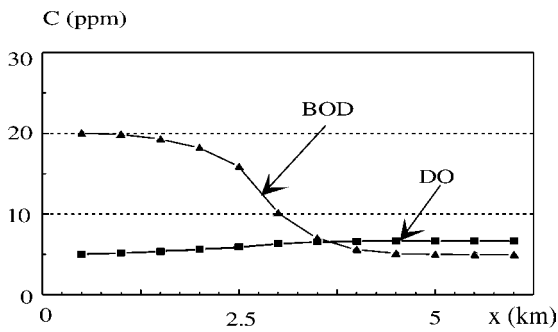
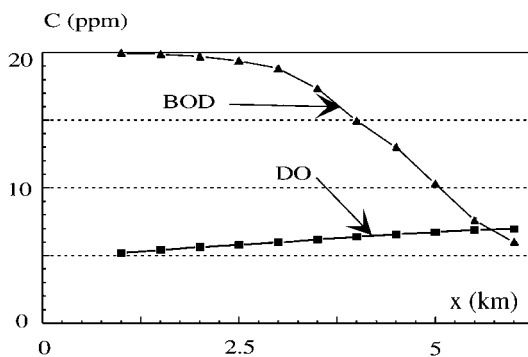


Figure 6.28 Seasonal sub-series of N-NO₃ concentrations at station III (Axios river).



(a) $t = 1 \text{ h}$



(b) $t = 2 \text{ h}$

Figure 6.29 Mathematical modelling of DO-BOD distribution due to wastewater discharge (Axios river, Lagrangian-Eulerian algorithm).

This corresponds to the initial design of sewage collection and disposal in the city of Thessaloniki (cf. Figure 6.4).

Results of numerical integration of the two coupled partial differential equations, Equations 4.63 and 4.64, for BOD and DO are shown in Figure 6.29 for two different times. The increase in DO due to reaeration and the decrease in BOD due to biodegradation and dispersion can be seen in this figure, from $x=0$ (sewage disposal) to $x=7$ km (river estuary).

6.3

Groundwater Pollution: the Campaspe Aquifer (Victoria, Australia)

The main problem of groundwater quality in the riverine plain, northern Victoria, is increasing salinisation near the soil surface. It has been recognised (Tickell and Humphrys, 1984) that the rise in the groundwater table in the upper aquifer (Shepparton formation) is one of the main causes of water logging and salinity increase near the top layer of the soil. As the groundwater moves upwards, the increasing salinity concentrations are caused by dissolution of natural salts that are contained in the soil. The rising groundwater table results from the combined effect of intensive actual irrigation practices together with the disruption of the natural equilibrium between plants, soil and groundwaters. In fact, intensive removal of deep rooted vegetation in the past has reduced the natural drainage capacity of the basin and destroyed the natural equilibrium between groundwater recharge and drainage. When the water table rises to a depth less than 2 m from the soil surface, salt concentrations are further increased by evaporation and damage to vegetation and soils are very likely.

The distribution of groundwater salinity concentrations are highly variable whether in a vertical direction or along the horizontal plane. In the Shepparton formation aquifer, which is the upper geological formation, the salinity is generally greater than that in the underlying Deep Lead formation. In this aquifer, which is mainly composed of gravels and sands, the water circulates more easily than in the Shepparton formation, which is composed mainly of clays. Salinity concentrations range between 300 and 1500 ppm TDS in the Deep Lead and between 500 and 20 000 ppm TDS in the Shepparton formation. Pumping in the Deep Lead formation has been recommended (Reid, 1988) as it will contribute to the lowering of the shallow water table in the Shepparton formation thus providing better drainage for the groundwaters in the basin. Furthermore, Nolan and Reid (1989) studied the salt redistribution resulting from pumping and re-use of groundwaters in the Deep Lead formation and concluded that further work is needed to obtain the 'best policy' for groundwater extraction from the Deep Lead formation in order to avoid any degradation in the quality of the groundwater.

Experience from pumping in the Deep Lead formation during the past few years has produced contradictory results as far as the time variation of the salinity of the pumping water is concerned (Reid, 1988): in some cases (Rochester) a systematic improvement in groundwater salinity has been associated with pumping, while in

some others (Loddon and Campaspe) pumping has led to a deterioration in the groundwater quality. These different behaviours are mainly related to the initial distribution of salinity around the pumping wells and the mechanism of saline redistribution during pumping. In all these cases, both advection and dispersion phenomena are important for the redistribution of salinity in the vertical as well as in the horizontal planes.

Control and management of the salinity in the basin is a complex process, involving several steps and actions, such as the evaluation of the present situation, mathematical modelling and the definition of various water disposal and treatment strategies. For salinity control plans, pumping in the Deep Lead formation, use and disposal of groundwaters, water treatment and mixing between waters of different salinity are some of the available options in a multi-objective optimisation and decision process. An important component of the whole process is the mathematical modelling and computer simulation of the fate of salt concentrations due to pumping in the Deep Lead formation.

Different models and modelling techniques have been used so far in the Campaspe river basin, to account mainly for the time variation of the piezometric head at various locations in the aquifer. Williamson (1984) developed a lump-type hydrodynamic model of the Campaspe Irrigation District, using a rather rough grid discretisation of the aquifer in space. After calibration of the model, the maximum allowable pumping rate was estimated. An increase of 1 to 7% per year in the deep aquifer was predicted by the same study, using an elementary salinity model. Chiew and McMahon (1990) used the AQUIFEM-N model in order to take into account both the surface hydrological processes and the groundwater flow in the Campaspe valley in an integrated framework. Emphasis was given to the hydrodynamics of the processes rather than to the quality of the groundwater.

The objectives of the present study are as follows:

- (1) To develop a relatively local scale mathematical model for simulating solute transport and dispersion near pumping bores in the Deep Lead formation.
- (2) To understand and quantitatively explain the apparently contradictory observations concerning the rate of variation of salinity associated with pumping in the Deep Lead formation.
- (3) To demonstrate the possibility of reliable computer simulations of transient salinity fronts, using a random-walk-based computerised mathematical model.

The model was developed in a vertical cross-section and can be readily extended into 3-D space by using similar algorithmic expressions.

6.3.1

The Study Area

The Campaspe river in north-central Victoria is one of the tributaries of the Murray river. It flows northwards from Lake Eppalock to the Murray river, in a basin of approximately 2100 km² (Figure 6.30).

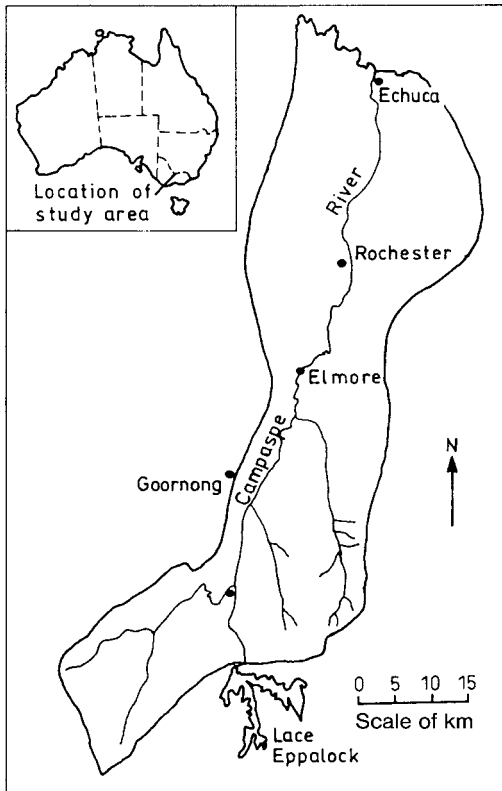


Figure 6.30 Location of the Campaspe river.

Two major aquifers are found in the Campaspe valley:

- (1) the Shepparton formation and
- (2) the Deep Lead formation.

The Shepparton formation aquifer is a sedimentary geological formation, mainly composed of clays. It extends vertically from the actual ground level to variable depths of about 70 m at Rochester and 55 m at Elmore. Hydraulically, the Shepparton formation behaves mainly as a phreatic aquifer in the north and partly as a semi-confined aquifer in the south. Hydraulic conductivities range from 25 to 55 m/day, specific yields from 0.02 to 0.2 and vertical conductivities from 0.001 to 0.03 m/year/m.

The Deep Lead formation extends vertically below the Shepparton formation to the Paleozoic bedrock, which, although capable of transmitting water within the fractures, is not active because the water pressures are almost the same as in the Deep Lead formation. The thickness of the Deep Lead formation is about 40 m at Rochester and 30 m at Elmore. It is composed mainly of gravels and sands and has up to four times greater hydraulic conductivity than the Shepparton formation. Values of hydraulic conductivity in the Deep Lead formation range from 25 to 200 m/day, with a typical value of 130 m/day in the south and lower values in the north. The Deep

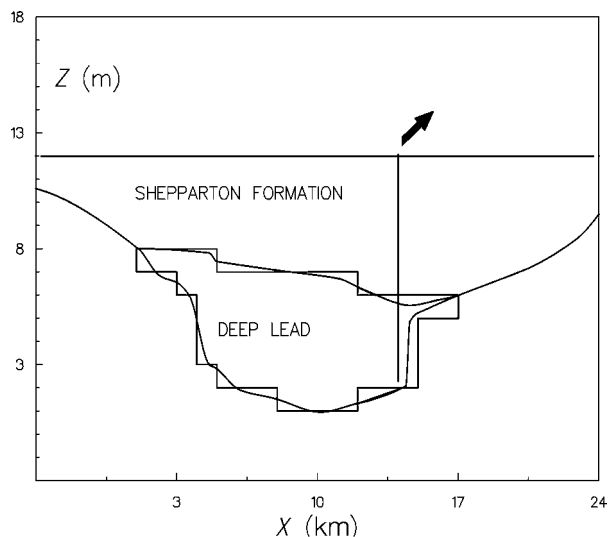


Figure 6.31 Typical geological cross-section.

Lead formation starts at Axedale, becoming larger and deeper from the south to the north. It varies from confined to semi-confined, with storage coefficients ranging from 10^{-4} to 10^{-2} . A typical geological cross-section of the Campaspe river basin is shown in Figure 6.31.

The study area was located in the Rochester irrigation district (Figure 6.30). Three cross-sections were chosen for mathematical modelling and computer simulations. Most of the computer runs were developed in the cross-section where the main production bore in the Deep Lead formation is located (Figure 6.31). Data for the salinity variation in this bore (RW10032-Houlihan) cover the period 1982–1990. As indicated in Figure 6.32 the data show a reduction in the observed salinity with time.

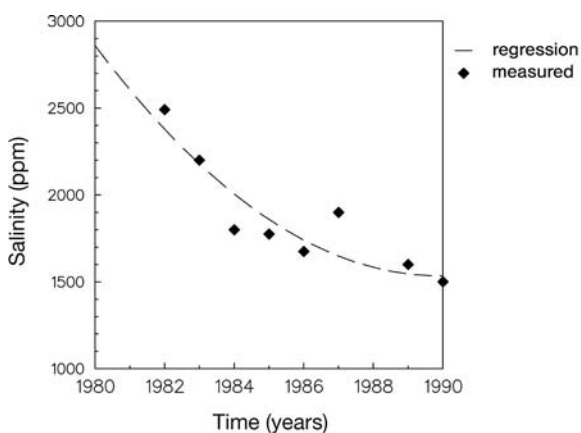


Figure 6.32 Time evolution of salinity levels in the RW10032 (Houlihan) bore.

This rather unexpected result seems to be systematic, as shown in Figure 6.32, where a polynomial regression of the data versus time was applied. It can be seen that the rate of decrease of the saline concentration associated with pumping is reduced with time and there is a tendency to reach an asymptotic limit. An explanation for this behaviour by mathematical modelling was the principal aim of this study.

6.3.2

Risk of Salinisation

6.3.2.1 Groundwater Hydrodynamics

Assuming a quasi-horizontal regional groundwater flow driven by gradients of total hydraulic head H , the application of the mass balance and Darcy's law in a semi-confined aquifer of constant porosity, gives the following partial differential equation

$$\frac{\partial H}{\partial t} = \nabla(T\nabla H) + k(H_0 - H) - q \quad (6.4)$$

where H is the total head, $T = T(x, y)$ is the transmissivity, k is vertical conductance, H_0 , the constant total head in the upper aquifer formation and q , the pumping rate in $\text{m}^3/\text{s}/\text{m}^2$.

By using over-relaxation and steady-flow conditions the piezometric head variation is shown in Figure 6.33 for $q = 10 \text{ l/s}$ and 100 l/s . Using the computed velocity fields, the salinity intrusion with pumping was simulated by developing a random walk numerical code (Ganoulis, 1993).

Taking into account the salinity distributions of the 1986 and 1987 data, the following initial distribution was assumed for 1982 (Figure 6.34): 2550 ppm around the well and 550 ppm in the remainder of the cross-section.

6.3.2.2 Random Walk Simulation

According to the random walk principle the probability of finding a particle at a given position after time Dt follows a Gaussian law of mean value 0 and variance $s^2 = 2DtD$, where D is the dispersion coefficient.

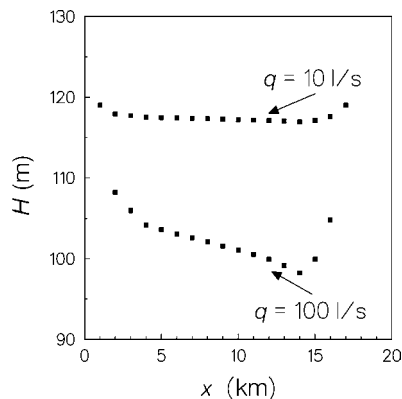


Figure 6.33 Piezometric head drawdown in the Deep Lead for $q = 10 \text{ l/s}$ and $q = 100 \text{ l/s}$.

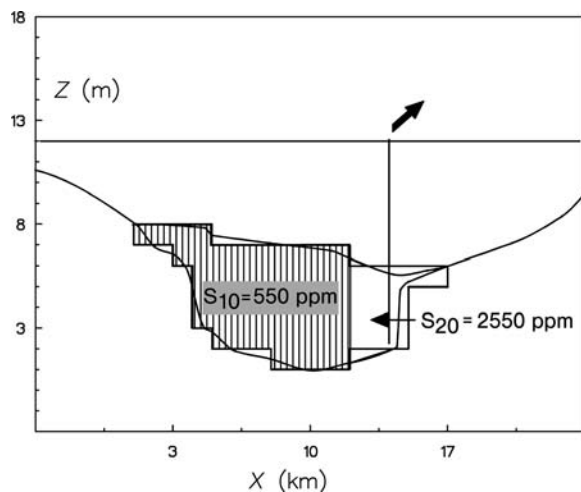


Figure 6.34 Assumed initial salinity levels for 1982.

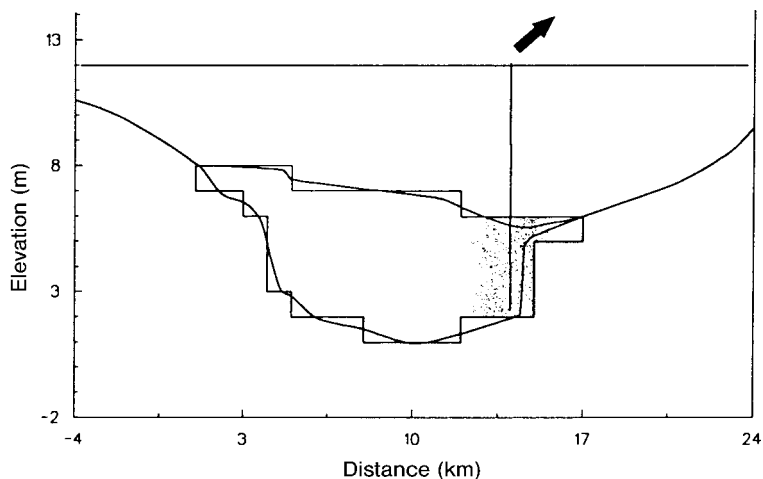


Figure 6.35 Particle concentrations after $t = 1$ year.

The probability of reaching a given grid cell and consequently the particle concentrations, can be evaluated by counting the number of particles which fell within the grid square.

The final salinity distribution was obtained by superposition of the value of 550 ppm salinity (initial difference). Figures 6.35 to 6.37 show the particle concentrations after 1, 2 and 7 years, as predicted by simulation of pumping for 6 months per year. Values of the dispersion coefficients are $Dx = 0.8 \times 10^{-3} \text{ m}^2/\text{s}$ and $Dy = 10^{-5} \text{ m}^2/\text{s}$. The respective salinity contours are shown in Figures 6.38 to 6.40 (Ganoulis, 1993).

The comparison between simulated results over 8 years and the data are presented in Figure 6.41. It is remarkable that good agreement was obtained with realistic

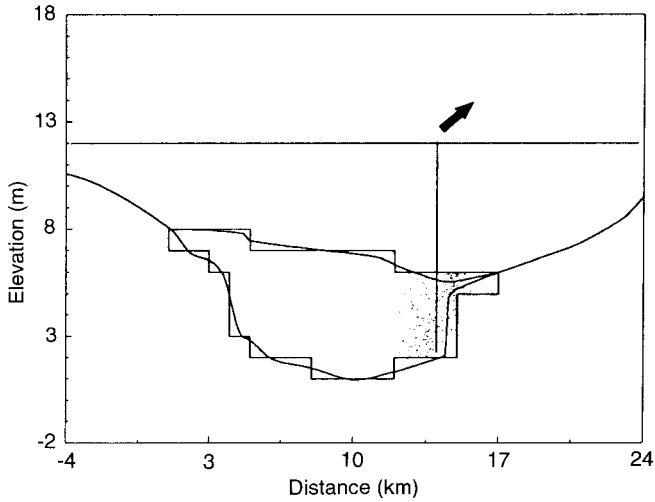


Figure 6.36 Particle concentrations after $t = 2$ years.

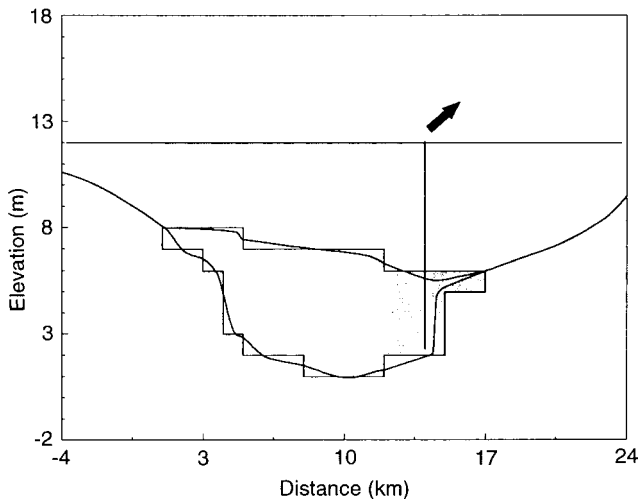


Figure 6.37 Particle concentrations after $t = 7$ years.

values of the dispersion coefficients and assuming the most probable initial condition regarding salinity distribution.

From the previous study of the Campaspe aquifer, the following general conclusions can be drawn:

- (a) For the assessment of risk of groundwater pollution and the design of groundwater development plans, reliable numerical simulation methods are needed. The risk of groundwater contamination may be quantified and the probability of meeting water quality standards can be determined by using Lagrangian

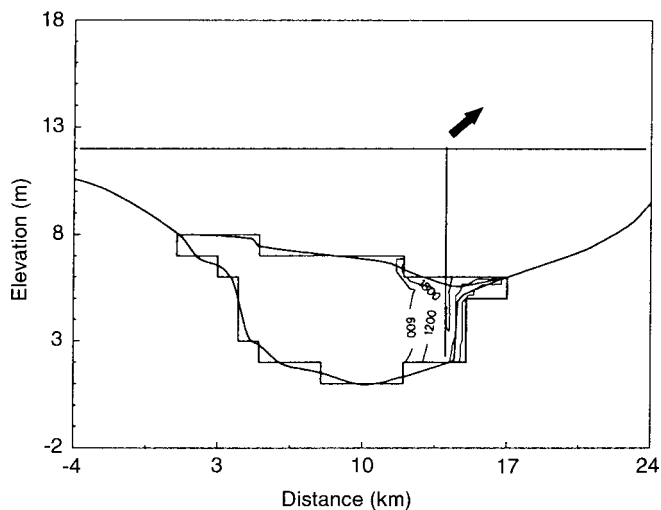


Figure 6.38 Salinity contours after $t = 1$ year.

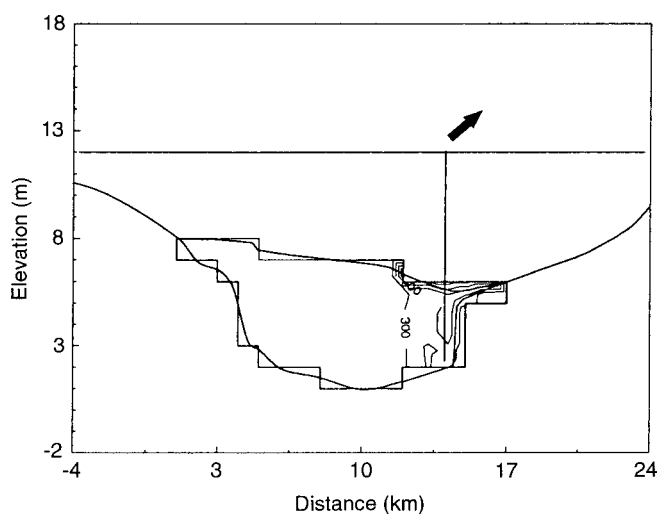


Figure 6.39 Salinity contours after $t = 2$ years.

algorithms based on random walk simulation. This methodology was applied in a case study in Victoria, Australia.

- (b) It was recommended that pumping should take place in the Deep Lead formation, which would contribute to lowering the shallow water table in the Sheperton formation and so provide better drainage of the groundwater and reduce the risk of salinisation. In order to understand and explain these recommendations quantitatively, a computerised mathematical model was developed, based

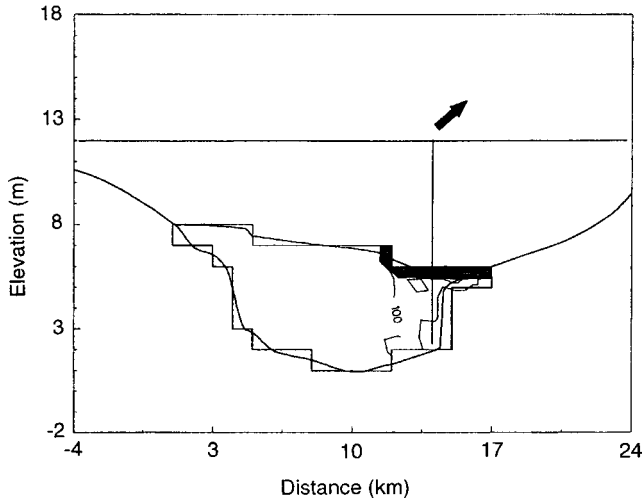


Figure 6.40 Salinity contours after $t = 7$ years.

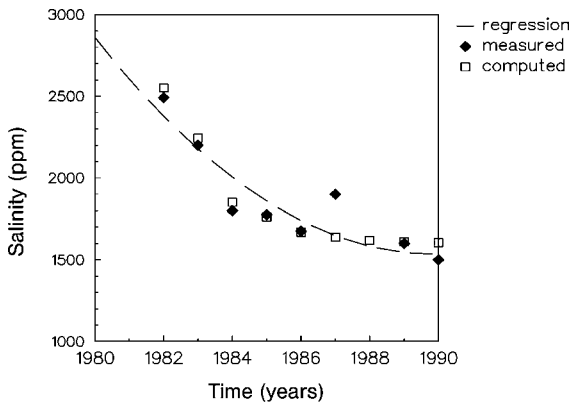


Figure 6.41 Comparison between measured and simulated results.

on random walk simulations. The model was applied in the RW10032 pumping bore (Houlihan), and the time evolution function of the salinity level was successfully simulated.

- (c) Further work comprised a sensitivity analysis of the computerised mathematical model and better validation of the results in other pumping wells. An extension into 3-D space would provide a powerful tool for the management and control of salinity in the Campaspe river basin.

Appendix A

The Probabilistic Approach

A.1

Basic Probability

According to the French mathematician Laplace, the ‘probabilistic approach is nothing more than the translation of common sense into calculus’. We will see in the next part of this appendix that the fuzzy set theory is also a mathematical approach to our everyday approximate reasoning.

Let us define as an *experiment* any observation or trial and as an *event* any possible outcome or result of an experiment. To account for probabilities first the *sample* space, or the *universal* set Ω , should be defined. This is the collection of all possible events. Every point belonging to Ω represents an elementary event.

Example A.1

Take the daily precipitation i as the total amount of rainfall water measured at one meteorological station. Every specific value of i is a positive number expressed, for example, in millimetres per day (mm/day). This number is one particular realisation or an elementary event belonging to Ω , which means here the set of all possible results of measurements, or all real numbers. It is shown in Figure A.1 by a single point (+) belonging to Ω . Take 30 such numbers, say the observations of rainfall during 1 month. This is a set A belonging to Ω , as shown in Figure A.1. If we consider the event ‘precipitation less than i_0 (mm/day)’ then the set of all real numbers less than i_0 is the subset B of Ω , as shown in Figure A.1.

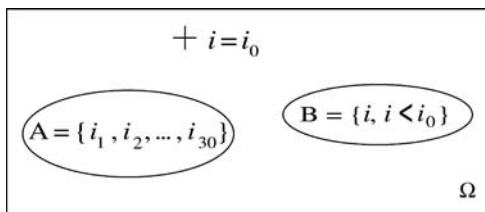


Figure A.1 Universal set Ω of daily precipitation i and particular realisations i_0 , A and B.

Example A.2

Instead of daily values of rainfall, consider two characteristic events:

- (1) D: dry day. This is when the daily rainfall is less than a certain amount.
- (2) W: wet day. This is when the daily rainfall is more than a certain amount.

It is obviously impossible to have a D and W day at the same time so in this case, the universal set is

$$\Omega = \{D, W\}$$

Example A.3

Consider now all possible pairs of two consecutive days DD, DW, WD, WW. Then, the universal set is given by

$$\Omega = \{DD, DW, WD, WW\}$$

The probability p_A of an event A is defined as the frequency of its occurrence during the repetition of a number of experiments. If N is the total number of times the experiment is performed and N_A the number of realisations of the event A, then p_A is the limit of the observed frequency N_A/N , as the number of experiments is repeated theoretically an infinite number of times

$$p_A = \lim_{N \rightarrow \infty} \frac{N_A}{N} \quad (\text{A.1})$$

If A and B are two events, then

- their union $A \cup B = (A \text{ or } B)$ occurs when A occurs, B occurs or both A and B occur;
- their intersection $A \cap B = (A \text{ and } B)$ occurs when both A and B occur simultaneously.

Relations between events are graphically presented in the form of Venn diagrams (after John Venn, 1834–1883), as in Figure A.2.

Events A and B are said to be *mutually exclusive* or *disjoint*, when the occurrence of one excludes the occurrence of the other (Figure A.3). In other words, the intersection of two mutually exclusive events A and B is the empty set, or $A \cap B = \emptyset$, where \emptyset is the empty set.

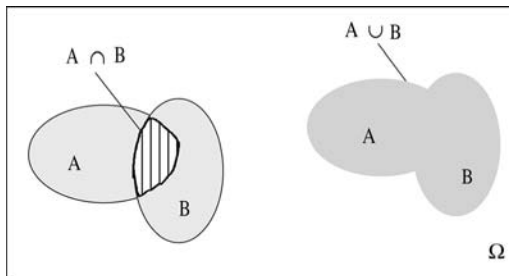


Figure A.2 Intersection $A \cap B$ and union $A \cup B$ of two probabilistic events A and B.

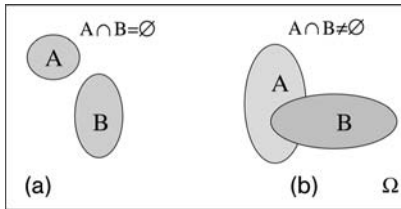


Figure A.3 Mutually exclusive or disjoint (a) and conditional or joint events (b).

Two events or sets may have common points, in which case the intersection is not empty, that is

$$A \cap B \neq \emptyset$$

As shown in Figure A.3, two non-mutually exclusive events have a common area. These are called *joint* or *conditional* events.

To compute the probability of events that are either dependent or independent in probability terms, first the basic rules for computing probabilities should be defined. There are three basic rules, or *axioms*, on which the calculus of probabilities is based (Papoulis, 1965; Ang and Tang, 1975)

$$(i) \ p_A = P(A) \geq 0 \quad (A.2)$$

$$(ii) \ P(\Omega) = 1 \quad (A.3)$$

$$(iii) \ P(A \cup B) = P(A) + P(B), \text{ if } A \text{ and } B \text{ are mutually exclusive} \quad (A.4)$$

Rules (i) and (ii) mean that probabilities are positive numbers between 0 and 1. In fact, for normalising the extent of probability variation, 1 is taken by definition as the probability of the universal set (i.e. when certainty and no doubt apply). The third rule concerns the probability of the union of two *mutually exclusive* events, which means that the event $(A \cup B) = (A \text{ or } B)$ occurs when either A or B occurs but not both simultaneously.

Using rules (ii) and (iii) for this case, we obtain the expression

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (A.5)$$

What is important to evaluate the so-called *joint probability*, that is

$$P(A \cap B) = P(A \text{ and } B) = \text{probability that both } A \text{ and } B \text{ occur.}$$

A.2 The Multiplicative Law

Let $P(B/A)$ represent the probability of event B, given that event A has already occurred. $P(B/A)$ is the *conditional probability* of B given that A occurred. From the basic rules the following relationship can be derived

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Similarly, the conditional probability of A given that B occurred can be deduced from the relationship

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

The above two relationships are applications of the *multiplicative probability law*, which can be written as

$$\begin{aligned} P(A \cap B) &= P(A)P(B/A) \\ &= P(B)P(A/B) \end{aligned} \quad (A.6)$$

A.3

Statistical Independence

Two events are statistically independent when the occurrence of one does not affect the probability of the occurrence of the other. This means that

$$P(A/B) = P(A) \text{ and } P(B/A) = P(B) \quad (A.7)$$

From Equation A.6 it follows that, for the statistically independent events A and B, we have

$$P(A \cap B) = P(A)P(B) \quad (A.8)$$

This result can be generalized for *n*-independent events as follows

$$P(A_1 \cap A_2 \dots \cap A_n) = P(A_1)P(A_2), \dots P(A_n) \quad (A.9)$$

A.4

Rare Events

If A and B are rare events, that is if the probabilities of A and B are very small and they are independent (but not necessarily mutually exclusive), then by applying Equation A.5 we have

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= P(A) + P(B) - P(A)P(B) \\ &\cong P(A) + P(B) \end{aligned}$$

The above result means that rare independent events behave approximately as if they were also mutually exclusive.

Example A.4

During a year of daily observations (360 measurements), the water quality of a river at a given station is defined as follows:

- D_i : at day i the water is *dirty*, because the concentration of a pollutant exceeds a certain value.
- C_i : at day i the water is *clear*, because the concentration of a pollutant is lower than this value.

Results of observations concerning two consecutive days are summarised in Table A.1.

The universal space Ω of two consecutive days is

$$\Omega = \{CC, CD, DC, DD\}$$

From the table it can be seen that over the year there are $90 + 126 = 216$ days of type C and $126 + 18 = 144$ days of type D, no matter what the type of the previous day was. This means that the probabilities of having a day C or a day D are

$$P(C) = \frac{216}{360} = 0.6 \quad \text{and} \quad P(D) = \frac{144}{360} = 0.4$$

Given that a day i is D, now determine the probability of the following day $i + 1$ being either of type C or type D. We have

$$P(C_{i+1}/D_i) = \frac{P(C_{i+1} \text{ and } D_i)}{P(D_i)} = \frac{126/360}{0.4} = \frac{0.35}{0.40} = 0.875$$

and

$$P(D_{i+1}/D_i) = \frac{P(D_{i+1} \text{ and } D_i)}{P(D_i)} = \frac{18/360}{0.4} = \frac{0.05}{0.40} = 0.125$$

It is easy to verify that the two events C_{i+1}/D and D_{i+1}/D are complementary, that is

$$P(C_{i+1}/D_i) + P(D_{i+1}/D_i) = 1$$

In fact, for every type of day i (D or C), it is certain that the following one should be either C_{i+1} or D_{i+1} .

Table A.1 Observed consecutive day pairs CC, CD, DC and DD.

i	$i + 1$	
	C_{i+1}	D_{i+1}
C_i	90	126
D_i	126	18

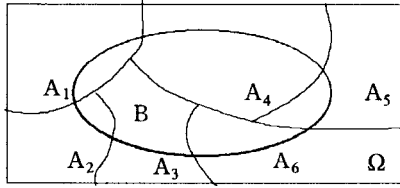


Figure A.4 Graphical representation of the theorem of total probability.

A.5

Theorem of Total Probability

Take now a set of mutually exclusive events $A_1, A_2, A_3, \dots, A_n$ such that their union gives the universal set Ω , that is

$$\bigcup_{i=1}^n A_i = \Omega$$

As shown in Figure A.4, an event B can be defined as the union of the following mutually exclusive events

$$B = (B \cap A_1) + (B \cap A_2) + \dots + (B \cap A_n)$$

Note that the above expression is also valid in cases where the union of A_i does not equal the whole sample space Ω .

By using the expressions shown in Equations A.4 and A.6 the *theorem of total probability* can be derived in the form

$$\begin{aligned} P(B) &= P(B/A_1)P(A_1) + P(B/A_2)P(A_2) + \dots + P(B/A_n)P(A_n) \\ &= \sum_{i=1}^n P(B/A_i)P(A_i) \end{aligned} \quad (\text{A.10})$$

A.6

Bayes' Theorem

From the theorem of total probability (Equation A.7) and the multiplicative law the well known Bayes' theorem can be obtained as follows

$$P(A_i/B) = \frac{P(A_i \cap B)}{P(B)} = \frac{P(A_i)P(B/A_i)}{\sum_{i=1}^n P(A_i)P(B/A_i)} \quad (\text{A.11})$$

In the above relationship A_1, A_2, \dots, A_n are mutually exclusive events and, at the same time, complementary, that is

$$\sum_{i=1}^n P(A_i) = 1$$

The physical interpretation of Equation A.11 is the following. To derive an estimate of the *conditional a posteriori* probabilities

$$P(A_1/B), P(A_2/B), \dots, P(A_i/B)$$

the *a priori* probabilities

$$P(A_1), P(A_2), \dots, P(A_i)$$

are introduced into Equation A.11 and are computed in terms of conditional probabilities

$$P(B/A_1), P(B/A_2), \dots, P(B/A_i)$$

Example A.5

The water quality of a river has been classified into two groups

$$-A_1 : \text{good} \quad -A_2 : \text{poor}$$

Considering different data, such as those on pollutant sources, agricultural activities in the river valley and the geology of the region, a *prior* estimation of the water quality in groups A_1 and A_2 gave the following probabilities

$$P(A_1) = 0.6 \quad P(A_2) = 0.4$$

A more quantitative result for water quality may be obtained by means of specific chemical analyses. A chemical water quality index I is then derived and used to classify the water quality into two groups I_1 and I_2 . Given that, in reality, the water quality belongs to the group A_j , the following table summarises the probabilities of finding a water quality index I_i (Table A.2).

Table A.2 Values of conditional probabilities $P(I_i/A_j)$.

A_j	I_i	
	1	2
1	0.7	0.3
2	0.1	0.9

In fact the values of $P(I_i/A_j)$ in the table are measures of the performance of the chemical quality index I : the closer the values of probabilities $P(I_i/A_j)$, $i=j$, are to 1, and those of (I_i/A_j) , $i \neq j$, are to 0, the more reliable the chemical index I_i is to represent water quality A_i . Note also the complementarity of the events (I_1/A_j) and (I_2/A_j) , that is

$$\sum_i I_i/A_j = 1 \quad i = 1, 2$$

Suppose now that, after sampling, a complete set of chemical analyses is carried out. There are two possible cases:

(1) The chemical water quality index takes the value I_1 .

The posteriori probabilities $P(A_1/I_1)$ and $P(A_2/I_1)$ can be computed by means of Bayes' theorem given by Equation 2.13. The results are

$$P(A_1/I_1) = \frac{P(A_1)P(I_1/A_1)}{\sum_i P(A_i)P(I_1/A_i)} = \frac{(0.6) \times (0.7)}{(0.6) \times (0.7) + (0.4) \times (0.1)} = 0.91$$

$$P(A_2/I_1) = \frac{P(A_2)P(I_1/A_2)}{\sum_i P(A_i)P(I_1/A_i)} = \frac{(0.6) \times (0.1)}{(0.6) \times (0.7) + (0.4) \times (0.1)} = 0.09$$

Comparing these results with the prior estimations: $P(A_1) = 0.6$ and $P(A_2) = 0.4$ it can be concluded that the water in the river is of better quality than estimated, because $P(A_1/I_1) > P(A_1)$.

(2) The chemical water quality index takes the value I_2 .

The posteriori probabilities $P(A_1/I_2)$ and $P(A_2/I_2)$ are also computed by means of the Bayes' theorem (Equation 2.13) and the following results are obtained

$$P(A_1/I_2) = \frac{P(A_1)P(I_2/A_1)}{\sum_i P(A_i)P(I_2/A_i)} = \frac{(0.6) \times (0.3)}{(0.6) \times (0.3) + (0.6) \times (0.9)} = 0.33$$

$$P(A_2/I_2) = \frac{P(A_2)P(I_2/A_2)}{\sum_i P(A_i)P(I_2/A_i)} = \frac{(0.4) \times (0.9)}{(0.6) \times (0.3) + (0.4) \times (0.9)} = 0.67$$

Comparing these results with the prior estimations: $P(A_1) = 0.6$ and $P(A_2) = 0.4$ it can be concluded that the water in the river is of poorer quality than estimated, because $P(A_1/I_2) < P(A_1)$.

One sampling is not sufficient to obtain a good estimation. At least two samples should be analysed and Bayes' rule applied again.

A.7

Random Variables

At every sample point $\omega \in \Omega$ a specific number x can be assigned. This is a value of the *random variable* $X(\omega)$ which is defined as a function mapping Ω into the set of real numbers R (Figure A.5), that is

$$X : \Omega \rightarrow R$$

The function $x = X(\omega)$ is not necessarily a one-to-one representation of the universal space but, as shown in Figure A.5, several points ω in the subset A may correspond to one value $x = X(\omega)$.

Let R_x denote the *range* of the random variable X , that is

$$R_x = \{x : x = X(\omega)\} \quad (\text{A.12})$$

R_x is called the *range space*.

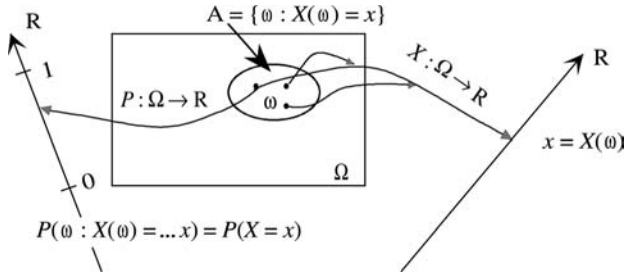


Figure A.5 Definition of a random variable as function $x = X(\omega)$ and probability $P(A) = P(X = x)$.

The probability $p(x) = P(X = x)$ is assigned as the probability of the subset A whose image is x , that is

$$p(x) = P(X = x) = P\{x : x = X(\omega)\} \quad (\text{A.13})$$

As far as notation is concerned, capital letters refer to random variables and lower case letters to the values of random variables, that is, real numbers.

A.7.1

Discrete Random Variables

Let suppose that we are concerned with an experiment whose outcomes are discrete values x_1, x_2, \dots , of a random variable X with the probabilities

$$P(X = x_i) = p(x_i) = p_i \quad i = 1, 2, 3, \dots, n \quad (\text{A.14})$$

The function assigning $p(x_i)$ to x_i , or the set of ordered pairs $(x_i, p(x_i))$, is called the *probability distribution* or probability mass function of the discrete random variable X .

Example 2.6

- (a) Let a coin and a die be tossed and the side appearing on top observed. In the case of the die, the sample space is composed of all possible outputs, $\Omega = \{1, 2, 3, 4, 5, 6\}$. When the coin is tossed, the sample space consists of two possible outcomes, that is $\Omega = \{H, T\}$ with H = head and T = tail.

We can see that sample points ω may correspond to numerical or non-numerical outputs of an experiment.

Now a random variable X is introduced taking values $x_i = 1, 2, 3, 4, 5, 6$ for the die and $x_i = 0, 1$ for the coin. According to the definition given in Equation A.12, the range of values is $R_x = \{1, 2, 3, 4, 5, 6\}$ for the die and $R_x = \{0, 1\}$ for the coin. Here there is a one-to-one correspondence between the sample points ω and the values x_i . In the case of a fair coin and die the probabilities are uniformly distributed, that is

$$P(X = x_i) = 1/2 \text{ for the coin } (i = 1, 2) \quad \text{and}$$

$$P(X = x_i) = 1/6 \text{ for the die } (i = 1, 2, 3, 4, 5, 6)$$

The corresponding probability distributions are shown in Figure A.6.

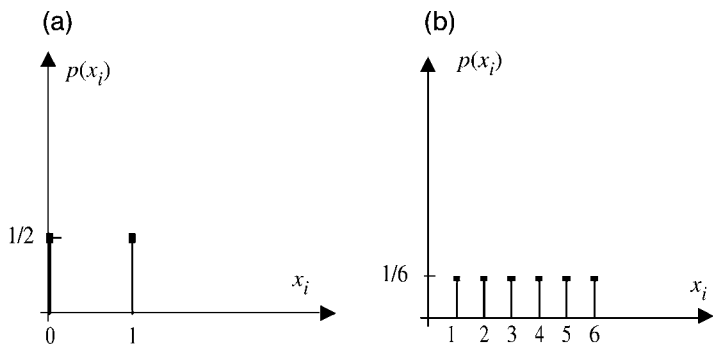


Figure A.6 Probability distribution functions from tossing a coin (a) and a die (b).

(b) A fair pair of dice is tossed. The sample space W consists of all possible pairs of values between 1 and 6. A total of 36 ordered pairs result in the form

$$W = \{(1, 2), (1, 3), (1, 4), \dots, (6, 6)\}$$

X is the random variable that assigns to each point of W the sum of the observed two numbers. The range space of X is thus

$$R_x = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

It can be seen that there is no one-to-one correspondence between W and R_x ; only one point $(1, 1)$ exists whose image is 2. There are two points $(1, 2)$ and $(2, 1)$ corresponding to 3, and three points $(1, 3)$, $(3, 1)$ and $(2, 2)$ having image 4. Concerning the corresponding probabilities, if the number of all possible outcomes is 36, then $P(X=2) = 1/36$, $P(X=3) = 2/36$ and $P(X=4) = 3/36$. The pairs of values $x_i, p(x_i)$ are given in Table A.3 and the probability distribution is shown in Figure A.7.

Table A.3 Probability distribution from tossing a pair of dice.

x_i	2	3	4	5	6	7	8	9	10	11	12
$p(x_i)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

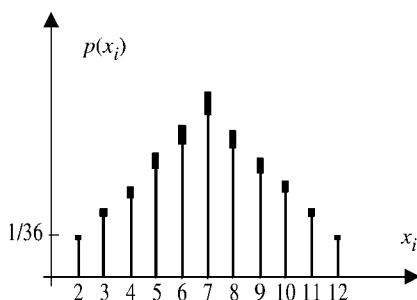


Figure A.7 Probability distribution function from tossing a pair of dice.

The probability function is subject to the following conditions

$$0 \leq p(x_i) \leq 1 \quad \sum_i p(x_i) = 1 \quad (\text{A.15})$$

The cumulative distribution function $F(x)$ of a discrete random variable is defined as follows

$$F(x) = \sum_{x_i \leq x} p(x_i) \quad (\text{A.16})$$

A useful property of $F(x)$ is that the probability of X being between x_i and $x_j > x_i$ is equal to the difference between values $F(x_j)$ and $F(x_i)$, that is

$$P(x_i \leq X \leq x_j) = F(x_j) - F(x_i) \quad (\text{A.17})$$

A.7.2

Continuous Random Variables

When the values of a random variable X are continuous, then we can compute the probability for x to lie within the elementary interval Δx as follows

$$P(x \leq X \leq x + \Delta x) = f_X(x) \Delta x \quad (\text{A.18})$$

The function $f_X(x)$ is called the *probability density function* of the continuous random variable X . It is a function which by integration gives a probability, and hence it is similar to the probability or mass distribution function of a discrete random variable. As in the case shown by Equation A.15, the probability density function is subject to the following conditions

$$f_X(x) \geq 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} f_X(x) dx = 1 \quad (\text{A.19})$$

The probability distribution, or cumulative distribution function, is obtained by integrating the probability density function as follows

$$F_X(x) = P(X < x) = \int_{-\infty}^x f_X(x) dx \quad (\text{A.20})$$

Equation A.20 may be differentiated to yield

$$f_X(x) = \frac{dF_X(x)}{dx} \quad (\text{A.21})$$

The general form of probability density and probability distribution functions is given in Figure A.8.

A.8

Expectation, Variance and Standard Deviation

The *expectation*, or *expected value*, or *mean value*, or *average* of a random variable X can be defined to be

$$E(X) = \langle X \rangle = \mu_X = \bar{X} = \sum_i x_i p(x_i) \quad i = 1, 2, \dots, k \quad (\text{A.22})$$

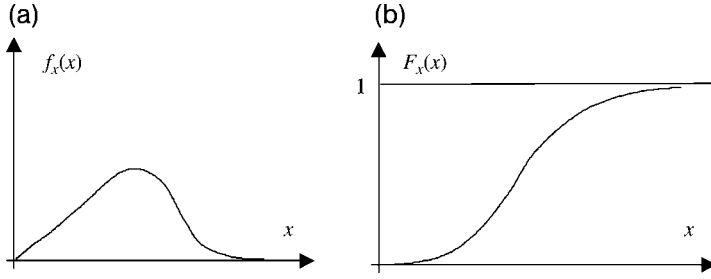


Figure A.8 Probability density (a) and probability distribution (b) functions.

for a discrete variable and

$$E(X) = \langle X \rangle = \mu_X = \bar{X} = \int_{-\infty}^{+\infty} x f(x) dx \quad (\text{A.23})$$

for a continuous random variable.

The *variance* of a random variable X is the degree of dispersion of the values x about its mean or expected value. For a discrete variable we have

$$\text{Var}(X) = \sum_i (x_i - \bar{X})^2 p(x_i) \quad i = 1, 2, \dots, k \quad (\text{A.24})$$

and for a continuous variable

$$\text{Var}(X) = \int_{-\infty}^{+\infty} (x - \bar{X})^2 f(x) dx = \langle X^2 \rangle - \langle X \rangle^2 \quad (\text{A.25})$$

The *standard deviation* σ_X is defined as the positive square root of the variance

$$\sigma_X = \sqrt{\text{Var}(X)} = \sqrt{\langle X^2 \rangle - \langle X \rangle^2} \quad (\text{A.26})$$

A.9

Derived Distributions

Let X and Y be two random variables with probability density functions $f_X(x)$ and $f_Y(y)$. As shown in Figure A.9, we suppose that Y is related to X by the functional relationship

$$Y = g(X) \quad (\text{A.27})$$

The question is, how to derive the probability density function $f_Y(y)$ in terms of $f_X(x)$ and $g(x)$. As shown in Figure A.9 for the case of increasing function $y = g(x)$, we consider the fact that the probability of finding X in the interval $(x, x + dx)$ is equal to the probability of Y being between y and $y + dy$, where y is related to x by the relationship $y = g(x)$. By means of probability density functions this is written as

$$f_Y(y) \Delta y = f_X(x) \Delta x \quad (\text{A.28})$$

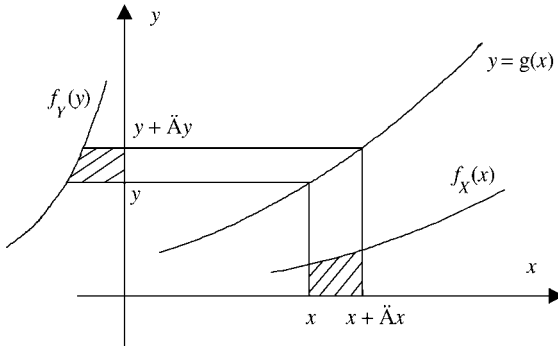


Figure A.9 Transformation of random variables.

As Δx and Δy tend to zero, the ratio $\Delta y/\Delta x$ tends to the derivative $g'(x) = dg/dx$. Taking into account the sign of this derivative in the case of a decreasing function $g(x)$, Equation A.28 yields the following general expression

$$f_Y(y) = \frac{1}{\left| \frac{dg(x)}{dx} \right|} f_X(x) \quad (\text{A.29})$$

Example 2.7

Let C be the mass concentration of a pollutant in a river. In a first approximation we assume that C is constant, that is, $C = C_0$. The flow rate of the river Q is assumed to follow a normal distribution with parameters Q_0 and σ_Q^2 . If $M = CQ$ is the mass rate of the pollutant, find the probability density function of M .

The probability density distribution of Q is $N(C, \sigma_Q^2)$. This notation means that

$$f_Q(q) = \frac{1}{\sigma_Q \sqrt{2\pi}} \exp\left(-\frac{(q - Q_0)^2}{2\sigma_Q^2}\right) \quad (\text{A.30})$$

By using Equation A.29 with $M = C_0 Q$ and $dM/dQ = C_0$, we find that M is also a normal variable with mean equal to $Q_0 \cdot C_0$ and variance $\sigma_M^2 = \sigma_Q^2 \cdot C_0^2$.

The distributions of Q and M are shown in Figure A.10, where $C = C_0 = 3$.

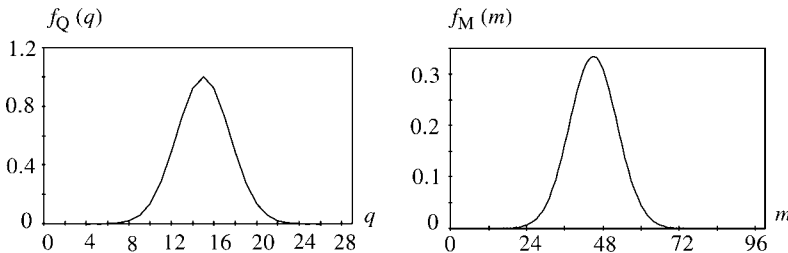


Figure A.10 Normal random variables Q and $M = CQ = C_0Q$ with $C_0 = 3$.

A.10**Two-dimensional Distributions**

A *joint probability density* of two random variables X and Y is a function which, by integration, yields the probability that X and Y are located in a given domain $(x_1, x_2) \times (y_1, y_2)$. This can be written as follows

$$P(x_1 \leq X \leq x_2, y_1 \leq Y \leq y_2) = \int_{x_1}^{x_2} \int_{y_1}^{y_2} f_{XY}(x, y) dx dy \quad (\text{A.31})$$

The function $f_{XY}(x, y)$ is subject to the following conditions

$$f_{XY}(x, y) \geq 0 \quad \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f_{XY}(x, y) dx dy = 1 \quad (\text{A.32})$$

The *cumulative distribution function* is defined as

$$F_{XY}(x, y) = \int_{-\infty}^x \int_{-\infty}^y f_{XY}(x, y) dx dy \quad (\text{A.33})$$

If X and Y are independent, then

$$f_{XY}(x, y) = f_X(x) f_Y(y) \quad (\text{A.34})$$

Two random variables X and Y may be considered as components of a random vector.

A.11**Functions of Random Vectors****A.11.1****Sum of Random Variables**

Let the random variable $Z = X + Y$. The probability distribution function of Z can be calculated as

$$F_Z(z) = P(Z < z) = P(X + Y < z)$$

In terms of the joint density distribution function of x and y we have

$$P(X + Y < z) = \iint_{x+y < z} f_{XY}(x, y) dx dy$$

As shown in Figure A.11, the integration should be extended over the lower left half-plane limited by the line $x + y = z$. In that domain we have $x + y < z$.

For fixed x we will integrate with respect to y over the vertical strip shown in Figure A.11 between $-\infty$ to $y = z - x$ and then over all x , so that

$$F_Z(z) = \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{z-x} f_{XY}(x, y) dy \right] dx$$

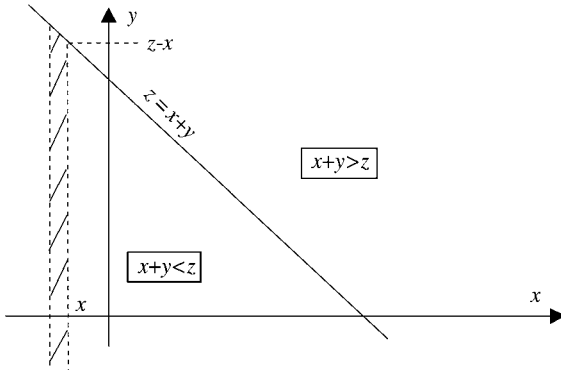


Figure A.11 Integration domain for the sum of two random variables.

Differentiating with respect to z under the first integral, the probability density function of Z is obtained as follows

$$\begin{aligned}
 f_Z(z) &= \frac{d}{dz} \left\{ \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{z-x} f_{XY}(x, y) dy \right] dx \right\} \\
 &= \int_{-\infty}^{+\infty} \left[\frac{d}{dz} \int_{-\infty}^{z-x} f_{XY}(x, y) dy \right] dx \\
 &= \int_{-\infty}^{+\infty} f_{XY}(x, z-x) dx
 \end{aligned}$$

This is the probability density function of the sum of two random variables. If X and Y are independent, this result reduces to the form

$$f_Z(z) = \int_{-\infty}^{+\infty} f_X(x) f_Y(z-x) dx \quad (\text{A.35})$$

The integral on the right-hand side is known as a *convolution* integral.

A.11.2

Difference of Random Variables

Let $Z = X - Y$ the difference of two random variables X and Y . The probability distribution function of Z can be calculated as

$$F_Z(z) = P(Z < z) = P(X - Y < z) = \iint_{x-y < z} f_{XY}(x, y) dx dy$$

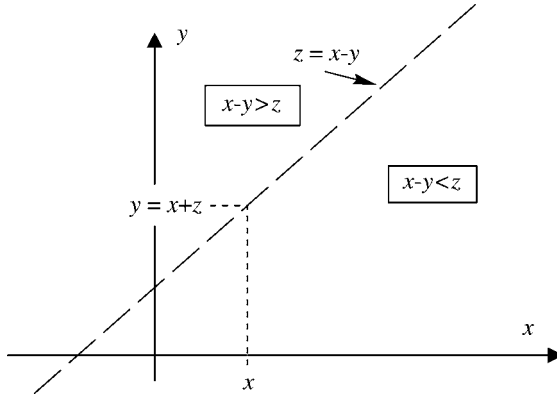


Figure A.12 Integration domain for the difference of two random variables.

As shown in Figure A.12, the integration domain is over the lower right half-plane limited by the line $z = x - y$. Integrating and then differentiating under the first integral we obtain

$$\begin{aligned}
 F_Z(z) &= \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{z+x} f_{XY}(x, y) dy \right] dx \\
 f_Z(z) &= \int_{-\infty}^{+\infty} f_X(x) f_Y(z+x) dx = \int_{-\infty}^{+\infty} f_Y(y) f_X(z+y) dy
 \end{aligned} \tag{A.36}$$

A.11.3

Product of Random Variables

If X and Y are independent random variables and have a continuous joint density, then the product $Z = XY$ also has a continuous cumulative distribution function. By integration over the domain shown in Figure A.13 and then differentiation under the integral, we obtain the following density distribution

$$f_Z(z) = \int_{-\infty}^{+\infty} f_X(x) f_Y\left(\frac{z}{x}\right) \frac{dx}{x} = \int_{-\infty}^{+\infty} f_Y(y) f_X\left(\frac{z}{y}\right) \frac{dy}{y}$$

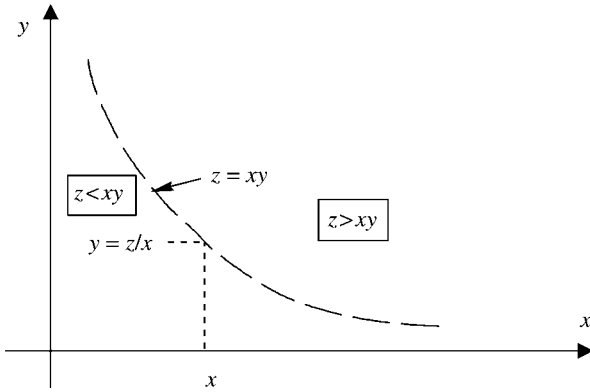


Figure A.13 Integration domain for the product of two random variables.

A.11.4

Ratio of Random Variables

If $Z = X/Y$ is the ratio of two independent random variables X and Y then we obtain

$$f_Z(z) = \int_{-\infty}^{+\infty} f_X(x) f_Y\left(\frac{x}{z}\right) |x| dx = \int_{-\infty}^{+\infty} f_Y(y) f_X(yz) |y| dy$$

Appendix B

The Fuzzy Set Theory

B.1

Basic Definitions

The uncertainty inherent in data, values of parameters, boundary conditions or variables used as inputs to mathematical models may be quantified by use of stochastic variables. As an example, let us consider the mortality of bacteria, which may be considered as a parameter useful to characterise the quality of a water sample. If the mortality has large values, then bacteria are eliminated and the water quality has a good chance of remaining acceptable. Mortality of bacteria is influenced by several factors, such as temperature, solar light, salinity and some biological characteristics. Usually, all these parameters are taken into consideration by means of the characteristic time t_{90} , that is the time necessary to eliminate 90% of bacteria. Because of the various uncertainties, t_{90} may be considered to be a random variable having a probability density distribution. As shown in Figure B.1, a log-normal probability density function may be used to fit the available data and represent uncertainties in the values of t_{90} .

Suppose now that the available data are not sufficient to fit a probability density distribution and that the information available is only scarce. We know, for example that t_{90} is greater than 0 h and that it is very unlikely to be greater than 25 h. Our

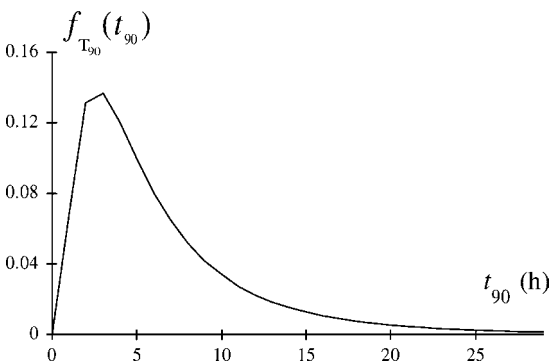


Figure B.1 Log-normal probability density distribution of t_{90} ($\mu = 1.6$, $\sigma = 0.8$).

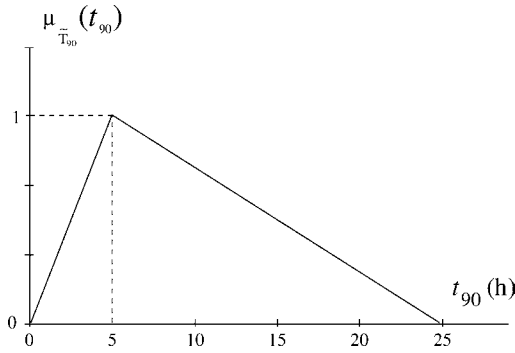


Figure B.2 Representation of t_{90} as a fuzzy number.

strongest belief is that t_{90} takes the value 5 h. This kind of information may be sufficient to represent t_{90} as a fuzzy number. As shown in Figure B.2, the fuzzy number \tilde{T}_{90} is composed of an interval of values between the minimum (0 h) and the maximum value (25 h) of t_{90} . At every point in the interval there is a corresponding value of function $\mu_{\tilde{T}_{90}}(t_{90})$. This function, called *membership function*, represents, within the interval (0,1), the degree of confidence one might have for a particular value of the fuzzy number. As shown in Figure B.2, the membership function has been chosen as triangular, with a maximum value corresponding to $t_{90} = 5$ h equal to 1, which is the maximum degree of confidence.

From the above it is possible to conclude that fuzzy numbers are equivalent to random variables, with membership functions corresponding to probability density functions. However, as we will see in the following, the basic rules of the arithmetic of fuzzy sets are different from those of probability theory. In a figurative manner, the difference in calculus between probabilities and fuzzy sets makes the two methodologies perpendicular to each other, rather than parallel.

B.2 Fuzzy Sets

Fuzzy set theory (Zadeh, 1965; Zimmermann, 1985) is a mathematical method used to characterise and quantify uncertainty and imprecision in data and functional relationships. Fuzzy sets are especially useful when the number of data is not sufficient to characterise uncertainty by means of standard statistical measures involving the estimation of frequencies (e.g. mean, standard deviation and distribution type).

Fuzziness represents situations where membership in sets cannot be defined on a yes/no basis, because the boundaries of sets are vague. The central concept of fuzzy set theory is the membership function which represents numerically the degree to which an element belongs to a set. In a classical or binary set, a sharp distinction exists between members and non-members of the set. In other words, the value of the membership function for each element in a classical or binary

is either 1, for members (those that certainly belong to it), and 0 for non-members (those that certainly do not). However, many of the concepts we commonly employ, such as the classes of water quality, or values of groundwater transmissivity, do not exhibit this characteristic. That is their members belong to these sets up to certain degree, which is expressed by a number between 0 and 1.

Since, in such cases, the transition from member to non-member appears gradual rather than abrupt, the fuzzy set introduces vagueness (with the aim of reducing complexity) by eliminating the sharp boundary which divides members of the class from non-members (Klir and Folger, 1988). Thus, if an element belongs to a fuzzy set to some degree, the value of its membership function can be any number between 0 and 1. When the membership function of an element may have values 0 or 1 only, the fuzzy set theory reduces to the classical set theory.

Consider Ω the *universal* or *reference* space consisting of points ω_i . We note this as follows

$$\Omega = \{\omega_1, \omega_2, \dots, \omega_i, \dots\}$$

As shown in Figure B.3, set membership in an ordinary set A is binary: an element either belongs to a set or does not. In a fuzzy set \tilde{F} , by contrast, there is some degree of membership associated with any element. The membership value $x = \mu_{\tilde{F}}(\omega)$ of an element can range from 0 to 1; the higher the membership value, the more the element belongs to the fuzzy set \tilde{F} . A fuzzy set then consists of a set of ordered pairs containing the element and its membership value.

Stated formally, a set \tilde{F} is called fuzzy in a universe Ω if it consists of ordered pairs such that

$$\tilde{F} = \{(\omega, \mu_{\tilde{F}}(\omega)) : \forall \omega \in \Omega; \mu_{\tilde{F}}(\omega) \in [0, 1]\} \quad (\text{B.1})$$

where $\mu_{\tilde{F}}(\omega)$ is the degree of membership of ω in the set \tilde{F} .

If ω were an ordinary or ‘crisp’ number, it would have had as membership function the set $\{0,1\}$. The extent of membership would be equal to 1, if ω were in A , and 0 if it were not. With fuzzy sets, therefore, it is possible to deal with set membership that can be decided only in a relatively more ‘vague’ manner.

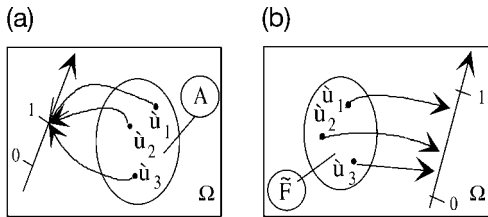


Figure B.3 An ordinary or binary set A (a) and a fuzzy set \tilde{F} (b).

Example B.1

Let the fuzzy set \tilde{F} be 'the water quality in a river' and be composed by three discrete elements 1, 2, 3 denoting the rating of the water quality in the river. The water quality has been classified into three classes

$$1 = \text{excellent} \quad 2 = \text{good} \quad 3 = \text{poor}$$

For example the water quality of the Axios river in Northern Greece, may have the membership function shown in Figure B.4, where

$$\mu_{\tilde{F}}(1) = 0.35, \quad \mu_{\tilde{F}}(2) = 0.84 \quad \text{and} \quad \mu_{\tilde{F}}(3) = 0.21$$

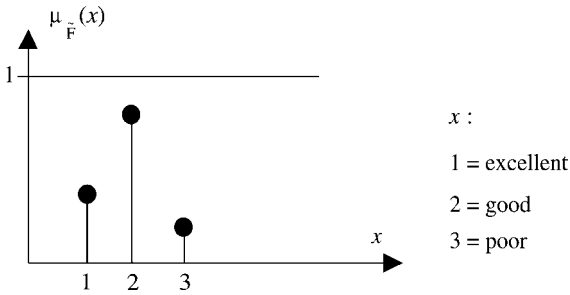


Figure B.4 Membership function of a discrete fuzzy set representing the water quality in the Axios river (N. Greece).

This is the case of a 'linguistic', or semantic, or non-numerical, fuzzy set with discrete membership function.

By means of the expression shown in Equation 2.42, the fuzzy set \tilde{F} may be written as follows

$$\tilde{F} = \{(1, 0.35), (2, 0.84), (3, 0.21)\}.$$

Example B.2

Let us define a fuzzy set \tilde{F} as: 'the pollutant concentration C is about an order of magnitude greater than 10 ppm'. This is the case of a continuous fuzzy set with membership function as shown in Figure B.5. Different membership functions

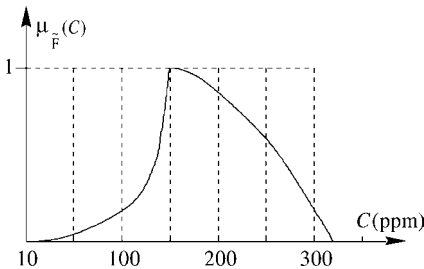


Figure B.5 Membership function of the fuzzy set \tilde{F} : 'about an order of magnitude greater than 10'.

may be used, depending on the available of information. In this particular example, the maximum degree of membership is 1 when C is equal to 150, but other values of C , such as 90, 110, 250 belong to \tilde{F} with a lesser degree of membership.

Up to this point the membership function should be considered as equivalent to a probability density function. However, in contrast to the probability basic rules, in fuzzy set theory the following rules are defined axiomatically for evaluating the membership function of the union $\tilde{A} \cup \tilde{B}$ and the intersection $\tilde{A} \cap \tilde{B}$ of the fuzzy sets \tilde{A} and \tilde{B} . As shown in Figure B.6, for every two sets \tilde{A} and \tilde{B} , belonging to Ω , we have

$$\forall \tilde{A}, \tilde{B} \subseteq \Omega \quad \mu_{\tilde{A} \cup \tilde{B}}(\omega) = \max(\mu_{\tilde{A}}(\omega), \mu_{\tilde{B}}(\omega)) \quad (\text{B.2})$$

$$\forall \tilde{A}, \tilde{B} \subseteq \Omega \quad \mu_{\tilde{A} \cap \tilde{B}}(\omega) = \min(\mu_{\tilde{A}}(\omega), \mu_{\tilde{B}}(\omega)) \quad (\text{B.3})$$

The above fuzzy rules appear arbitrary, but fuzzy logic is compatible with human reasoning. In fact, when available information is approximate, we tend to use rules based on maxima and minima rather than on complicated operations.

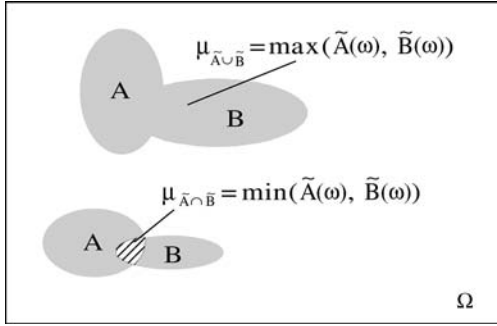


Figure B.6 Basic rules for membership function composition of fuzzy sets.

Example B.3

Let the following two fuzzy sets \tilde{A} and \tilde{B} describe the groundwater quality due to different sources of pollution.

\tilde{A} = ‘the groundwater quality due to an accidental spill of pollutants’,

\tilde{B} = ‘the groundwater quality due to agricultural activities’.

Both \tilde{A} and \tilde{B} are composed of three discrete elements rating the groundwater quality from 1 to 3 as follows

1 poor 2 good 3 excellent

From the available information, \tilde{A} and \tilde{B} have membership functions shown in Figure B.7.

The two fuzzy sets may be written formally as

$$\tilde{A} = \{(1, 0.9), (2, 0.4), (3, 0.1)\} \quad \text{and} \quad \tilde{B} = \{(1, 0.2), (2, 0.7), (3, 0.3)\}$$

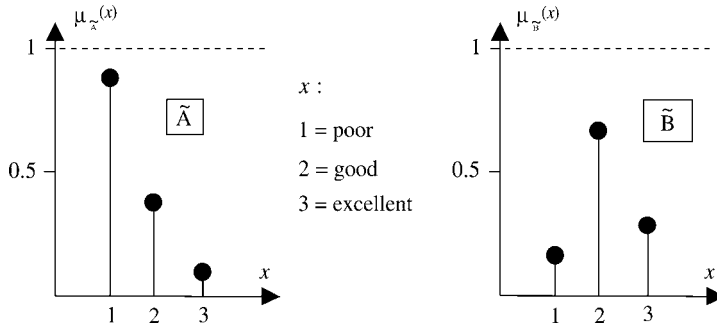


Figure B.7 Membership functions of fuzzy sets \tilde{A} and \tilde{B} .

The union of \tilde{A} and \tilde{B} is the fuzzy set \tilde{C} , where

$\tilde{C} = \tilde{A} \cup \tilde{B}$ = 'the groundwater quality is affected either by an accident or agriculture'.

By means of Equation B.2 the membership function of \tilde{C} is computed as follows:

$$\begin{aligned}\mu_{\tilde{C}}(1) &= \max(\mu_{\tilde{A}}(1), \mu_{\tilde{B}}(1)) = \max(0.9, 0.2) = 0.9 \\ \mu_{\tilde{C}}(2) &= \max(\mu_{\tilde{A}}(2), \mu_{\tilde{B}}(2)) = \max(0.4, 0.7) = 0.7 \\ \mu_{\tilde{C}}(3) &= \max(\mu_{\tilde{A}}(3), \mu_{\tilde{B}}(3)) = \max(0.1, 0.3) = 0.3\end{aligned}$$

\tilde{D} is the intersection of \tilde{A} and \tilde{B} , that is

$\tilde{D} = \tilde{A} \cap \tilde{B}$ = 'the groundwater quality is affected both by accident and agriculture'.

By means of Equation B.3 the membership function of \tilde{D} is computed as follows:

$$\begin{aligned}\mu_{\tilde{D}}(1) &= \min(\mu_{\tilde{A}}(1), \mu_{\tilde{B}}(1)) = \min(0.9, 0.2) = 0.2 \\ \mu_{\tilde{D}}(2) &= \min(\mu_{\tilde{A}}(2), \mu_{\tilde{B}}(2)) = \min(0.4, 0.7) = 0.4 \\ \mu_{\tilde{D}}(3) &= \min(\mu_{\tilde{A}}(3), \mu_{\tilde{B}}(3)) = \min(0.1, 0.3) = 0.1\end{aligned}$$

Membership functions of \tilde{C} and \tilde{D} are shown in Figure B.8.

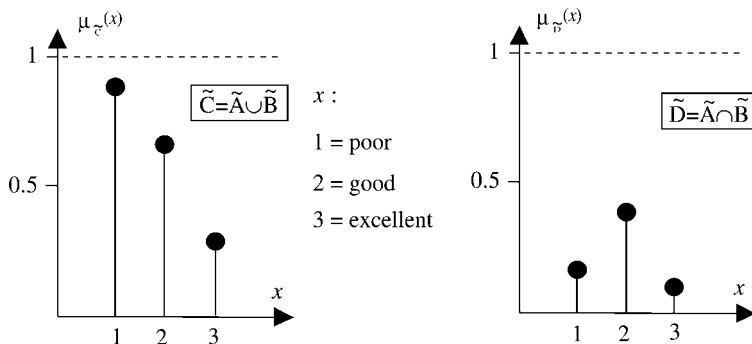


Figure B.8 Membership functions of fuzzy sets $\tilde{A} \cup \tilde{B}$ and $\tilde{A} \cap \tilde{B}$.

Note that rules represented by Equations B.2 and B.3 are respectively the fuzzy equivalents of the probabilistic rules for the *union* of two events A and B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

and their *intersection* (multiplicative probability law), that is

$$P(A \cap B) = P(A)P(B/A) = P(B)P(A/B).$$

According to the fuzzy rule in Equation B.2 the membership function of the union is estimated as the maximum between the corresponding membership functions of \tilde{A} and \tilde{B} . This is consistent with the physical notion of *possibility of occurrence: the realisation of either \tilde{A} and \tilde{B} is equivalent with the realisation of the easier one or of that having the maximum possibility measure*. The rule shown in Equation B.2 is not necessarily applied only for disjoint sets. It has been proved that if it is valid for every pair of disjoint sets ($\tilde{A} \cap \tilde{B} = \emptyset$), then it is true for every pair of events (Dubois and Prade, 1987).

The rule of intersection (Equation B.3) means that realising both \tilde{A} and \tilde{B} at the same time is equivalent to the realisation of the set with the *minimum measure of necessity*.

Another basic rule in fuzzy set theory is *complementation*. \tilde{A}_c is a fuzzy set, the complement of \tilde{A} , when

$$\mu_{\tilde{A}_c} = 1 - \mu_{\tilde{A}}$$

The rules of union, intersection and complementation in fuzzy set theory do not necessarily imply that

$$\tilde{A} \cup \tilde{A}_c = \emptyset \quad \text{and} \quad \tilde{A} \cap \tilde{A}_c = \Omega$$

as is true for the probabilistic approach. Ω is here the universal or referential or ‘always sure event’ and \emptyset is the empty set, which is identified with the ‘always impossible event’. This means that

$$\forall \omega \in \Omega \quad \mu_{\Omega}(\omega) = 1 \quad \text{and} \quad \mu_{\emptyset}(\omega) = 0$$

Take as an example the fuzzy set

$$\tilde{A} = \{(1, 0.9), (2, 0.4), (3, 0.1)\}$$

shown in Figure B.7, which is taken from the finite referential set $\Omega = \{1, 2, 3\}$. We have $\tilde{A}_c = \{(1, 0.1), (2, 0.6), (3, 0.9)\}$ and according to the rules of Equations B.2 and B.3 for the union and intersection

$$\tilde{A} \cup \tilde{A}_c = \{(1, 0.9), (2, 0.6), (3, 0.9)\} \neq \Omega \quad \text{and}$$

$$\tilde{A} \cap \tilde{A}_c = \{(1, 0.1), (2, 0.4), (3, 0.1)\} \neq \emptyset$$

The above defined rules of union, intersection and complementation are the more commonly used rules in fuzzy set theory. However, they are not unique. For example the use of the t -norm for the intersection and the s -norm for the union has been proposed (Dubois and Prade, 1980). More information about the mathematical

principles of fuzzy logic and the theory of possibility or necessity may be found in Zadeh (1978) and Dubois and Prade (1987).

B.3

***h*-Level Sets, Normal and Convex Fuzzy Sets**

For each element w belonging to \tilde{A} , there is an associated level h of membership. Inversely, consider all elements having the same level of membership h . Thus, the h -level set of a fuzzy set \tilde{A} will be defined as the *ordinary* set of all elements whose value of membership is h or higher, that is

$$A(h) = \{(\omega, \mu_{\tilde{A}}(\omega) \geq h); \omega \in \Omega, h \in [0, 1]\} \quad (\text{B.4})$$

A *normal fuzzy set* is one where at least one value of $\omega \in \Omega$ exists, such that $\mu_{\tilde{A}}(\omega) = 1$. In other words, the maximum value of the membership function is unity.

A *convex fuzzy set* is one, where for every real number a, b, c with $a < b < c$, it applies that

$$\mu_{\tilde{A}}(b) \geq \min(\mu_{\tilde{A}}(a), \mu_{\tilde{A}}(c)) \quad (\text{B.5})$$

The interpretation of Equation B.5 is that the membership function possesses no local extrema.

These concepts of normality and convexity are used in the definition of a fuzzy number.

B.4

Fuzzy Numbers

A fuzzy number \tilde{X} is a special case of fuzzy set, having the following properties:

- (a) It is defined on the set of real numbers R , rather than a set of linguistic properties.
- (b) Its membership function always reaches the maximum value of 1, that is it is a normal fuzzy set.
- (c) Its membership function is unimodal, that is it consists of an increasing and a decreasing part.
- (d) As we will see in the following, a complete arithmetic is available (e.g. Kaufmann and Gupta, 1985) to combine fuzzy numbers; furthermore, multi-dimensional functions of fuzzy numbers can be defined and computed. Thus, fuzzy uncertainty analysis, fuzzy model predictions and fuzzy risk and reliability analysis can be carried out.

From the above properties, a fuzzy number may be formally defined as

$$\tilde{X} = \{(x, \mu_{\tilde{X}}(x)) : x \in R; \mu_{\tilde{X}}(x) \in [0, 1]\}$$

The closer $\mu_{\tilde{X}}(x)$ is to 1, the more ‘certain’ one is about the value of x .

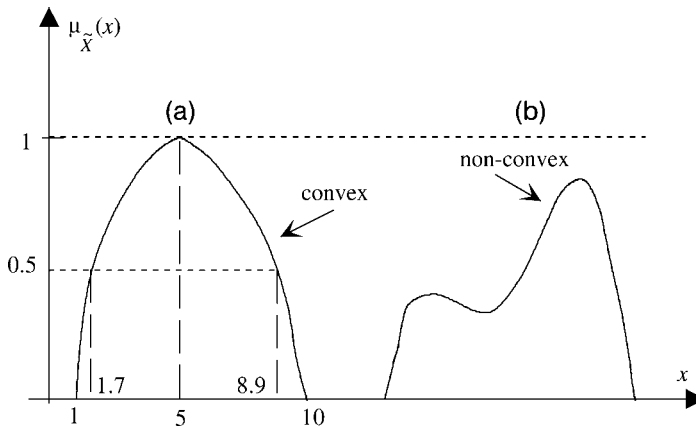


Figure B.9 Convex (a) and non-convex (b) fuzzy numbers.

Thus, a fuzzy number \tilde{X} is a normal, convex fuzzy subset of the set of real numbers. Figure B.9 represents membership functions of fuzzy numbers, one convex (a) and one non-convex (b). The property of convexity limits the shape that a fuzzy number can take: it always increases to the left of the peak, and decreases to the right.

Referring to Figure B.9, the width of the membership function of a fuzzy number 'about 5 but no less than 1 and no more than 10' is $10 - 1 = 9$. This width is an internal length. If a so-called credibility level $h = 0.5$ is defined, corresponding to a value 0.5 for the strength of acceptance, then the level set $h = 0.5$ is defined as the set $[1.7, 8.9]$. If $h = 0$, the level set is $[1, 10]$.

From the above it appears that fuzzy sets and fuzzy numbers may be used to characterise input uncertainty whenever variables (and/or relationships) are not defined precisely. For example the concentration of a pollutant may be described by the phrase 'about 20 ppm but no less than 12 and not more than about 30'. In general, an extreme loading (flood, drought, oil spill) or an abnormally low resistance (low DO, high water temperature, weakened foundation) are hazards that may be imprecisely defined; in such cases, a fuzzy number representation may be used, specifying a range of real numbers x and the fuzzy set membership function.

As can be seen in Figure B.9, there are two values of x where the membership function reaches zero, and at least one where it reaches a value of 1. A fuzzy number can be characterised by these three points and the shape of the curve defined by a pair of functions, one to the left and one to the right of the peak, since left-right symmetry is not a necessary condition.

A real, or *crisp* number, is a fuzzy number whose elements comprise only one number with a non-zero membership value, which is equal to 1.

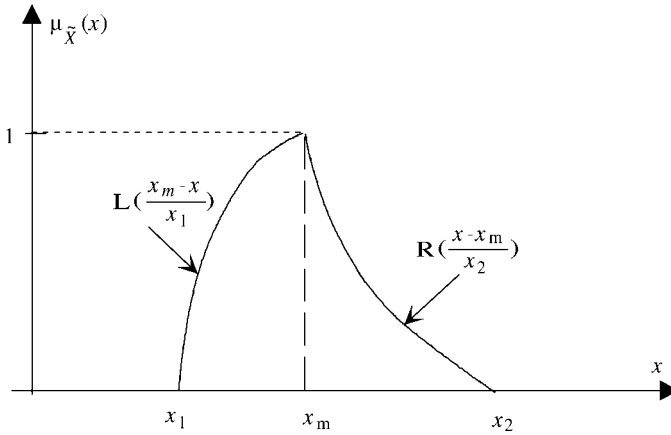


Figure B.10 L-R representation of a fuzzy number.

B.4.1

L-R Representation of a Fuzzy Number

The membership function of a fuzzy number may be described mathematically by means of two strictly decreasing functions L and R (Dubois and Prade, 1980). As shown in Figure B.10, for a convex fuzzy number, the part of a membership function left to the peak may be expressed in terms of the non-dimensional variable $(x_m - x)/x_1$ as function L . The corresponding part, to the right of the peak, is the function R , which is expressed in terms of the non-dimensional variable $(x - x_m)/x_2$. We have

$$\mu_{\tilde{X}}(x) = \begin{cases} L\left(\frac{x_m - x}{x_1}\right) & x \leq x_m \quad x_1 > 0 \\ R\left(\frac{x - x_m}{x_2}\right) & x > x_m \quad x_2 > 0 \end{cases}$$

By use of L-R notation, a fuzzy number \tilde{X} is symbolically expressed as

$$\tilde{X} = (x_m, x_1, x_2)_{LR}$$

B.4.2

Triangular and Trapezoidal Fuzzy Numbers

The simplest type of fuzzy number is triangular, that is one having linear membership functions on either side of the peak. Figure B.11a gives an example of a triangular fuzzy number (TFN). This may be described by the values of x at points x_1 , x_2 and x_3 .

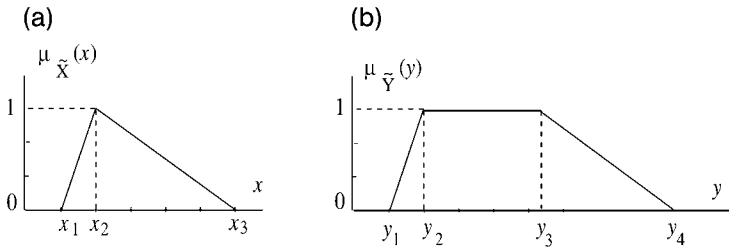


Figure B.11 Triangular and trapezoidal fuzzy numbers.

Thus, $\tilde{X} = (x_1, x_2, x_3)$ completely characterises a triangular number. It follows that any real or ‘crisp’ number can be defined as a triangular fuzzy number, with $x_1 = x_2 = x_3$.

Similarly, a trapezoidal fuzzy number \tilde{Y} may be defined by four values y_1, y_2, y_3 and y_4 as shown in Figure B.11b.

A more general definition and further details about fuzzy numbers may be found in Dubois and Prade, (1980) and Zimmermann (1985).

B.4.3

Support and h -Level of a Fuzzy Number

The support of a fuzzy number \tilde{X} is the ordinary set defined as follows

$$S(\tilde{X}) = \{x : \mu_{\tilde{X}}(x) > 0\} \quad (\text{B.6})$$

Because of the convexity assumption, the support of a fuzzy number is an interval (Figure B.12).

The h -level set of a fuzzy number \tilde{X} is the ordinary set or interval $\bar{X}(h)$, defined as

$$\bar{X}(h) = \{x : \mu_{\tilde{X}}(x) \geq h\}. \quad (\text{B.7})$$

Figure B.12 illustrates the above definitions.

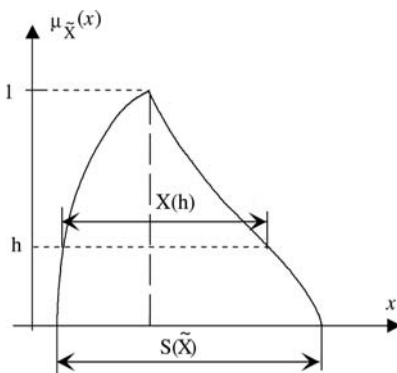


Figure B.12 Support and h -level of a fuzzy number.

B.5

Cartesian Product

Let $\tilde{X}_1, \tilde{X}_2, \dots, \tilde{X}_n$ be fuzzy numbers defined on the corresponding real numbers x_1, x_2, \dots, x_n . More generally $\tilde{X}_1, \tilde{X}_2, \dots, \tilde{X}_n$ should be fuzzy sets with corresponding universal spaces x_1, x_2, \dots, x_n .

As is known, the Cartesian product $A \times B$ of two ordinary sets A and B is a set composed of all possible pairs of the members of A and B , that is

$$A \times B = \{(A_i, B_i) : A_i \in A \text{ and } B_i \in B\}$$

The Cartesian product of the n fuzzy sets $\tilde{X}_1, \tilde{X}_2, \dots, \tilde{X}_n$ is also a fuzzy set \tilde{X} in the Cartesian product of x_1, x_2, \dots, x_n . A fuzzy relation between fuzzy sets, which is currently used in human discourse as expressing similarities between different sets, corresponds to a Cartesian product. The question is how to evaluate the membership function of $\tilde{X} = \tilde{X}_1 \times \tilde{X}_2 \times \dots \times \tilde{X}_n$.

As illustrated in Figure 2.32, for the case of two fuzzy numbers, according to the fuzzy rules described previously we should have

$$\mu_{\tilde{X}}(x) = \min\{\mu_{\tilde{X}_k}(x_k); k = 1, 2, \dots, n\} \quad (\text{B.8})$$

where $\mu_{\tilde{X}_k}(x_k); k = 1, 2, \dots, n$ is the membership function of fuzzy set $\tilde{X}(k)$ (Figure B.13).

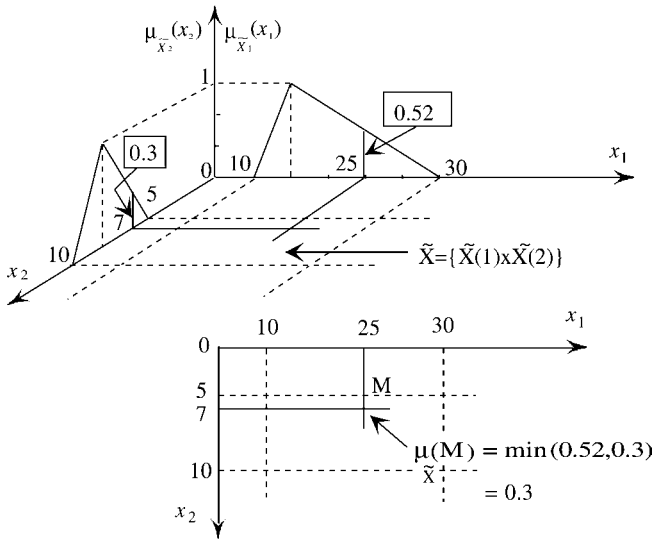


Figure B.13 Membership function of a Cartesian product.

B.6

Extension Principle

The extension principle is a method of computing membership functions of fuzzy sets which are functions of other fuzzy sets. Using this principle, which is a basic tool of fuzzy arithmetic, we can perform point-to-point operations on fuzzy sets.

Let X and Y be two ordinary sets and f a point-to-point mapping from X to Y that is

$$f = x \rightarrow y \quad \forall x \in X, y = f(x), y \in Y \quad (\text{B.9})$$

Function f is deterministic and can be extended to the fuzzy set situation as follows.

Let \tilde{X} be a fuzzy set in X with membership function $\mu_{\tilde{X}}(x)$. The image of \tilde{X} in Y is the fuzzy set \tilde{Y} with membership function given by the extension principle as follows

$$\mu_{\tilde{Y}}(y) = \begin{cases} \sup\{\mu_{\tilde{X}}(x); & y = f(x), x \in X, y \in Y \\ 0 & \text{otherwise} \end{cases} \quad (\text{B.10})$$

If, in turn, \tilde{X} is defined as a Cartesian product, then $\mu_{\tilde{X}}(x)$ in Equation B.10 must be replaced by its expression in Equation B.8

$$\mu_{\tilde{Y}}(y) = \begin{cases} \sup\{\min(\mu_{\tilde{X}_k}(x_k)); & y = f(x), x \in X, y \in Y \\ 0 & \text{otherwise} \end{cases} \quad (\text{B.11})$$

Figure B.14 illustrates the extension principle (Zadeh (1965) and Jan Lukasiewicz in the 1920s), when $y = f(x)$ is one-to-one mapping and (Figure B.15) the similar transformation when the correspondence is not unique.

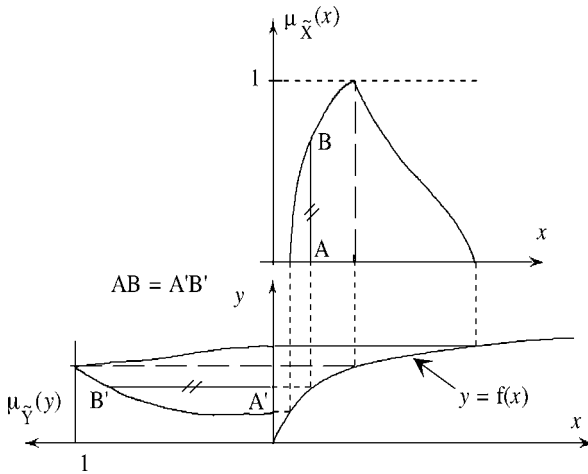


Figure B.14 Illustration of the extension principle when $y = f(x)$ is one-to-one mapping.

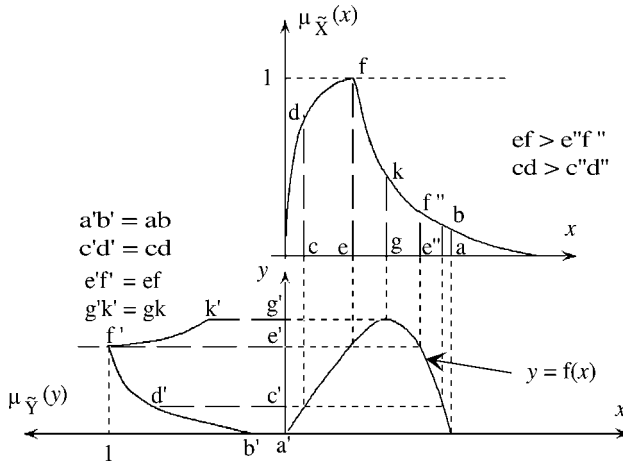


Figure B.15 Illustration of the extension principle when $y=f(x)$ is not one-to-one mapping.

B.7

Arithmetic Operations on Fuzzy Numbers as Extension of Interval Analysis

The simplest method of considering uncertainty in model prediction is to perform an interval analysis. An uncertain parameter in a flow model, such as the dispersion coefficient, may take any value within such an interval. With more information on this uncertain parameter the interval model can be 'sharpened', that is we determine the possibility that the parameter may take certain value(s) within the interval. If the axioms and hypotheses of probability theory are verified, then the probabilistic procedure is simply an extension of interval analysis.

The propagation of uncertainty characteristics measured as intervals is the basis of traditional sensitivity analysis. The arithmetic of intervals is straightforward and functions of interval numbers are easy to calculate (Dong and Wong, 1986). To utilise additional information on an uncertain model parameter, multi-intervals can be defined in the form of fuzzy numbers. For example if we believe that a model parameter is definitely greater than 1 and less than 10, about 5 but possibly somewhere else in the interval 1 to 10, then the multi-intervals sketched in Figure 2.28 (a), in the form of a fuzzy number, may be appropriate.

B.8

Arithmetic Operations on Intervals

Fuzzy calculus is an extension of operations performed on intervals of real numbers. Let us then briefly review interval arithmetics for positive real numbers.

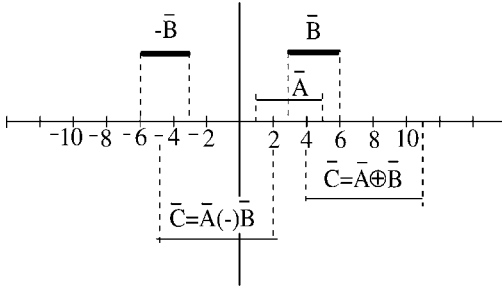


Figure B.16 Addition and subtraction of intervals.

B.8.1

Addition and Subtraction of Intervals

An interval is defined by an ordered pair in brackets as

$$\bar{A} = [a_1, a_2] = \{a : a_1 \leq a \leq a_2\}$$

where a_1 is the left limit and a_2 the right limit of \bar{A} .

As indicated in Figure B.16, take two intervals $\bar{A} = [a_1, a_2]$ and $\bar{B} = [b_1, b_2]$. The opposite interval of \bar{B} is the mirror image of \bar{B} taking point 0 as origin, that is

$$-\bar{B} = [-b_2, -b_1] \quad (\text{B.12})$$

The sum of intervals \bar{A} and \bar{B} is simply

$$\bar{C} = \bar{A} \oplus \bar{B} = [a_1 + b_1, a_2 + b_2] \quad (\text{B.13})$$

By means of Equation B.12 the difference between intervals \bar{A} and \bar{B} is computed as follows

$$\bar{C} = \bar{A} (-) \bar{B} = \bar{A} \oplus (-\bar{B}) = [a_1 - b_2, a_2 - b_1] \quad (\text{B.14})$$

Figure B.16 illustrates the arithmetic operations represented in Equations B.13 and B.14.

B.8.2

Multiplication and Division of Intervals

The product of two intervals $\bar{A} = [a_1, a_2]$ and $\bar{B} = [b_1, b_2]$ is given by the simple relationship

$$\bar{C} = \bar{A} \otimes \bar{B} = [a_1 b_1, a_2 b_2] \quad (\text{B.15})$$

The inverse interval of \bar{B} is defined as follows

$$\bar{B}^{-1} = \left[\frac{1}{b_2}, \frac{1}{b_1} \right] \quad (\text{B.16})$$

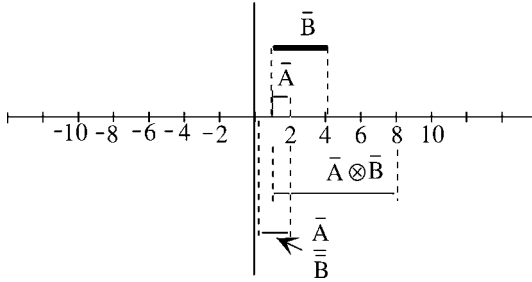


Figure B.17 Product and ratio of intervals.

By means of Equation B.16, the ratio between intervals \bar{A} and \bar{B} is evaluated as follows

$$\bar{C} = \frac{\bar{A}}{\bar{B}} = \bar{A} \otimes (\bar{B}^{-1}) = \left[\frac{a_1}{b_2}, \frac{a_2}{b_1} \right] \quad (\text{B.17})$$

Figure B.17 illustrates the arithmetic operations of Equations B.15 and B.17.

Operations on fuzzy numbers may be performed by considering their h -level intervals and then applying the corresponding operations on intervals. Generally speaking, let us take two fuzzy numbers \tilde{A} and \tilde{B} and the corresponding h -level intervals

$$\bar{A}(h) = [a_1(h), a_2(h)] \text{ and } \bar{B}(h) = [b_1(h), b_2(h)]$$

B.8.3

Addition of Fuzzy Numbers

Let us define the sum of two positive fuzzy numbers as

$$\tilde{C} = \tilde{A} \oplus \tilde{B} \quad (\text{B.18})$$

By considering h -level arithmetic we have

$$\bar{C}(h) = \bar{A}(h) \oplus \bar{B}(h) = [a_1(h) + b_1(h), a_2(h) + b_2(h)] = [c_1(h), c_2(h)] \quad (\text{B.19})$$

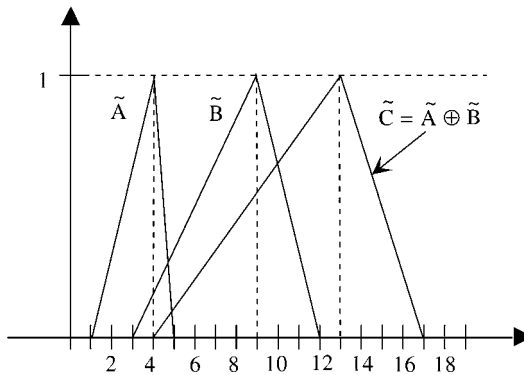


Figure B.18 Addition of two triangular fuzzy numbers.

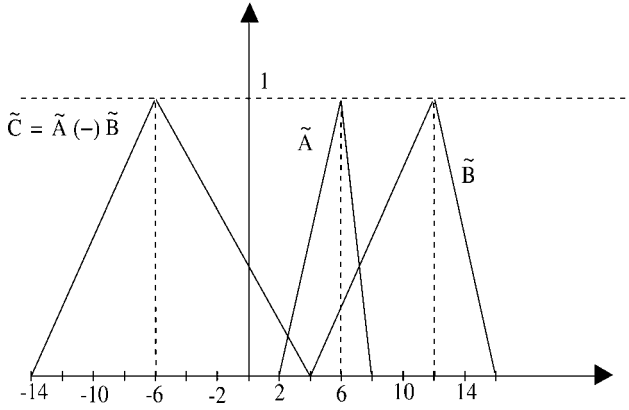


Figure B.19 Subtraction of two triangular fuzzy numbers.

In the case of Triangular Fuzzy Numbers (TFN) the operation is simpler and, as shown in Figure B.18, interval transformation is not needed. The rule is to add the three numbers characterising each TFN, that is

$$\begin{aligned}\tilde{C} &= \tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \\ &= (c_1, c_2, c_3)\end{aligned}\quad (\text{B.20})$$

B.8.4

Subtraction of Fuzzy Numbers

Let us define the opposite of a positive fuzzy number \tilde{B} as

$$\bar{\tilde{B}}(h) = [-b_2(h), -b_1(h)] \quad (\text{B.21})$$

For a TFN we have $\bar{\tilde{B}} = (-b_3, -b_2, -b_1)$

By considering the h -level arithmetic we have

$$\begin{aligned}\bar{\tilde{C}}(h) &= \bar{\tilde{A}}(h)(-)\bar{\tilde{B}}(h) = \bar{\tilde{A}}(h) \oplus \bar{\tilde{B}}^-(h) \\ &= [a_1(h) - b_2(h), a_2(h) - b_1(h)] \\ &= [c_1(h), c_2(h)]\end{aligned}\quad (\text{B.22})$$

For TFNs we have (Figure B.19)

$$\begin{aligned}\tilde{C} &= \tilde{A}(-)\tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1) \\ &= (c_1, c_2, c_3)\end{aligned}\quad (\text{B.23})$$

B.8.5

Multiplication of Fuzzy Numbers

Let us define the product of two positive fuzzy numbers \tilde{A} and \tilde{B} as

$$\tilde{C} = \tilde{A} \otimes \tilde{B} \quad (\text{B.24})$$

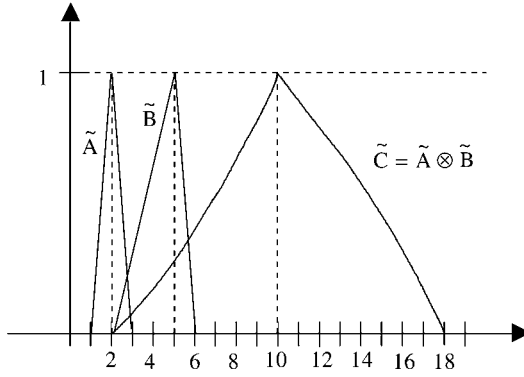


Figure B.20 Multiplication of two triangular fuzzy numbers.

By considering the h -level arithmetic we have

$$\begin{aligned}\bar{C}(h) &= \bar{A}(h) \otimes \bar{B}(h) = [a_1(h)b_1(h), a_2(h)b_2(h)] \\ &= [c_1(h), c_2(h)]\end{aligned}\quad (\text{B.25})$$

It is worth noting that the product of two TFNs is not necessarily another TFN. This is illustrated in the example shown in Figure B.20.

B.8.6

Division of Fuzzy Numbers

Let us define, by means of the h -level interval, the inverse of a positive fuzzy number \tilde{B} as

$$\bar{B}^{-1}(h) = \left[\frac{1}{b_2(h)}, \frac{1}{b_1(h)} \right] \quad (\text{B.26})$$

By considering the h -level arithmetic again we have

$$\begin{aligned}\bar{C}(h) &= \bar{A}(h) (:) \bar{B}(h) = \bar{A}(h) \otimes \bar{B}^{-1}(h) \\ &= \left[\frac{a_1(h)}{b_2(h)}, \frac{a_2(h)}{b_1(h)} \right] \\ &= [c_1(h), c_2(h)]\end{aligned}\quad (\text{B.27})$$

Again, the ratio of two TFNs is not necessarily another TFN. This is illustrated in the example shown in Figure B.21.

B.8.7

Minimum and Maximum of Fuzzy Numbers

Let us define

$$\tilde{C} = \max(\tilde{A}, \tilde{B}) = \tilde{A} \vee \tilde{B} \quad (\text{B.28})$$

$$\tilde{D} = \min(\tilde{A}, \tilde{B}) = \tilde{A} \wedge \tilde{B} \quad (\text{B.29})$$

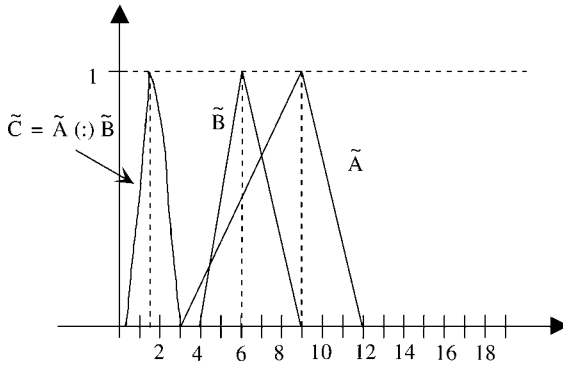


Figure B.21 Ratio between two triangular fuzzy numbers.

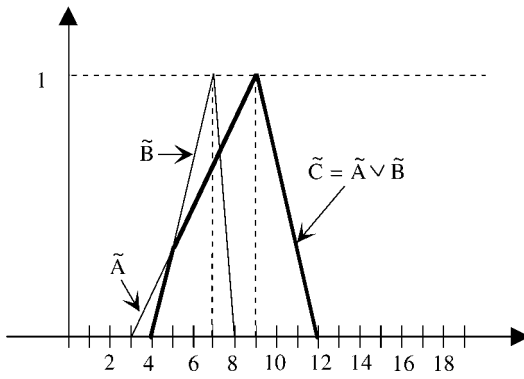


Figure B.22 Maximum of two triangular fuzzy numbers.

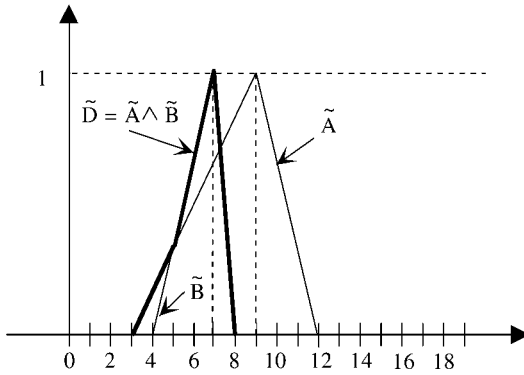


Figure B.23 Minimum of two triangular fuzzy numbers.

By considering the h -level arithmetic we obtain for the maximum (Figure B.22)

$$\begin{aligned}\bar{C}(h) &= \bar{A}(h) \vee \bar{B}(h) = [a_1(h) \vee b_1(h), a_2(h) \vee b_2(h)] \\ &= [c_1(h), c_2(h)]\end{aligned}$$

and for the minimum (Figure B.23)

$$\begin{aligned}\bar{D}(h) &= \bar{A}(h) \wedge \bar{B}(h) = [a_1(h) \wedge b_1(h), a_2(h) \wedge b_2(h)] \\ &= [d_1(h), d_2(h)]\end{aligned}$$

B.8.8

Mean and Width of Fuzzy Numbers

The mean of a fuzzy number \tilde{A} may be defined as (Figure B.24)

$$FM(\tilde{X}) = \frac{\int_{-\infty}^{+\infty} x \mu_{\tilde{X}}(x) dx}{\int_{-\infty}^{+\infty} \mu_{\tilde{X}}(x) dx} \quad (\text{B.30})$$

The width of a fuzzy number \tilde{A} is defined as

$$FW(\tilde{X}) = \max\{x, \mu_{\tilde{X}}(x) > 0\} - \min\{x, \mu_{\tilde{X}}(x) > 0\} \quad (\text{B.31})$$

B.8.9

Convolution of Fuzzy Numbers

The operations on fuzzy numbers defined above refer to *positive numbers*. The extension principle given by Equation B.11 can be used to generalise the formulas presented above (Zadeh, 1965, 1987). Let (\cdot) represent an arithmetic operation, that is any of the operations \oplus , $(-)$, \otimes , $(:)$ between two fuzzy numbers. Let \tilde{X} and \tilde{Y} be two fuzzy numbers on which the operation (\cdot) is to be performed, and \tilde{Z} be the result of that operation.

Then $\tilde{Z} = \tilde{X}(\cdot)\tilde{Y}$ is the fuzzy number with membership function:

$$\mu_{\tilde{Z}}(z) = \begin{cases} \sup\{\min \mu_{\tilde{X}}(x), \mu_{\tilde{Y}}(y)\} & \text{such that } z = x(\cdot)y \\ 0 & \text{otherwise} \end{cases} \quad (\text{B.32})$$

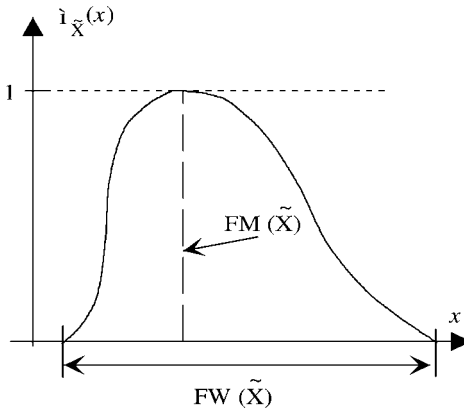


Figure B.24 Mean and width of a fuzzy number.

Appendix C

Hints for Answering Questions and Solutions to Problems

C.1

Answers to Questions and Problems – Chapter 1

A Systemic View of Water Resources

- (a) 70%
- (b) 3%
- (c) 0.3%
- (d) 30%
- (e) Water is the origin of all kinds of life on Earth. There is no substitute for water. Furthermore, water is limited, finite and a very fragile natural resource. It is necessary for all living ecosystems, for plants, for producing energy and all kinds of manufactured products.
- (f) The main characteristic of the hydrological cycle is that it is renewable.
- (g) Efficient precipitation is defined as the difference between precipitation and evapotranspiration. It is equal to surface plus groundwater runoff (total runoff).
- (h) The water balance equation is generally valid. Limitations are due to the difficult estimation of various terms at global scale.

Problems

1. Catchment area of $A = 0.7 \text{ Gm}^2 = 0.7 \times 10^9 \text{ m}^2 = 0.7 \times 10^3 \text{ km}^2$

- (a) For negligible storage $\Delta S = 0$, the water budget equation becomes:

$$\text{Total Runoff (TR)} = \text{Precipitation (P)} - \text{Evapotranspiration (ET)}$$

$$\text{TR} = 670 \text{ mm} - 520 \text{ mm} = 150 \text{ mm}$$

$$\text{TR} = (150 \times 10^{-6} \text{ km}) \times (0.7 \times 10^3 \text{ km}^2) = 0.105 \text{ km}^3$$

$$\text{P} = (670 \times 10^{-6} \text{ km}) \times (0.7 \times 10^3 \text{ km}^2) = 0.469 \text{ km}^3$$

(b) $\text{ET} = (520 \times 10^{-6} \text{ km}) \times (0.7 \times 10^3 \text{ km}^2) = 0.364 \text{ km}^3$

$$\text{TR} = \text{P} - \text{ET} = 0.469 - 0.364 = 0.105 \text{ km}^3$$

(c) $\text{Ratio} = ET/P = 0.520/0.670 = 77.6$

This ratio is greater than the global average on Earth, which is 60%. This means that the area has a warm climate and is probably in tropical Africa.

2. Daily evaporation rate (DE) = annual rate/365 = $1500/365 = 4.11 \text{ mm}$

(a) In m^3 we have:

$$\begin{aligned} \text{DE} &= (4.11 \times 10^{-3} \text{ m}) \times (\text{surface area}) \\ &= (4.11 \times 10^{-3} \text{ m}) \times (0.9 \times 10^6 \text{ m}^2) = 3699 \text{ m}^3 \end{aligned}$$

Change in storage per year

$$\begin{aligned} \Delta S &= \text{inflow} - E = (0.15 \text{ m}^3/\text{s}) \times (365 \times 24 \times 3600) \text{ s} - 1.5 \text{ m} \\ &\quad \times (0.9 \times 10^6 \text{ m}^2) = (4.73 - 1.35) \times 10^6 \text{ m}^3 \\ &= 3.38 \times 10^6 \text{ m}^3 \text{ or } (3.38/0.9) \times 10^3 \text{ m} \\ &= 3755 \text{ mm (increase)} \end{aligned}$$

(b) $T = (1000 \text{ mm}/3755 \text{ mm}) \times 365 \text{ days} = 97.20 \text{ days}$

The New Paradigm of Water Quality

- (a) Water quality deteriorates with decreasing water quantity, because the level of concentration of pollutants increases with water scarcity.
- (b) The new water quality paradigm means that water quality depends not only on the physico-chemical properties of the water but also on the health and biodiversity of the aquatic ecosystems (bio-ecological status of water resources).
- (c) According to the new definition of water quality, joint investigations of abiotic and biotic components of water are required (EU-Water Framework Directive 2000/60).

Integrated Water Resources Management (IWRM)

- (a) The river basin.
- (b) Conflicts between different water uses. Coordination for more reliable use of different natural resources, preservation of the environment, equitable use of water and achieving economic efficiency. Ultimately, IWRM contributes to sustainable development.
- (c) Examples of benefits may be given from your experience, considering issues like:
 - (c.1) Coordinated management of different resources such as land, groundwater and water (e.g. joint management of groundwater and surface water may reduce the over-pumping of groundwater resources, which in turn affects the availability of surface water and threatens ecosystems).
 - (c.2) Management at the catchment scale: (e.g. coordination of water uses upstream and downstream may reduce pollution problems at the river delta from pollutant sources located upstream).

- (c.3) Coordination between institutions: (e.g. local authorities responsible for water supply may improve water quantity and quality in their network by collaborating with institutions responsible for groundwater resources).
- (c.4) Interdisciplinary approach.
- (c.5) Public participation.

Also benefits may be considered by different sectors, like

- (c.6) Environment: provide environmental flows for ecosystems.
 - (c.7) Agriculture: save irrigation water.
 - (c.8) Water supply and sanitation: improve network's efficiency and maintenance.
- (d) Water is indispensable for human life, which means that everyone is entitled to a sufficient amount of good quality water in order to satisfy vital needs. Although many international forums and declarations support the concepts of the 'human right to water' and 'water vital needs' these are not expressly established in international law.
- (e) The notions of 'water as social good' and 'water as a commodity' refer to water pricing. Water should be considered as a 'social good' and be distributed at as low a price as is necessary to ensure that people's vital need for water can be satisfied. In order to be able to develop the necessary infrastructure and services for water distribution and also to regulate excessive water demand, when consumed for purposes that go beyond vital needs, water should be treated as a commodity.

Water Pollution in Transboundary Regions

- (a) Fair transboundary water allocation refers to the equitable sharing of water between riparian countries. This should be planned in such a way that not only are basic human water rights satisfied but also that sustainable socio-economic growth is ensured, by satisfying as far as possible the water demand of all users in both upstream and downstream countries. Furthermore, sufficient quantity of water and also adequate water quality should be ensured for water ecosystems. Generally speaking, efficient transboundary water management has to satisfy demand from ecosystems and overcome conflicts not only between different sectors in the same country, but also between sectors from different countries.
- (b) Equitable access to water does not necessarily mean access to equal quantities of water but rather equal opportunity to access water. Equity deals with the distribution of wealth or resources among sectors or individuals in riparian countries. The wider definition of equity and efficiency also calls for suitable institutional, economic and legal arrangements, which provide users with sufficient security of water tenure and support sustainable socio-economic development.
- (c) Not necessarily. Higher value uses (such as urban water supply) often have the potential to mobilise sufficient financial resources to secure a reliable supply. Higher value uses often require higher levels of reliability, meaning larger dams

and hence much larger investments compared with lower value uses (e.g. irrigation). The obvious economic advantage to society of not giving priority to various non-primary uses is that sectors have to fend for themselves, and will not, in all but the most extreme periods of drought, damage each other.

The EU Water Framework Directive

- (a) The 'good status' is determined by a 'good ecological' and a 'good chemical' status.
- (b) This is determined by hydro-morphological (e.g. the habitat conditions), physico-chemical and biological monitoring and analysis. The WFD aims to establish a framework for the protection of inland surface waters, transitional waters, coastal waters and groundwater.
- (c) Key elements of the WFD include:
 - (c.1) Technical considerations: monitoring, river basin planning and management.
 - (c.2) Institutional: adopt a single system for water management based on the river basin.
 - (c.3) Environmental: water quality and ecosystems.
 - (c.4) Water economics.
 - (c.5) Public participation.

Uncertainties in Water Resources Management (WRM)

- (a) Uncertainties are mainly due to the spatial and temporal variability associated with hydrological variables. In addition to these uncertainties, which arise from the definition of the physical problem, there are also other types of uncertainties such as those related to the use of methodologies and tools to describe and model physical, ecological and social problems (i.e. sampling techniques, data acquisition, data analysis and hydrological and ecological modelling, conflict resolution).
- (b) Risk may be important when uncertainties occur. If uncertainties are negligible or not present, the risk is zero.
- (c) Types of Uncertainty on WRM:
 - (c.1) Hydrologic Uncertainty
 - (i) This refers to the various hydrological events such as precipitation, river flow, coastal currents, water quality, and so on.
 - (c.2) Hydraulic Uncertainty
 - (ii) These are uncertainties related to hydraulic design and hydraulic engineering structures.
 - (c.3) Economic Uncertainty
 - (iii) This refers to all fluctuations in prices, costs and investments that may affect the design and optimisation processes.
 - (c.4) Structural Uncertainty
 - (iv) This means all deviations due to material tolerances and other possible technical causes of structural failure.

Environmental Risk Assessment (ERA) and Environmental Risk Management (ERM)

- (a) The assessment of environmental risk is mainly based on data, information and modelling of different types of hazards and possible consequences from these on the physical, human and ecological system, including forecasting of its behaviour under risk. Environmental risk management is the investigation of incremental costs and benefits under variable risk-based scenario.
- (b) Four steps for ERA:
 - (b.1) Step 1: Risk or hazard identification
 - (b.2) Step 2: Assessment of loads and resistances
 - (b.3) Step 3: Uncertainty analysis
 - (b.4) Step 4: Risk quantification
- (c) Five steps for ERM:
 - (c.1) Step 1: Identification of alternatives and associated risks
 - (c.2) Step 2: Assessment of costs involved in various risk levels
 - (c.3) Step 3: Technical feasibility of alternative solutions
 - (c.4) Step 4: Selection of acceptable options according to the public perception of risk, government policy and social factors
 - (c.5) Step 5: Implementation of the optimal choice

C.2**Answers to Questions and Problems – Chapter 2****Definitions of Risk**

- (a) 'Hazard' is the source of potential loss or damage and 'risk' is the possibility of loss or damage caused by exposure to a hazard.
- (b) Risk is the product of 'hazard' and 'vulnerability'. It may also be defined as the product of the 'probability of failure' and the 'consequences of failure'.
- (c) The risk of water pollution is the probability of exceeding the acceptable pollutant concentration, set by regulation. It may also be defined as the product of the above probability and the consequences due to non-acceptable pollutant concentrations.

Typology of Risks and the Precautionary Principle

- (a) There are natural and anthropogenic risks.
- (b) Acceptable risks are those causing affordable damage to communities. The challenge is how and by whom levels of 'affordable' damages can be set.
- (c) If risk is considered to be the product of (Probability) \times (Damage), then for very high or very low values of (Probability) or (Damage), the same risk will be attributed to an event of high probability and low damage (e.g. overflow of a

secondary drainage pipe) as to an event of high damage and low probability (e.g. explosion of a nuclear facility like the Chernobyl accident). This does not seem realistic.

Uncertainties in Water Pollution Problems

- (a) Aleatory uncertainties are due to natural variability, both in time and space (natural randomness). They cannot be reduced by human intervention. Epistemic uncertainties are man-induced (e.g. data, modelling, technological and operational uncertainties).
- (b) Only epistemic uncertainties can be reduced by improving data collection and management methodologies, developing more effective modelling techniques or inventing better technology.
- (c) To determine pollution frequency, the number of years in which pollution actually occurs is divided by the total number of years in the time series. This ratio converges to probability for very long time series.

Water Quality Specifications

- (a) Water quality standards are defined statistically in order to take into account various uncertainties, such as natural and human-induced variabilities.
- (b) It is necessary to distinguish and respect both effluent and receiving water body standards, because accumulation and interaction between acceptable effluents may produce adverse effects in the receiving waters.
- (c) The capacity of a water body to receive a specific pollutant (e.g. urban sewage) is the amount of pollutant that the water body can absorb by hydrodynamic, physico-chemical and biological interactions, without having any adverse effects (pollution).

Probabilistic Risk and Reliability

- (a) The annual assassination risk of a USA president is: $4/200 = 2 \times 10^{-2}$.
Per 100 000 of population this risk is: $(2 \times 10^{-2}) \times (10^+5) = 2 \times 10^+3$, which is 200 times the annual accidental mortality risk per 100 000 inhabitants of an airline pilot.
- (b) The general expression of risk:
 - (b.1) When load and resistance are two independent variables, risk is

$$p_F = \int_0^{\infty} f_L(\ell) \left\{ \int_0^{\ell} f_R(r) dr \right\} d\ell,$$

where $f_L(\ell)$ and $f_R(r)$ are the probability density distributions of the load L and the resistance R .

(b.2) If the resistance R is constant and equal to R_0 , then risk is

$$p_F = \int_{R_0}^{\infty} f_R(r) dr = F_R(R_0)$$

(c) For a 5-kg baby:

Allowed daily dose of nitrates: $(5 \text{ mg/kg/day}) \times (5 \text{ kg}) = 25 \text{ mg/day}$.

Absorbed daily dose of nitrates: $(70 \text{ mg/l}) \times (0.40 \text{ l/day}) = 28 \text{ mg/day} > 25 \text{ mg/day}$.

The situation is risky because the water quality is poor and the daily water dose is high.

Fuzzy Risk and Reliability

(a) The interval of safety margin is

$$\tilde{Z} = \tilde{R} - \tilde{L} = [2, 6] - [-3, 4] = [\min(2+3, 2-4, 6+3, 6-4) - \max(2+3, 2-4, 6+3, 6-4)] = [-2, 9]$$

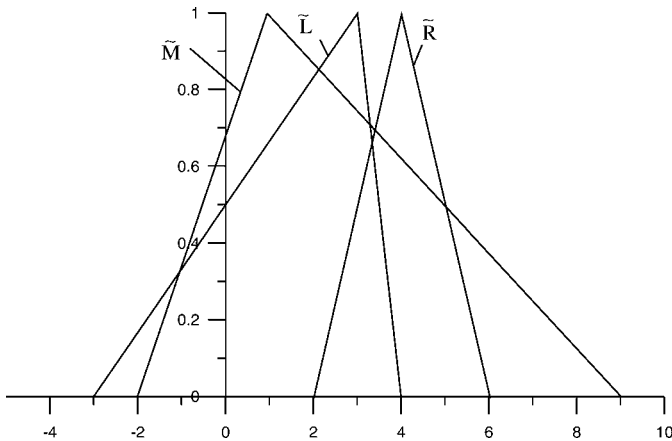
$$\text{Risk index} = (\text{length of interval at risk or } M < 0) / (\text{length of safety margin interval}) = 2/11 = 22\%$$

$$\text{Reliability index} = 1 - 2/11 = 9/11 = 88\%$$

(b) Using the notation for triangular fuzzy numbers $\tilde{R} = (2, 4, 6)$ and $\tilde{L} = (-3, 3, 4)$, the safety margin is the fuzzy number $\tilde{M} = \tilde{R} - \tilde{L} = (-2, 1, 9)$.

(c) The risk index may be defined as the ratio between the surface where \tilde{M} takes negative values over the total surface of \tilde{M} . From the result obtained above and the related figure we find

$$\text{Risk} = \frac{0.5 \times 2 \times 0.67}{0.5 \times 11 \times 1.0} = 12.18\%$$



C.3

Answers to Questions and Problems – Chapter 3

Stochastic Approach

(a) Urban water demand and supply

(a.1) Normal distributions: Demand = Load = L: $N(3, 1^2)$; Supply = Resistance = R: $N(5, 0.75^2)$ (i) Reliability index $= \beta = (R_0 - L_0) / \sqrt{\sigma_R^2 + \sigma_L^2} = 2 / \sqrt{0.5625} = 2.67$ (ii) Reliability $= \Phi(2.67) = 0.9962 = 99.62\%$ (iii) Risk $= 0.38\%$

(a.2) log-normal distributions:

$$\mu_{\ln R} = \frac{1}{2} \ln \left(\frac{R_0^2}{1 + \left(\frac{\sigma_R}{\mu_R} \right)^2} \right) = 0.5 \ln \left(\frac{25}{1 + (0.75/5)^2} \right) = 0.5 (\ln 24.445) = 1.598$$

$$\sigma_{\ln R}^2 = \ln \left(1 + \left(\frac{\sigma_R}{\mu_R} \right)^2 \right) = \ln(1 + 0.0225) = 0.0222$$

$$\mu_{\ln L} = \frac{1}{2} \ln \left(\frac{L_0^2}{1 + \left(\frac{\sigma_L}{\mu_L} \right)^2} \right) = 0.5 \ln \left(\frac{9}{1 + (1/3)^2} \right) = 0.5 (\ln 8.10) = 1.0459$$

$$\sigma_{\ln L}^2 = \ln \left(1 + \left(\frac{\sigma_L}{\mu_L} \right)^2 \right) = \ln(1 + (1/3)^2) = \ln(1.111) = 0.1053$$

Reliability index $= \beta = (1.598 - 1.0459) / \sqrt{0.0222 + 0.1053} = 0.5521 / \sqrt{0.1275} = 1.546$ Reliability $= \Phi(1.546) = 0.9389 = 93.89\%$ Risk $= 6.11\%$

(b) Toxic contamination of a lake

(b.1.1) If A is the event of lethal contamination, the risk may be defined as the probability of exceeding the lethal concentration. If $f(C)$ and $F(C)$ are the probability density and the cumulative probability distribution functions, then

$$\text{Risk} = P(A) = P(C \geq C_\lambda) = \int_{C_\lambda}^{\infty} f(C) dC = F(C_\lambda)$$

- (b.1.2) If $D(C)$ is the damage (cost or loss or consequence) for every value of concentration c exceeding the lethal one, the risk may be defined as the expected damage, that is,

$$\text{Risk} = \int_{C_k}^{\infty} D(C)f(C)dC = E[D] = \text{expected damage}$$

- (b.2) E_1, E_2 and E_3 are disjoint and complementary events

$$\begin{aligned} P(A) &= P(A \cap E_1) + P(A \cap E_2) + P(A \cap E_3) = \\ &= P(A/E_1)P(E_1) + P(A/E_2)P(E_2) + P(A/E_3)P(E_3) \\ &= 0.75 \times 0.25 + 0.25 \times 0.50 + 0.05 \times 0.25 = \\ &= 0.1875 + 0.125 + 0.0125 = 0.3250 = 32.50\% \\ P(A \cap E_1) &= P(A/E_1)P(E_1) + 0.75 \times 0.25 = 18.75\% \end{aligned}$$

- (b.3) $P(A \cap E_2) = P(A/E_2)P(E_2) + 0.25 \times 0.50 = 12.50\%$
 $P(A \cap E_3) = P(A/E_3)P(E_3) + 0.05 \times 0.25 = 1.250\%$

- (c) The correct answer is (c.2).

Fuzzy Modelling

- (a) If \bar{V} is the velocity interval, then the time interval $\bar{t} = L/\bar{V}$

$$\begin{aligned} \bar{V} &= (\bar{K}\bar{i})/(\bar{\eta}\bar{R}), \bar{K}\bar{i} = [10^2-10^3][10^{-4}-10^{-3}] = [10^{-2}-1], \\ \bar{\eta}\bar{R} &= [0.1-0.3][5-80] = [0.5-24] \\ \bar{V} &= \left[\frac{10^{-2}}{24} - \frac{1}{0.5} \right], \bar{t} = L/\bar{V} = 100 / \left[\frac{10^{-2}}{24} - \frac{1}{0.5} \right] = [50-24 \times 10^4] \end{aligned}$$

the time the pollution reaches the well ranges between 50 and 240 000 (!) years.

If K, i, η , and R are fuzzy numbers rather than intervals, we could repeat the above calculations for four or five different confidence h-level intervals.

- (b) $\bar{V} = (\bar{K}\bar{i})\{\bar{\eta}(1 + \bar{\rho}(K_d/\bar{\eta}))\}$,

$$\begin{aligned} \bar{\rho}(K_d/\bar{\eta}) &= K_d(\bar{\rho}/\bar{\eta}) = 10^{-3} \frac{[1500-1700]}{[0.1-0.3]} = 10^{-3} \left[\frac{1500}{0.3} - \frac{1700}{0.1} \right] \\ &= \left[\frac{1.5}{0.3} - \frac{1.7}{0.1} \right] = [5-17] \\ \bar{\eta}\{1 + \bar{\rho}(K_d/\bar{\eta})\} &= [0.1-0.3][6-18] = [0.6-5.4] \quad (C.1) \\ \bar{V} &= [10^{-2}-1]/[0.6-5.4] = \left[\frac{10^{-2}}{5.4} - \frac{1}{0.6} \right] = [1.85 \times 10^{-3}-1.67] \\ \bar{t} &= L/\bar{V} = 100/[1.85 \times 10^{-3}-1.67] = [59.88-5.4 \times 10^4] \end{aligned}$$

Pollution is expected between 60 and 54 000 (!) years.

(c)

$$\bar{\eta} + \bar{p}K_d = [0.1-0.3] + [1.5-1.7] = [1.6-2.0] \quad (C.2)$$

$$\bar{V} = [10^{-2}-1]/[1.6-2.0] = \left[\frac{10^{-2}}{2.0} - \frac{1}{1.6} \right] = [0.50 \times 10^{-2} - 0.625]$$

$$\bar{t} = L/\bar{V} = 100/[0.5 \times 10^{-2} - 0.625] = [160-20000]$$

Pollution is expected between 160 and 20 000 years.

When comparing results (C.1) and (C.2) it can be seen that when multiple occurrence of an interval variable occurs (as in Equation C.1, the variable $\bar{\eta}$), the result is a wider interval than in the case of just one occurrence C.2. This is the well known *subdistributivity rule* of interval arithmetics, that is, if A, B, C are intervals, then $A(B + C) \subseteq AB + AC$.

For the numerical application the result (C.2) produces a shorter interval, which is a better approximation.

Time Dependence and System Risk

(a) The ‘failure function’ is the exponential function $F(t) = 1 - \exp(-\lambda t)$. The ‘reliability function’ is $R(t) = \exp(-\lambda t)$.

(a.1) Two pumps are connected ‘in series’ and have a reliability function $R_{12}(t) = R_1(t) \times R_2(t) = \exp\{-(\lambda_1 + \lambda_2)t\}$, for $\lambda_1 = 4.10^{-4} \text{ h}^{-1}$, $\lambda_2 = 6.10^{-4} \text{ h}^{-1}$:
 $R_{12}(t) = \exp(-10^{-3} t)$.

(a.2) For $t = 2000 \text{ h}$, $R_{12} = \exp(-2) = 1/e^2 = 0.135 = 13.5\%$.

(a.3) Mean time to failure $= 1/(\lambda_1 + \lambda_2) = 1000 \text{ h}$.

(b) If the risk of failure of the i pipe is p_i

(b.1) n pipes connected in series: Reliability $= r_1 \cdot r_2 \cdot r_3 \dots r_n = \prod_{i=1}^n (1-p_i)$,

$$\text{Risk} = 1 - \prod_{i=1}^n (1-p_i)$$

(b.2) n pipes connected in parallel: Risk $= p_1 \cdot p_2 \cdot p_3 \dots p_n = \prod_{i=1}^n p_i$,

$$\text{Reliability} = 1 - \prod_{i=1}^n p_i$$

(b.3) for $n = 10$ pipes in series and $p_i = p = 0.01 = \text{Cte}$:

$$\begin{aligned} \text{Reliability} &= \prod_{i=1}^{10} (1-p_i) = (1-p)^{10} = (1-0.01)^{10} = (0.99)^{10} = 0.9044 \\ &= 90.44\%, \text{ Risk} = 9.56\% \end{aligned}$$

for $n = 10$ pipes in parallel and $p_i = p = 0.01 = \text{Cte}$:

$$\text{Risk} = p^{10} = 0.01^{10} = 10^{-20}, \text{ Reliability} = 1 - 10^{-20} \cong 99.99\%$$

- (c) Reliability $(1-2) = (1-p)^2$, Risk $(1-2) = 1 - (1-p)^2$, Risk $(3-4) = (1-p)^2$,
 Risk $(1-2-3-4) = \{(1-p)^2\}^2 = \{1-0.99^2\}^2 = 0.04\%$, Reliability = 99.96%

C.4

Answers to Questions and Problems – Chapter 4

Risk in Coastal Water Pollution

- (a) 1. The transport of pollutants by coastal currents, 2. the turbulent dispersion, and 3. the biochemical interactions.
- (b) The pollutant flux, that is, the flow rate of pollutant mass per unit area, is proportional to the gradient in space ∇C of the pollutant concentration C .
- (c) These are similar processes occurring at different scales: (i) at the molecular scale for diffusion, (ii) at macroscopic scales for turbulent diffusion and dispersion.
- (d) The biological growth or decay rate in wastewaters dC/dt may be described by mathematical expressions showing the linear or non-linear character of the phenomenon.
- (e) The tides, the wind, the density variations and the Coriolis forces.
- (f) By integrating the flow equations vertically (2-D models) or by using a vertical coordinate transformation (3-D models).

Risk in River Water Quality

- (a) The pollutant mass balance equation takes the form:

$$QC_1 + q_0 C_0 = (Q + q) C_2. \text{ For } C_1 = 0, C_2 = C_0 \{q_0 / (Q + q_0)\}$$
- (b) The oxygen sag curve describes the oxygen deficit variation downstream from the site of wastewater disposal. The form of this curve may be explained as follows: near the wastewater disposal site the BOD concentration is high and as a consequence the oxygen deficit will increase downstream. This deficit will gradually decrease because of re-aeration.
- (c) A mixed Lagrangian–Eulerian algorithm may be used for effective numerical simulation of a river's water quality

Risk in Groundwater Pollution

- (a) Using as a criterion the boundary conditions, aquifers may be classified as
 - (a.1) *phreatic*, when groundwater is in direct contact with the atmosphere, or
 - (a.2) *confined*, when the aquifer is overlain by an almost impermeable geological formation (hydro-geological classification). Confined aquifers may be distinguished as *fully-confined*, when the upper geological formation is impermeable, *semi-confined*, when the cover layer has non-zero permeability and as *artesian*, when aquifer pressure exceeds the atmospheric pressure.

- (b) Using as a criterion the aquifer's geological characteristics (geological classification), aquifers may be classified as (b.1) *porous aquifers* (sedimentary and alluvial), when groundwater circulates in successive layers, which consist mostly of gravels, sands, clays and silts, (b.2.1) *limestone karstic aquifers*, (b.2.2) *limestone non-karstic aquifers*, and (b.3) *crystalline rock fractured aquifers*.
- (c) The variogram of an aquifer's property is defined as the quadratic mean of the difference of property values between two points at distance ξ . This is related to the auto-covariance function of the aquifer's property.
- (d) The groundwater flow is described in Figure 4.41 and the flow rate is given in Equation 4.80 as follows: $Q = \frac{Km}{2L}(h_1^2 - h_2^2)$. For $K = 10^{-3}$ cm/s, $m = 50$ m, $L = 30$ m, $h_1 = 20$ m and $h_2 = 1$ m we obtain $Q = 10^{-5} \times 50(20^2 - 1)/(2 \times 30) = (399 \times 50/60) 10^{-5} = 3.32 \times 10^{-3} \text{ m}^3/\text{s} = 3.32 \text{ l/s}$

C.5

Answers to Questions and Problems – Chapter 5

Performance Indices and Figures of Merit

- (a) *Performance indices* (PIs) are indicators of the system behaviour under external stresses. *Figures of merit* (FMs) are defined as functions of the performance indices (super-indicators).
- (b) The engineering risk (RI) may be generally expressed as a function of different PIs. If RE is the reliability and D the expected damage of the system, then generally $RI = g(\text{RE}, D, \dots)$, where $g(\cdot)$ is a suitable function. Different definitions of risk follow, such as $RI = 1 - \text{RE}$ or $RI = D$.
- (c) *Vulnerability* is the sensitivity of the system to sustain a given hazard. It may be quantified by an index measuring the degree of damage an incident may cause to the system. *Resilience* is the capacity of the system to recover under a given hazard. It may be measured by the time the system needs in order to return to initial safe operating conditions.

These two FMs interact in opposite directions, that is, a highly vulnerable system may be expected to have low resilience and vice versa. *Sustainability* is a much more general performance indicator showing the long-term safe behaviour of the system. It may be expressed as a combination of high resilience and low vulnerability.

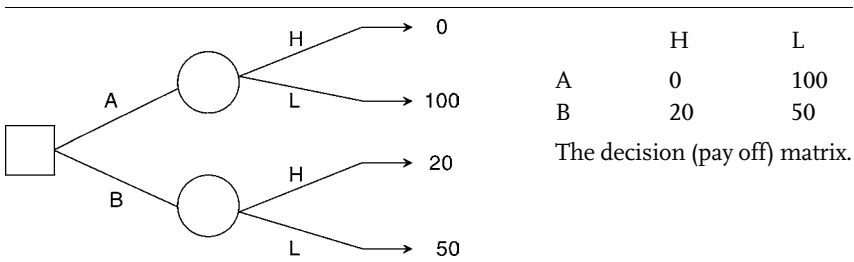
Objective Function and Optimisation

- (a) After defining various criteria (e.g. economic, technical, environmental and social) the objective function may be described as a mathematical function minimising or maximising one criterion or non-dimensional combinations of some criteria.

- (b) A system can be mathematically optimised only when one objective or criterion is taken into consideration. When multiple conflicting criteria are considered, a negotiated acceptable or compromise or composite solution may be found.

Basic Decision Theory

- (a) Two alternatives: A (small scale) and B (full scale).

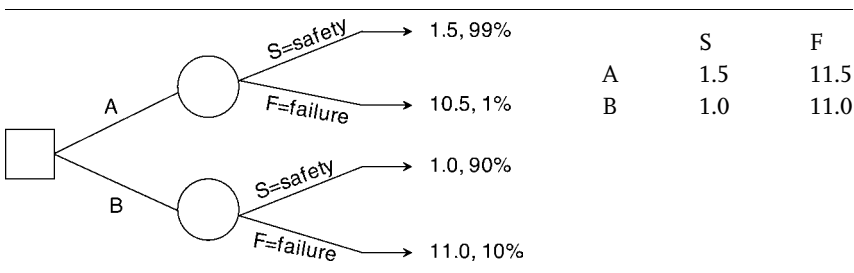


- (a.1) The decision tree.
(a.2) Decision under uncertainty.

MiniMax rule: B is decided				MiniMin rule: A is decided			
	H	L	MiniMax (row max)		H	L	MiniMin (row min)
A	0	100	100	A	0	100	0
B	20	50	50	B	20	50	20

- (a.3) Decision under risk: $p(H) = 0.30$, $p(L) = 1 - p(H) = 0.70$
 Expected utilities: $L(A) = 0 \times 0.30 + 100 \times 0.70 = 70$,
 $L(B) = 20 \times 0.30 + 50 \times 0.70 = 41 < 70$
 Alternative B is chosen.

- (b) Two alternatives are suggested: A (traditional) and B (new material).



- (b.1) The decision tree and the decision matrix are shown above.
- (b.2) Decision under risk: rule of minimum expected cost.

$$L(A) = 1.5 \times 0.99 + 11.5 \times 0.01 = 1.59$$

$$L(B) = 1.0 \times 0.90 + 11 \times .10 = 2.0 > L(A)$$

Alternative A is chosen.

Utility Theory

- (a) Benefit is the monetary value of gains. Utility is the personal appreciation of benefits under risk. Utilities coincide with benefits when there is no risk.
- (b) Expected benefits: $L(A) = 100 \times 1.0 = 100$, $L(B) = 100 \times 0.89 + 500 \times 0.10 + 0 \times 0.01 = 139 > 100$, which means that B is better. Most people choose A, because they are risk adverse (better to have less with certainty than more under risk). This is the so-called 'Allais paradox'.
- (c) The transitivity property of preferences is not always applied in people's choice.

Multiobjective Decision Analysis

- (a) The reason is that when dealing with two or more conflicting objectives, an increase in one objective usually results in a deterioration of some other(s). In such circumstances, trade-offs between the objectives are made in order to reach solutions that are not simultaneously optimum but still acceptable to the decision-maker with respect to each objective.
- (b) A 'feasible' or acceptable solution is a solution for which all values of the objectives satisfy the constraints.

'Dominant' or 'non-dominated' solutions are those located along a surface where there can be no increase in one objective without a decrease in the value of other objectives. The surface is called the 'feasibility frontier'.

- (c)
 - (c.1) Value or utility-type.
 - (c.2) Distance-based techniques.
 - (c.3) Outranking techniques.
 - (c.4) Direction-based interactive or dynamic techniques.
 - (c.5) Mixed techniques, which utilise aspects of two or more of the above four types.

In 'Compromise' or 'Composite Programming' methodology ranking between different strategies or options is carried out by using the range of composite distances from the ideal solution. Composite distances are trade-offs between objectives.

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